# APPRECIATIONS ABOUT A VARIABLE DISPLACEMENT STIRLING ENGINE

Asist.ing. Vlad Mario HOMUTESCU\*, Ş.l.ing Condrat Adrian HOMUTESCU\*, ing. Adrian HOMUTESCU

\*UNIVERSITATEA TEHNICĂ "Gheorghe Asachi" Iași

REZUMAT. Lucrarea prezintă concepția constructivă a unui motor Stirling cu cilindree variabilă cu două pistoane coaxiale. Variația cilindreei se obține cu un mecanism motor derivat din mecanismul romboidal, care modifică (sub sarcină) lungimea echivalentă a două din biele.

Urmează analiza cinematică a mecanismului și calculul termic și se analizează în ce măsură se poate regla puterea motorului prin modificarea cilindreei.

#### 1. INTRODUCTION

Named as the man that patented them in 1816, the Stirling engines are external combustion reciprocating movement machines using heat regeneration. The Stirling engines run on a thermodynamic cycle made of two isothermal processes linked by two isochoric processes. After experiencing an important development around 1900, the Stirling engines have been overrun by the internal combustion engines and, in effect, forgotten.

In the last fifty years, due mostly to the impetuous development of science and technology, the interest for Stirling engines arose again as a new range of specific utilization fields came up for them [3], [4], [5], [6]. In this context different configurations of Stirling engines developed.

The Stirling engine presented hereby brings something new for this growing on domain. The engine features a motive drive allowing displacement modifications on load and thus continuous variation of the mechanical power.

In this paper are evaluated the performance of such an engine and the efficiency of the adjustment method proposed.

## 2. VARIABLE DISPLACEMENT STIRLING ENGINE

In the Stirling engine schematic diagram (fig. 1) we can see the cylinder 1, the power piston 2, the displacer 3 and three heat exchangers, 4 - the low temperature one, 5 - the high temperature one and the regenerator 6.

The low temperature heat exchanger is cooled with water (or air, the case of small Stirling engines) and the high temperature heat exchanger is placed in the engine burning chamber.

The expansion space 7 is found between the cylinder head and the displacer and the compression space 8 between the two pistons. The motive drive has two crankshafts 9 which spin in opposite directions. The crankshaft movements are synchronized by the gear wheels 10. The displacer 3 is equipped with a stem 11 which pierces through the power piston 2 in the middle. The stem 11 finishes at the lower yoke 12, at the ends of

which the lower rods 13 are socketed. The power piston 2 is equipped with a cylindrical stem 14 which finishes at the upper yoke 15, at which's ends the upper rods 16 are socketed. Between the upper rods 16 and the crankshafts 9 have been introduced the sides MN (fig. 3) of the triangular plates 17. The ends T of the triangular plates are socketed with the adjustment bars 18. The adjustment bars themselves are socketed in the leaning bars 19 which can oscillate in the fixed sockets V. The sockets U between the 17 and 18 bars are able to move along circle arcs thanks to the adjustment screw

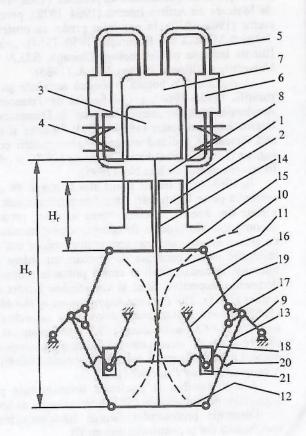


Fig. 1

20 which has two distinct regions of opposite threading. The screw 20 is spun from outside through the nuts 21 which stand for the U sockets. The spinning effect is that the ends T of the triangular plates 17 shift and thus the lengths of the "equivalent rods" MP are modified resulting in the displacement variation of the Stirling engine analyzed here.

#### 3. KINEMATIC CALCULATION

The kinematic calculation of the variable displacement Stirling engine must establish the piston movement laws and the volume variation laws for the two engine chambers with respect to the angle  $\alpha$  and the adjustment gear position (the  $\psi$  angle).

On fig. 2 were used: e - offset; r - crankshaft radius;  $l_{1e}$  and  $l_{1r}$  - rod lengths;  $l_2$ ,  $l_3$ ,  $l_4$ ,  $l_5$  - bar dimensions;  $d_1$ ,  $d_2$ ,  $d_3$  - distances between specific mechanism sockets;  $\varphi_{1e}$ ,  $\varphi_{1r}$ ,  $\varphi_2$ ,  $\varphi_3$ ,  $\varphi_4$ ,  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$  - position angles of bars and distances;  $\gamma$  - triangular plate angle. Also, the following auxiliary angles were introduced:  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ ,  $\theta_4$ . The dimensions e, r,  $l_{1r}$ ,  $l_{1c}$ ,  $l_2$ ,  $l_3$ ,  $l_4$ ,  $l_5$ ,  $d_1$ ,  $\delta_1$ ,  $\gamma$  are known and then we successively calculate:

$$\theta_1 = \pi + \delta_1 - \psi; \tag{1}$$

$$d_2 = \sqrt{d_1^2 + l_5^2 - 2 d_1 l_5 \cos(\theta_1)}; \qquad (2)$$

$$\theta_2 = \arccos\left(\frac{l_5^2 + d_2^2 - d_1^2}{2d_2l_5}\right);$$
 (3)

$$\delta_2 = \pi + \psi - \theta_2 \,; \tag{4}$$

$$\theta_3 = \delta_2 - \alpha - \pi \tag{5}$$

$$d_3 = \sqrt{r^2 + d_2^2 - 2rd_2\cos(\theta_3)}$$
; (6)

$$\delta_3 = \arccos\left(\frac{d_3^2 + d_2^2 - r^2}{2d_2d_3}\right) + \delta_2;$$
 (7)

$$\delta_4 = \arccos\left(\frac{l_3^2 + d_3^2 - l_4^2}{2l_3 d_3}\right); \tag{8}$$

$$\varphi_3 = \delta_3 - \theta_4 - \psi; \tag{9}$$

$$\varphi_2 = \varphi_3 - \gamma. \tag{10}$$

Projecting the OMNP<sub>r</sub> contour line on the Oy axis we can write

$$\varphi_{1r} = \arcsin\left(\frac{e - r\sin(\alpha) - l_2\sin(\varphi_2)}{l_{1r}}\right). \tag{11}$$

The current position of the point P<sub>r</sub> (the power piston movement law) is calculated projecting the OMNP<sub>r</sub> contour line on the Ox axis:

$$x_{Pr}(\alpha, \psi) = r\cos(\alpha) + l_2\cos(\phi_2) + l_{1r}\cos(\phi_{1r})$$
. (12)

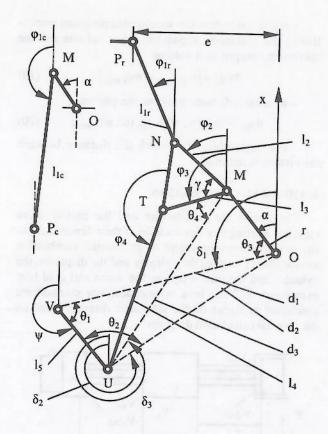


Fig. 2

The current position of the point P<sub>c</sub> (and i.e. the displacer movement law) is calculated from the next relation:

$$x_{Pc}(\alpha, \psi) = r\cos(\alpha) + l_{1c}\cos(\varphi_{1c}), \qquad (13)$$

$$\phi_{1c} = \arcsin\left(\frac{e - r\sin(\alpha)}{l_{1c}}\right).$$
(14)

The volumes of the two chambers are calculated, for geometrical reasons, from the formulas

$$V_r(\alpha, \psi) = \frac{\pi (D^2 - d^2)}{4} (H_c - H_r + x_{Pc} - x_{Pr});$$
 (15)

$$V_c(\alpha, \psi) = \frac{\pi D^2}{4} (x_{Pc,pmi} - x_{Pc}),$$
 (16)

where D and d are cylinder and the displacer stem diameter,  $H_c$  and  $H_r$  are the piston stem dimensions (fig. 2.) and  $x_{Pc, pmi}$  is the position of point  $P_c$  in which the displacer is found in the upper dead center.

Having the dimensions resulted from 12, 13, 15 and 16 we are able to obtain the following parameters of the variable displacement Stirling engine:

- displacement

$$V(\psi) = (V_c(\alpha, \psi) + V_r(\alpha, \psi))_{max}; \qquad (17)$$

- volumetric compression ratio

$$\varepsilon(\psi) = \frac{(V_c(\alpha, \psi) + V_r(\alpha, \psi))_{max}}{(V_c(\alpha, \psi) + V_r(\alpha, \psi))_{min}};$$
(18)

- phase angle between mechanism positions respective to the maximum expansion space volume and the maximum compression volume

$$\Phi(\psi) = \left| \alpha_{\text{Vcmax}} - \alpha_{\text{Vrmax}} \right|; \tag{19}$$

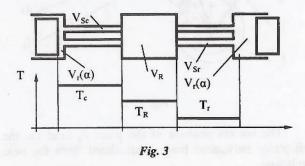
- minimum distance between the pistons

$$d_{Pm} = (x_{Pc}(\alpha, \psi) - x_{Pr}(\alpha, \psi))_{min};$$
 (20)

– the angle  $\alpha(d_{Pm})$  for which the distance between the pistons is minimal.

#### 4. THERMAL CALCULATION

No matter the architecture and the motive drive type, all Stirling engines include in their functional entity construction (equivalent to an internal combustion monocylinder engine) the cylinder and the displacer, the cylinder and the power piston, the warm and cold heat exchangers and the heat regenerator, key components connected as shown on the schematic diagram 2 and on the thermal calculation diagram 3.



Mathematical models for thermal calculation [3], [5] are based on the conditions referred to as G. Schmidt hypotheses: the working fluid is the ideal gas, all processes occur ideally, the temperature inside the expansion space and the warm heat exchanger is constant and equal to the one of the displacer and the cylinder around its warm zone, the temperature inside the compression space and the cold heat exchanger is constant and equal to the one of the power piston and the cylinder around its cold zone, the heat regenerator temperature is constant and equal to the two spaces temperature arithmetical mean, the momentary pressure is the same inside all engine chambers and the pistons move accordingly to sinusoidal laws.

The hypotheses concerning the temperatures inside the engine are shown as well in graphical manner below on fig. 3. In the paper are followed the upper hypotheses except for considering the real movement laws for the two pistons, laws differing of course from the sinusoidal ones.

On fig. 3 were used the letters V, T and M to mark the volumes, temperatures and masses and the subscripts c, r, Sc, Sr, R refer to the expansion and the compression spaces, heat exchangers and regenerator (they all correspond to Romanian, not English, initials).

Having made the notations on fig. 1, the gas masses inside the functional spaces are determined with the aid of the state equation:

$$m = \frac{p V}{R T} \tag{21}$$

Summing the relations given by the state equation and assigning m to the total mass of agent, the momentary pressure relation is obtained

$$p(\alpha, \psi) = \frac{m R}{\frac{V_c(\alpha, \psi) + V_{Sc}}{T_c} + \frac{2 V_R}{T_c + T_r} + \frac{V_r(\alpha, \psi) + V_{Sr}}{T_r}}$$
(22)

where R is the working fluid constant,  $\alpha$  is the crank-shaft rotation angle and  $\psi$  is the power adjustment gear position angle.

Using the relation (22) it is possible to calculate the unfolded indicator diagram  $p(\alpha, \psi) = p(\alpha)$  with  $\psi$  as parameter. Instantly follows the indicator diagram  $p(V(\alpha))$  and  $p(V_c(\alpha))$ ,  $p(V_r(\alpha))$  diagrams for which a newly introduced notation  $V(\alpha)$  stands for the total volume of the engine.

The work yielded per cycle is calculated with:

$$L(\psi) = \oint p(V(\alpha, \psi)) d(V(\alpha, \psi)) =$$

$$= \int_{0}^{2\pi} p(V(\alpha, \psi)) \left[ \frac{d}{d\alpha} (V(\alpha, \psi)) \right] d\alpha.$$
(23)

Relations similar to the precedent expression are used to obtain the work done by the expansion space  $L_c(\psi)$  and the work done by the compression space  $L_c(\psi)$ .

To obtain the mean indicated pressure is used

$$p_{e}(\psi) = \frac{L(\psi)}{(V_{c}(\alpha, \psi) + V_{r}(\alpha, \psi))_{max} - (V_{c}(\alpha, \psi) + V_{r}(\alpha, \psi))_{min}}$$
(24)

The working fluid pressure when the engine is rested and cold (has he same temperature as the environment) is given by the relation

$$p_0(\psi) = \frac{m R T_0}{(V_c(\alpha, \psi) + V_r(\alpha, \psi))_{max}}.$$
 (25)

# 5. CHARACTERISTICS OF THE VARIABLE DISPLACEMENT STIRLING ENGINE

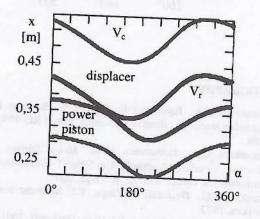
Up to now were calculated all the parameters given by all previous relations for an engine described by the following values: r = 0.0385 m; e = 1.6 r;  $l_{1c} = 3$  r;  $l_{1r} = 2.5$  r;  $l_2 = l_3 = l_5 = 2$  r;  $l_4 = 3$  r;  $d_1 = 2.5$  r;  $\gamma = 50^\circ$ ;  $\delta_1 = 100^\circ$ ; D = 0.073 m; d = 0.02 m;  $H_r = 5$  r;  $H_c = 12.5$  r;  $V_{Sc} = V_{Sr} = 0.05$   $V_{cmax}$ ;  $V_R = 1.2$   $V_{cmax}$ ; m = 0.002 kg  $H_2$ ;  $T_c = 750$  K;  $T_r = 300$  K, for adjustment angle between  $160^\circ$  ...  $205^\circ$ .

On fig. 4 are shown the piston position and the volume variation at 180° gear adjustment.

On fig. 5 and 6 are shown the curves corresponding to the extreme positions 160° and 205° and to a mean position 180° of the adjustment gear.

The unfolded indicator diagrams (fig. 5) show that as the adjustment angle  $\psi$  increases so does the

maximum pressure, phenomenon due to the increase of the volume occupied by the working fluid. We also notice that the angular positions for which the maximum and the minimum pressures are reached during the cycle are closing. Moreover, only in the interval  $\psi=180^\circ$  ... 205° can we inflict a considerable grow or drop of work done per cycle.



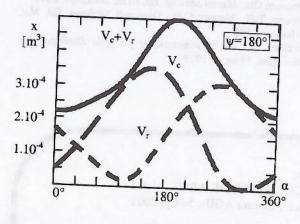


Fig. 4

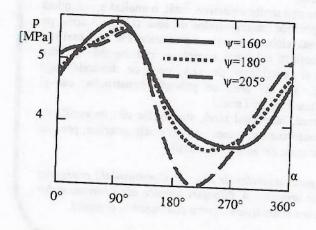
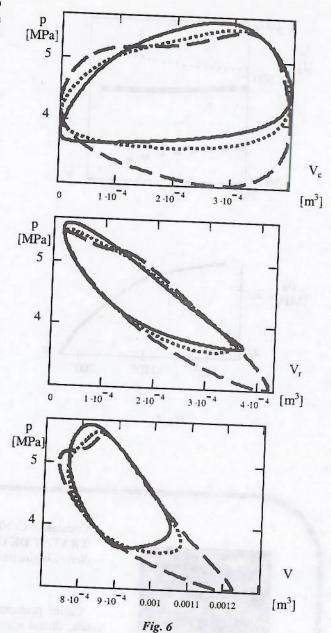


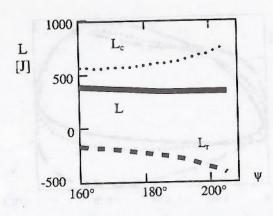
Fig. 5



As expected, once with the displacement increase comes the increase of the work per cycle (fig. 7) and the decrease of the initial and the mean indicated pressure.

### 6. CONCLUSIONS

We analyzed the on load performances of a variable displacement Stirling engine. This variation was made possible by gearing the engine with a modified rhombic drive [1], [2]. Thermal calculations were accomplished using an improved Schmidt analysis method. Imposing a set of initial data (validated in previous tests) it came out that the displacement variation of our method is a fair and reasonable way to adjust the indicated work of a full cycle in a range of (1 ... 1,3) x its minimum value.



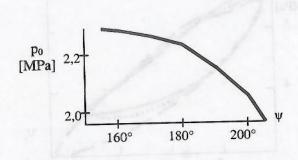
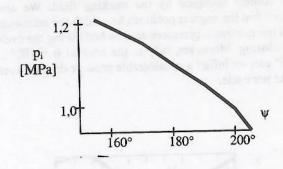


Fig. 7



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Cartea presupune o nouă abordare a problemelor specifice tematicii tratate, ținând seama de materialul amplu acumulat în domeniu, atât în țară, cât și pe plan mondial.

Lucrarea își propune mai multe obiective. Întâi, o analiză aprofundată a fenomenelor fizice specifice generatoarelor de abur (GA), ca apoi, pe această bază, să se poată elabora modele matematice cât mai realiste; soluțiile acestora evidențiază cele mai complexe interdependențe între mărimile constructive și funcționale ale GA, ceea ce deschide largi posibilități de acțiune, atât în ceea ce privește construcția, cât și exploatarea lor, cu cheltuieli cât mai mici.

Concepută în mai multe volume, lucrarea se adresează, în primul rând, studenților din universitățile tehnice; ea poate fi însă utilă și cercetătorilor științifici, doctoranzilor, proiectanților, constructorilor, precum și celor preocupați de funcționarea în condiții cât mai economice ale acestor instalații.

Cuprinsul volumului I: 1. Introducere în ingineria generatoarelor de abur. 2. Combustibilii energetici organici. 3. Bilanțuri materiale ale arderii combustibililor organici. 4. Bilanțuri termice ale generatoarelor de abur. Randamentul și consumul de combustibili. 5. Focare și arzătoare pentru combustibili organici.