STEADY-STATE THERMAL ANALYSIS OF A PIPE INTERSECTION WITH A CYLINDRICAL TANK

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Abstract. *In this report we use the finite element method (FEM) to execute a steady-state thermal analysis, whose principal results are the determination of temperature and heat flux vector inside of a metal structure (like the junction of a pipe with a cylindrical tank). This problem was solved using the first law of thermodynamics, which was applied in classical form to each node of each finite element resulted after an adequate discreteness inside of work boundaries. All of these aspects will be discussed in the following.*

1. THE PROBLEM DESCRIPTION

As is shown in fig. 1, a cylindrical tank is penetrated in the radial direction by a pipe at the point where its axis remotes from the ends of the tank. The inside of the tank is exposed to a fluid of 503 K (230°C). The pipe experiences a steady flow of 303 K (30°C) fluid, and the two flow regimes are isolated from each other by a thin tube. The film coefficient in the tank is a steady 1420 W/(m²·K). The film coefficient in the pipe varies with the metal temperature and is given in the material property table 1.

Table 1

T [K]	294	366	422	476	533
ρ [kg/m³]	7800	7800	7800	7800	7800
K [W/(m·K)]	14.453	15.405	16.184	16.963	17.708
c [J/(kg·K)]	473.13	489.87	498.25	510.81	523.37
h [W/(m²·K)]	2418.8	2299.5	1998.6	1561.4	1254.8

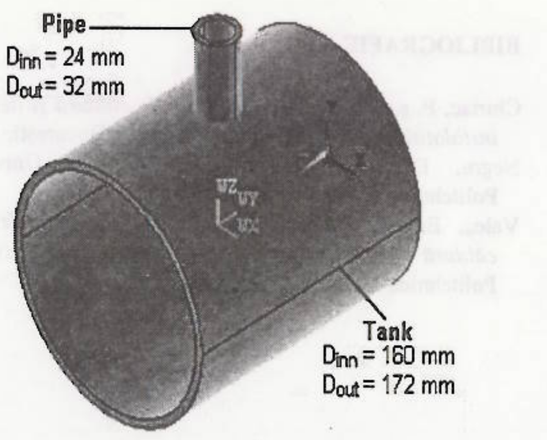

Fig. 1

The purpose of this problem is to determine the temperature distribution and the flow heat flux round the pipe-tank junction.

2. THE MATHEMATICAL ASSUMPTIONS

The steady-state thermal analysis of this case can be solved using the first law of thermodynamics, which states that thermal energy is conserved. Applying this principle to a differential control volume, we have

$$\rho \cdot c\left(\frac{\partial T}{\partial t}+\{V\}^{tr}\{L\}T\right)+=\{L\}^{tr}\{q\}=\ddot{q} \qquad (1)$$

where: ρ = density of materials [kg/m³]; c = specific heat [J/(kg·K]; T = absolute temperature [K]; t = time [s]; $\{L\}=\{\partial/\partial x\ \ \partial/\partial y\ \ \partial/\partial y\}^{tr}$ = vector (column) operator; $\{V\}=\{V_x\ \ V_y\ \ V_z\}^{tr}$ = velocity (column) vector for mass transport of heat; $\{q\}$ = heat flux vector; $\ddot{q}$ = heat generation rate per unit volume.

It should be realized that the terms $\{L\}^{tr}$ and $\{L\}^{tr}\cdot\{q\}$ may be interpre-ted as ∇T and $\nabla\cdot\{q\}$, respectively, where ∇ represents the *gradient operator* and $\nabla\cdot$ represents the *divergence operator*.

Next, Fourier's law (1) is used to relate the flux vector to the thermal gradients. So, we have

$$\{q\}=-[D]\cdot\{L\}T \qquad (2)$$

where

$$[D]=\begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{bmatrix} = \text{conductivity matrix;}$$

K_{xx}, K_{yy}, K_{zz} are the conductivities in the element x, y and z directions respectively.

Combining equations (1) and (2), we obtain

$$\rho\cdot c\left(\frac{\partial T}{\partial t}+\{V\}^{tr}\cdot\{L\}T\right)==\{L\}^{tr}\left([D]\cdot\{L\}T\right)+\ddot{q} \qquad (3)$$

This expression can be written in the next familiar form

$$\rho c\left(\frac{\partial T}{\partial t}+V_x\frac{\partial T}{\partial x}+V_y\frac{\partial T}{\partial y}+V_z\frac{\partial T}{\partial z}\right)=\ddot{q}+\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial T}{\partial x}\right)+$$
$$+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial T}{\partial z}\right) \tag{4}$$

which is available in the global Cartesian system.

In our problem two types of boundary conditions are conside-red. In addition, it is presumed that these cover the entire element.

a) Specified temperatures acting over surface:

$$T=T^* \tag{5}$$

where T^* is the specified temperature.

b) Specified convection surfaces acting over surface (Newton's law of cooling):

$$\{q\}^{tr}\{n\}=-h_f\left(T_B-T_S\right) \tag{6}$$

where $\{n\}$= unit outward normal vector; h_f = film coefficient, evaluated at temperature $(T_B+T_S)/2$ unless otherwise specified for the element; T_B = bulk tempe-rature of the adjacent fluid; T_S = temperature at the surface of the model.

We note that positive specific heat flow is into the boundary (i.e., in the direction opposite of $\{n\}$), which accounts for the negative signs in equation (6).

Combining equation (2) with equation (6), we obtain

$$\{n\}^{tr}[D]\{L\}T=h_f\left(T_B-T\right) \tag{7}$$

Finally, premultiplying equation (3) by a virtual change in temperature, integrating over the volume of each element and combining with equation (7) with some manipulation yields, results:

$$\iiint_{vol}\left[\rho c\,\delta T\left(\frac{\partial T}{\partial t}+\{V\}^{tr}\{L\}T\right)+\{L\}^{tr}\delta T\left([D]\{L\}T\right)\right]d(vol)=$$
$$=\iint_{surf}\delta T\,h_f\left(T_B-T\right)d(surf)+\iiint_{vol}\delta T\,\ddot{q}\,d(vol) \tag{8}$$

where vol = volume of the element; $surf$ = surface of the element; $\delta T=\delta T(x,y,z,t)$ an allowable virtual temperature.

3. THE ANALYSIS APPROACH

The problem treated in this work was solved using the **Multiphysics** product of **ANSYS** software - **version 5.5/1998,** following the next four main steps proposed in any typical ANSYS analysis: build the model, apply loads, obtain the solution and review the corresponding results.

The geometrical model uses in our case quarter-symmetry to represent the entire pipe-tank junction (fig. 2). The tank is assumed to be long enough for its remote end to be held at a constant temperature of 503 K. A similar assumption is made at the $y=0$ plane of the tank. Building the model involves defining two cylinder primitives and a Boolean overlap operation.

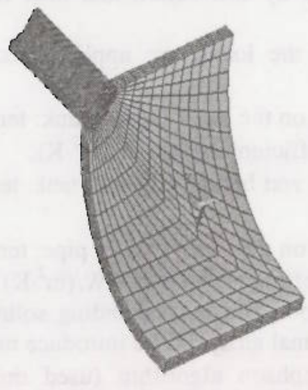

Fig. 2

The model of junction exposed below was finally meshed (in style 'mapped' and after a few essays) into 1,056 finite elements, classified by ANSYS as 'Thermal Solid Brick 20 node 90'. The mesh is represented in fig. 2.

As is shown in fig. 3, each element of this type has 20 nodes with a single degree of freedom – temperature at each node. This element, recommended by ANSYS for all three-dimensional steady-state or transient thermal analysis, has compatible temperature shapes and as well suited to model curved boundaries.

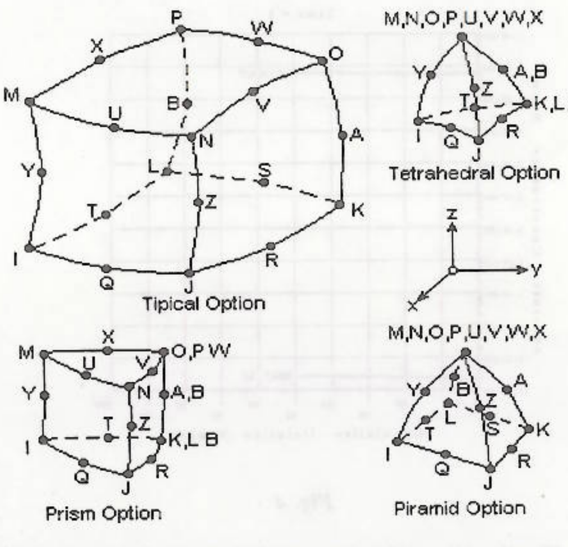

Fig. 3

The main input data for each element are:

-nodes:I, J, K, L, M N, O, P, Q, R, S, T, U, V, W, X, Y, Z, A, B;

-degrees of freedom (DOF): temperature;

-material properties: density, thermal conductivity, specific heat, convection film coefficient, enthalpy etc;

-surface loads: convection faces and heat flux faces;

On the other part, the corresponding output main data are:

-geometrical characteristics regarding nodes and faces; *-thermal corresponding properties*: film coefficient, average tempera-ture fluid bulk temperature, heat flow rate across face by convection, heat flow rate per unit area across face by convection and heat flux at each node.

As regards the loads, we applied next boundary conditions:

-convection on the inner face of tank: temperature = = 503K film coefficient = 1420 W/(m^2·K);

-on the right and bottom edges of tank: temperature = = 503 K;

-convection on the inner face of pipe: temperature = = 303 K film coefficient =–1635.4 W/(m^2·K).

Finally, to obtain a corresponding solution in this steady-state thermal analysis, we introduce next options:

-Newton–Raphson algorithm (used only in non-linear analysis), which specifies how often the tangent matrix is updated during finding solution;

-number of substeps per load step = 50 (having in view that any nonlinear analysis requires multiple substeps within each load step).

4. DISPLAY OF RESULTS

In conditions exposed above, the evolution can be accounted convergent, as is shown in fig. 4. Depending of computer system configuration and of complexity of model, the duration of computing process can be evaluated at 10 to 120 min.

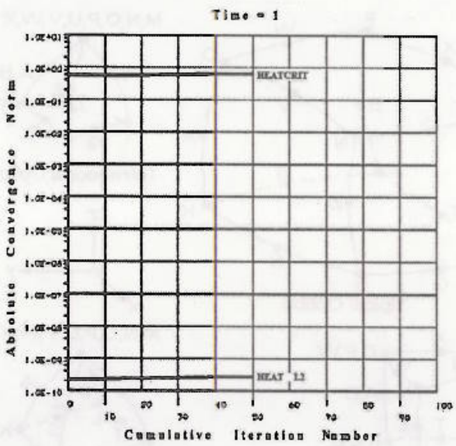

Fig. 4

Finally, all results data can be resumed in numerical or graphical form. E.g., next are shown the complete maps of temperature and flow thermal flux distributions round the analyzed pipe-tank junction (fig. 5, respec-tively fig. 6).

5. CONCLUSIONS

The problem exposed above can be met, as was exposed, in many cases, like exhaust-steam boiler drums, bottom headers of exhaust-steam boilers, inlet flows of steam turbines etc, in which the complete knowledge of temperature distributions on body boundaries is a very important objective.

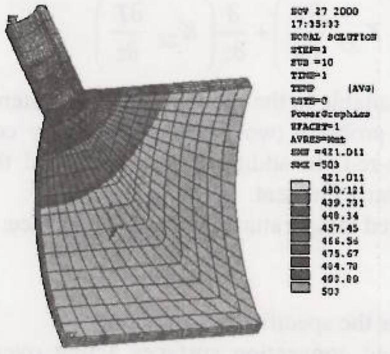

Fig. 5

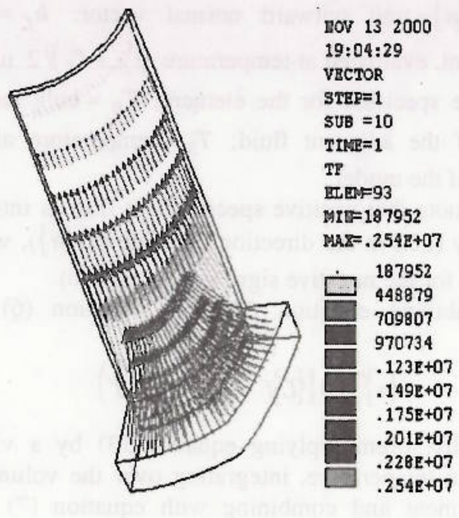

Fig. 6

This study presents all necessary job steps to achieve any steady-state thermal analysis and shows the CAD capabilities of ANSYS Inc. software to solve any similar problems.

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