CONSTRUCTAL DESIGN: OPTIMAL FLOW-SYSTEM GEOMETRY DEDUCED FROM THERMODYNAMIC OPTIMIZATION AND CONSTRAINTS

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Rezumat. *Această analiză este destinată să atragă atenția asupra unui ansamblu de lucrări referitoare la optimizarea termodinamică globală care urmărește stabilirea arhitecturii curgerii (teoria constructală). Analiza exergetica stabilește limitele performanței teoretice. Optimizarea termodinamică (sau minimizarea generării de entropie) apropie proiectarea cât se poate de mult de limita performanței teoretice. Proiectarea este sortită să rămână imperfectă datorită restricțiilor existente în sistem (de exemplu, prezența rezistențelor la curgere). Determinarea configurației optime presupune găsirea distribuției spațiale și temporale, a formei geometice, periodicității, topologiei și geografiei sistemului. Arhitectura sistemului iese de sub incidența optimizării globale cu restricții [1]. Sunt prezentate două clase de aplicații. În curgeri care leagă un volum de o suprafață, structura care rezultă este de forma unui copac cu ramuri având conductivitate termică mare, respectiv interstiții având conductivitate termică mică. În lucrare mai este ilustrată optimizarea proiectării energetice a avioanelor: viteza de croazieră corespunzătoare distrugerii minime de exergie și structura schimbătoarelor de căldură din sistemul de control al mediului înconjurător.*

CONSTRUCTAL DESIGN AND THEORY

The most common method of modeling, analysis and optimization in thermal engineering begins with assuming the system configuration — the geometry, architecture, components and manner in which the components are connected. Next, the conceived model is a simplified facsimile of the assumed system, and the analysis is the mathematical description of the model and its performance. The optimization is the simulation of system operation under various conditions, and the search for the operating conditions most favorable for maximum performance.

The search for optimal design is considerably more challenging than optimizing the operation of an assumed configuration. In principle, one may contemplate a large number of configurations, all the way to large numbers of duct shapes and aspect ratios in every detail of a fluid flow network. In practice, one may assume two or three configurations, optimize their performance, and compare in the end the optimized alternatives. Selecting the best among two or three is to optimize the configuration. Configuration in space and time (geometry rhythm) is the chief unknown and major challenge in design [1]. An engineering flow system owes its imperfection (irreversibility) to several mechanisms, notably the flow of heat, fluid and electric current against finite resistances. The entropy generated by each current is proportional to the resistance overcome by the current. In simple terms, the entire effort to optimize thermodynamically the greater system rests on the ability to minimize all the internal flow resistances, *together*. Resistances cannot be minimized individually and indiscriminately because of constraints: space is limited, streams must connect components, and components must "fit" inside the greater system. The route to improvements in global performance is by *balancing* the reductions in the competing resistances. This amounts to spreading the imperfection through the system in an optimal way, so that the total irreversibility is reduced. Optimal spreading is achieved by properly sizing, shaping and positioning the components. Optimal spreading means geometry. In the end, the geometric structure of the system emerges as the chief result of global optimization subject to constraints. Geometry is not random — it results from principle. Geometry makes systems achieve their best. Geometry matters.

The generation of system structure is, of course, complicated to the observer's eye when the systems and its functions are complex. It is simpler when the system houses only a few streams, as in the examples treated in the next section. These examples belong to the wide class of engineering and natural flows that connect an infinity of points (volume, area) to one or more discrete points (sources, sinks). All the volume-point flows are shaped as trees. Natural examples are the river basins and deltas, lungs, vascularized tissues, botanical trees, and leaves [1-5]. Manmade flows shaped as trees are found in the cooling systems of electronics packages and windings of electric machines, regenerative heat exchangers, traffic patterns, and networks for distributing city water and for collecting rainwater and sewage [1].

This work goes well beyond the calculation of the flow resistance associated with the flow path: the new and primary focus is on *optimizing geometrically* the flow path, so that the global volume-point resistance is minimum. What flows along the tree links is not nearly as important as how the geometric form "tree" is *generated* by the optimization principle. The generation of trees for heat flow, fluid flow, combined heat and fluid flow, electricity (e.g., lightning, circuitry), people and goods (e.g., streets, highways), and communications is reviewed in a new book [1]. The thought that the geometric optimization principle accounts not only for engineered flows but also for the billions of tree-shaped flows of nature was named *constructal theory*.

TREE-SHAPED POINT-VOLUME FLOWS

In the constructal optimization of the volume-to-point path for heat flow it was shown that a volume subsystem of any size can have its external shape and internal details optimized such that its own volume-to-point resistance is minimal [1]. This principle is repeated in the optimization of increasingly larger volumes, where each new volume is an assembly of previously optimized volumes. The construction spreads, as the assemblies cover larger spaces.

As an example, consider the flow of heat from a volume to one point. The heat-flow geometry is two-dimensional. The low-conductivity material, k_0, generates heat at the uniform rate q''' per unit volume. A small (fixed) amount of high-conductivity material, k_p, is to be distributed through the k_0 material. The construction begins with the smallest volume scale (the elemental system), which is represented by the rectangular area $A_0 = H_0 L_0$. The start of this sequence of volume sizes is shown in the upper-right corner of Fig. 1. The size A_0 is known and fixed by manufacturing and electrical design constraints. The system is elemental because it has only one insert of high-conductivity material. This blade has the thickness D_0, and is positioned on the long axis of the $H_0 \times L_0$ rectangle. The heat current $q'''A_0$ is guided out of A_0 through one end of the D_0 channel, which is the heat sink. The hot spots occur in the farthest corners relative to the sink.

The global volume-to-point resistance is the ratio $\Delta T_0 / (q''' A_0)$, where ΔT_0 is the temperature difference between the hot spot and the heat sink. This measure is "global" because the heat current $q'''A_0$ is integrated over the system A_0, and the maximum temperature difference ΔT_0 is the excess temperature of the hot spot (or hot spots) of the elemental system: the location of the hot spots is not an issue, as long as the hot spots reside inside the system.

The composite material that fills A_0 is characterized by two numbers, the conductivity ratio $\tilde{k} = k_p / k_0$, and the volume fraction of high-conductivity material, $\phi_0 = D_0 / H_0$. When $\tilde{k} >> 1$ and $\phi_0 << 1$ the global volume-to-point resistance is given by the two-term expression [1]

$$\frac{\Delta T_0 k_0}{q''' A_0} = \frac{1}{8} \cdot \frac{H_0}{L_0} + \frac{1}{2\tilde{k}\phi_0} \cdot \frac{L_0}{H_0} \qquad (1)$$

This resistance is minimal when the ratio H_0 / L_0 is equal to $2\left(\tilde{k}\phi_0\right)^{-1/2}$. The construction continues with the first assembly ($A_1 = H_1 L_1$ in Fig. 1), which contains n_1 elemental systems, $A_1 = n_1 A_0$. The heat currents produced by the elemental systems are collected by a new high-conductivity insert of thickness D_1, the left end of which is the heat sink. The hot spots are in the two right-hand corners. The global volume-to-point resistance is $\Delta T_1 / (q''' A_1)$, where ΔT_1 is the excess temperature at the hot spot. The resistance can be minimized with respect to the shape parameter H_1 / L_1 and the internal ratio D_1 / D_0.

The same geometric optimization principle applies at larger scales. The next scale is the second construct A_2, which contains a number of first constructs (n_2, Fig. 2). The optimal external aspect ratio of the second construct is $H_2 L_2 = 2$. In addition, it is possible to optimize the internal ratios of high-conductivity blade thicknesses, D_1 / D_0 and D_2 / D_1, where D_2 is the thickness of the central (thickest, newest) blade. In the optimized structure, the high-conductivity blades form a tree. The k_p blades are in optimal balance with the k_0 interstices: at the elemental scale, the k_p and k_0 spaces are equally important from the point of view of minimizing flow resistance.

Constructal trees look more natural if their freedom to provide easier access to their internal currents is expanded. In the elemental system of Fig. 1 (upper-right corner) it was assumed that the k_p channel stretches all the way across the volume. When this assumption is not made, we find numerically that there is an optimal spacing between the tip of the k_p channel and the adiabatic boundary of the elemental volume [1]. The associated reduction in global resistance is small (a few percentage points).

Another example is when the angle formed between each tributary channel and its central stem is allowed to vary. Numerical optimization of the tree structure show that there exists an optimal angle [1]. The reduction in global flow resistance is again small.

In Fig. 2 we see the results of a fully numerical optimization of the second construct with perpendicular and constant-thickness k_p inserts (D_0, D_1, D_2), where all the other geometric parameters were allowed to vary—the aspect ratios of all the rectangles, large and small, the number of elemental volumes in each first construct (n_1), and the number of first constructs in each second construct (n_2). The three designs of Fig. 2 have been optimized with respect to all the parameters except n_2, and they have been drawn to scale. The figure shows the effect of fine-tuning the number n_2. This number has almost no effect on the external shape of the construct, and on the overall thermal resistance

$\tilde{T} = (T_{max} - T_{min})_{min} k_0 / (q''' A_2)$. From left to right in Fig. 2, $\tilde{T}$ has the values 0.0379, 0.0354, and 0.0374.

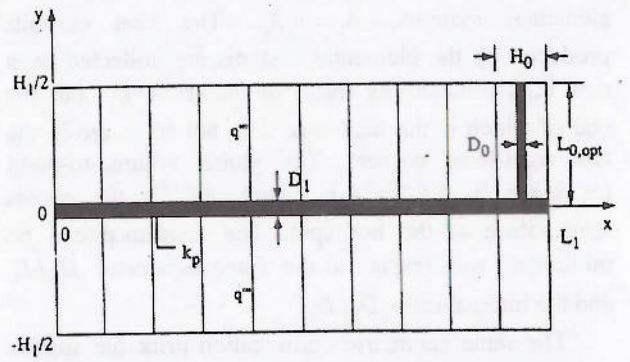

Fig. 1. The first construct: a large number of elemental volumes connected to a central high-conductivity path [1].

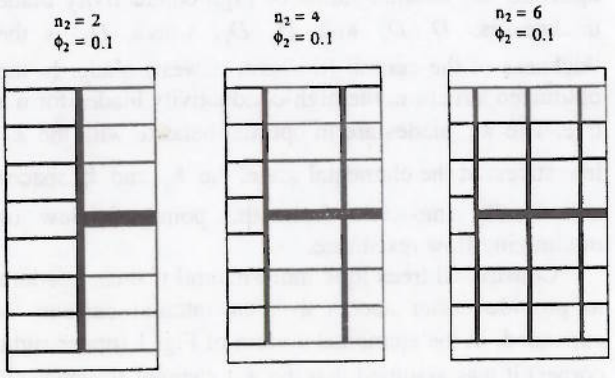

Fig. 2. The second construct optimized numerically, and the effect of changing the number of first constructs, n_2 [1].

All these minor improvements tell a very important story: the tree design is *robust* with respect to various modifications in its internal structure. This means that the global performance of the system is relatively insensitive to changes in some of the geometric details. Trees that are not identical have nearly identical performance, and nearly identical macroscopic features such as the external shape. The same holds for other complex flow systems, as the class discussed in the next section.

EXERGY DESTRUCTION ON AIRCRAFT

On an aircraft there are many systems and processes that contribute to the ultimate destruction of all the exergy furnished by the fuel. Fig. 3 presents a very brief summary of the distribution of losses. Proceeding in the direction of fuel flow, the first exergy loss (about 30 percent [6]) is due to the combustion process. Next is the loss due to the irreversible operation of the engine. The remaining exergy fraction is the power produced by the engine, which drives the subsystems and meets all the functions of the aircraft. Chief among functions is the power required to maintain the flight $(\dot{W})$, i.e. to overcome drag and hold the aircraft at its altitude (to lift it repeatedly). The power W is destroyed completely in the turbulent flow around the aircraft and in the wake.

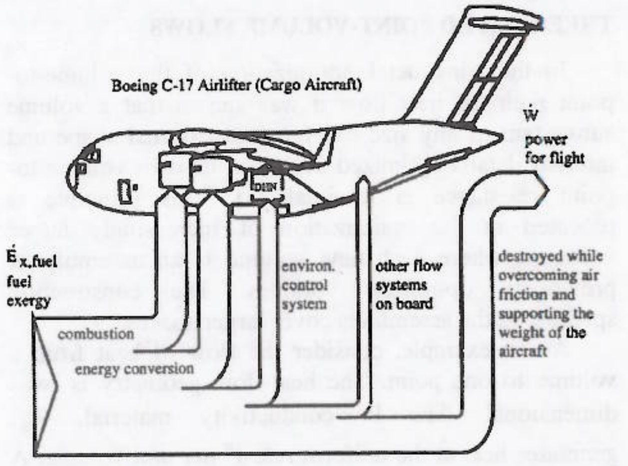

Fig. 3. The destruction of exergy on an aircraft [7].

The rest of the power generated by the engine is destroyed in the auxiliary systems, which are numerous. The dominant user of exergy is the environmental control system (ECS) which on a military transport plane such as the C-17 accounts for 64.6 percent of the engine power at cruise condition. The ECS itself is an assembly of subsystems, as shown in the example of Fig. 4. In the "bootstrap air cycle" a fraction of the compressed airstream is bled fro he engine compressor and compressed further in a compressor driven by its own turbine. In between, the compressed "engine air" is cooled in a crossflow heat exchanger by a strea of "ram air". The engine-air stream expanded through the turbine becomes the room temperature air needed to ventilate and cool the cabin and avionics. On the ram-air side, the heat exchanger core (H) is fitted with a diffuser (D) and a nozzle (N), to decelerate and reaccelerate the ram air that initially enters the ECS at cruising speed.

The exergy destruction rates of Fig. 3 can be minimized by allowing the configuration to vary. Each optimum represents the tradeoff between two or more irreversibility mechanisms, and the result is structure in time and in space. Consider, for example, the tradeoff between overcoming aerodynamic drag and supporting the weight of the aircraft. The flying body of mass M has the linear dimension D and average density ρ_b. This leads to the global requirement that the net vertical body force $Mg \sim \rho_b D^3 g$ must be supported by other forces. The generation of such forces is achieved through flight. The scale analysis of the conservation of mass, momentum and energy through the control volume occupied by the flying body leads to the conclusion that the power required for flight ($\dot{W}$ in Fig. 3) is [1]:

$$\dot{W} \sim \rho_b^2 g^2 D^4 / (\rho_a V) + \rho_a D^2 V^3 \qquad (2)$$

Here V, ρ_a, and g are the cruising speed, air density, and gravitational acceleration. The first term accounts for the power required to maintain the body in the air, and the second for the power needed to overcome drag. The competition between the two terms leads to the minimum $\dot{W}$, which occurs at the optimal cruising speed

$$V_{opt} \sim [(\rho_b / \rho_a) g D]^{1/2} \quad (3)$$

The cruising speed for minimum exergy destruction is proportional to $D^{1/2}$, or to the body mass raised to the power 1/6. The engineering and biology literatures [1] have shown convincingly that virtually all the known flying bodies (airplanes, birds, insects) follow the thermodynamic optimum predicted by Eq. (3).

Another example is the selection of the architecture (sizes, aspect ratios, passages) of heat exchangers, such as the heat exchanger shown in Fig. 4. The irreversibility of the crossflow configuration is treated in detail in Ref. [8], and the counterflow configuration in Ref. [9]. The essential tradeoffs are more evident if the heat transfer surfaces are simpler, e.g., parallel-plate channels with spacings D_a and D_e for streams $\dot{m}_a$ and, respectively, $\dot{m}_e$ (Fig. 5). Geometric features such as D_a / D_e can be selected such that the global entropy generation rate N_S of the entire ECS is minimized. Fixed in this optimization are the total heat transfer area $\tilde{S}$, the ratio of capacity rates, C_e/C_e and the external aspect ratios W/L and H/L.

Similar geometric features are deduced if the calculated entropy generation rate refers strictly to the space occupied by the heat exchanger. In other words, the optimized architecture of the heat exchanger is robust with respect to how we delineate the flow system the performance of which is impacted by the imperfection of the heat exchanger.

In some applications the function of the heat exchanger is to extract exergy from the hot stream and to deliver it to the cold stream. In such cases the spent hot stream is not useful anymore, and is discharged into the ambient. The mixing triggered by the discharge serves as an additional source of irreversibility. In such cases we find that in addition to the optimized geometric features noted above, there is an optimal match (a ratio) between the capacity rates of the two streams [10]. This optimum is a characteristic of power systems driven by exergy drawn from a stream of hot gas, which is eventually discharged into the ambient.

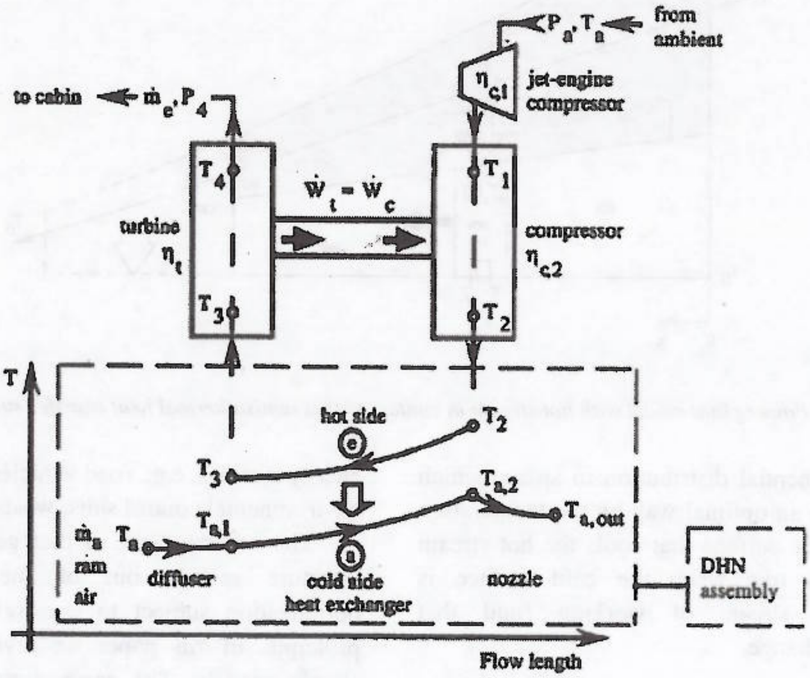

Fig. 4. Environmental control system (ECS) [7].

This observation is useful in a fundamental sense, because it invites us to adopt a different perspective on power generation. Recall that the limits to the conversion of heat into work are traditionally discussed with reference to two temperature reservoirs, the high-temperature reservoir that serves as source of heat, and the low-temperature ambient that serves as heat sink. The existence of "reservoirs" of high temperature is questionable. What exist, and drive power plants, are finite amounts of fuel or hot materials. The hot stream produced by combustion becomes cooler gradually, as its exergy is extracted by the power plant. The hot stream offers an entire range of temperatures, and the heating effect that it provides is proportional to its flow rate. The problem is to extract most exergy from a given stream of fuel and oxidant that interacts through a power plant of fixed size with the ambient. Variational calculus showed that the temperature of the hot stream

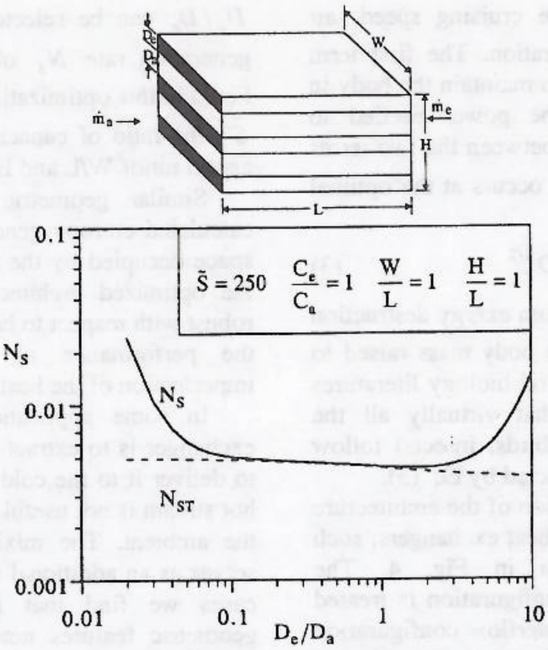

Fig. 5. The thermodynamic optimization of the ratio of channel spacings in a parallel-plates heat exchangers [9].

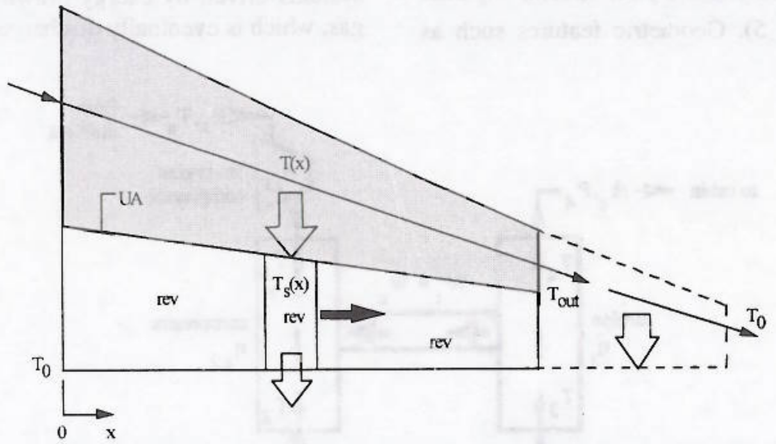

Fig. 6. Power plant model with hot stream in contact with a nonisothermal heat transfer surface [10].

must have an exponential distribution in space, which must be matched in an optimal way by the temperature distribution along the surface that cools the hot stream [10]. This is also true when the cold surface is represented by a stream of working fluid that experiences phase change.

CONCLUSION: STRUCTURE FROM PRINCIPLE

This paper reviewed a direction of thermodynamic optimization, where the system structure is the chief unknown. The global performance of the system is maximized subject to global constraints. The examples given in this paper showed the progress from single components and assemblies (Figs. 1 and 2), to the aircraft as a whole (Figs. 3 and 4). The constructal design approach can be extended to related classes of energy systems, e.g., road vehicles, ships, portable tools, environmental control suits, weapons systems, etc.

The broader issue is that geometric and temporal structure springs out of the process of global optimization subject to constraints. To illustrate the principle, in this paper we reviewed work based on simple models. The same optimization opportunities deserve to be pursued based on more realistic models, and in designs of greater complexity. The challenge that we must face is the number of degrees of freedom, which increases as the complexity of the system increases. Relevant to this is whether the optima of simpler systems are robust enough to reappear at least approximately in larger assemblies. In such cases the results obtained for simpler systems can be used as shortcuts in the optimization of larger and more complex constructs.

The relevance of these engineering conclusions to understanding the design and origin of natural flow structures also demands attention [1].

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CONFERENCE ROUMAINE DE THERMOTECHNIQUE
Édition la XI^{e}, Galați 17 – 19 mai 2001

Il est maintenant de longue tradition que la Société roumaine de Thermotechnique (S.R.T.) organise un colloque national annuel. La onzième édition a eu lieu du 17 au 19 mai 2001 à Galați ; que les organisateurs de la chaire de Thermotechnique et Machines Thermiques de l'Université « Dunărea de Jos » de Galați en soit profondément remercier !

Cette édition de la manifestation a connu un succès mérité, comme en témoigne les quatre volumes des comptes rendus édités. Le premier volume est dédié à la thermodynamique, au transfert de chaleur et de masse, à la gazodynamique et la thermoeconomie (48 articles ; 306 pages). Le deuxième volume est relatif aux bouilleurs, turbines à vapeur et à gaz, turbocompresseurs, centrales thermiques, automatisations et optimisation des centrales, protection de l'environnement (51 articles ; 372 pages). Le troisième volume rapporte des moteurs à combustions interne, des compresseurs, des combustibles et lubrification, et de l'automatisation et optimisation des systèmes énergétiques à moteurs à combustion interne (31 articles ; 212 pages). Enfin, le dernier volume (le quatrième) concerne les machines à froid et pompes à chaleur, la cryogénie, le conditionnement d'air, puis l'automatisation et l'optimisation des installations à machines à froid ou pompes à chaleur (45 articles ; 312 pages).

Depuis quelques années nous avons le prévilège de participer et en conséquence de voir : • Le succès croisant de cette manifestation en participation ; • Le développement de la qualité des travaux soumis et leur pertinence ; • l'ouverture internationale grandissante, comme en témoigne, plus particulièrement cette année, la présence d'un « ENFANT DU PAYS », Adrian Bejan.

Une autre tradition est en train de naître par la publication de quelques travaux suite à cette manifestation, et conjointement dans les revues « Termotehnica » et « Entropie ». Ainsi, auprès quatre ans de parution, la revue « Termotehnica », dans une version nouvelle, a eu une première parution à l'occasion de la 11^{e} conférence de S.R.T., suite au colloque de Sibiu en 2000. De même le no. 232 de la revue « Entropie » a été aussi entièrement dédié au colloque de Sibiu. Le présent numéro de « Termotehnica » matérialise la deuxième ; souhaitons, un troisième à venir, selon le proverbe « jamais deux sans trois ».

Que les participants qui n'apparaissent pas dans ces numéros spéciaux, n'y voient pas un désaveu, mais plutôt un encouragement pour les manifestations à venir. Que les membres de comité scientifique et des relecteurs soient profondément remercies pour le travail consciencieux effectue et les choix difficiles. Que les auteurs soient félicités pour leur promptitude et soin dans les travaux de qualité qu'il font.

Il semble que cette première année du millénaire a été prometteuse. Que les suivantes le soient encore plus, au sein d'une Europe émergente. Et pourquoi pas une Europe de la Thermodynamique et de la Thermotechnique !

LA MULȚI ANI 2002 !

Nancy, le 29.12.2001

Michel FEIDT
Prof. Université « Henri Poincaré »