CONVECTIVE HEAT TRANSFER IN MICROTUBES FOR GASEOUS FLOW

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Rezumat. Lucrarea studiază transferul termic convectiv în regim staționar pentru curgeri complet dezvoltate hidrodinamic în microcanale. Condițiile la limită sunt de tip temperatură la perete constantă sau flux termic constant iar pentru rezolvarea ecuațiilor curgerii s-au folosit metode numerice. Sunt luate în considerare saltul de temperatură și desprinderea de curgere la perete, precum și încălzirea datorată disipației vâscoase. Soluțiile sunt verificate pentru cazul limită în care se neglijează încălzirea datorată disipației vâscoase. Pentru aceste cazuri, considerând un număr Brinkman dat și lungimi axiale date, sunt prezentate efectele datorate vâscozității, ca funcție de lungimea termică de intrare. De asemenea, încălzirea datorată disipației vâscoase este studiată atât pentru cazul în care fluidul este încălzit, cât și pentru cazul în care fluidul este răcit. Efectele modificării numerelor Knudsen, Brinkman și Prandtl sunt prezentate în condiții de curgere complet dezvoltată și în regiunea de intrare termică, sub formă grafică și tabelară.

LIST OF SYMBOLS

A	–	temperature coefficient, K/m
$\bar{A}$		dimensionless temperature coefficient
Br	–	Brinkman number, Ec/Pr, $\mu u_m^2/k\,\Delta T$
c_p	–	specific heat at constant pressure, J/kgK
c_v	–	specific heat at constant volume, J/kgK
D	–	diameter of the tube, m
D_h	–	hydraulic diameter, m
Ec	–	Eckert number, $u_m^2/c_p\Delta T$
F	–	momentum accommodation coefficient
F_T	–	thermal accommodation coefficient
Gz	–	Graetz number, $RePrD_h/L$
h	–	heat transfer coefficient, W/m^{2}K
K	–	thermal conductivity, W/mK
$\bar{k}$	–	Boltzman const., 1.38×10^{-23}J/K mol
Kn	–	Knudsen number, λ/D_h
L	–	channel length, m
Nu	–	Nusselt number, hD_h/k
P	–	pressure, Pa
Pr	–	Prandtl number, ν/α
R	–	tube radius, m
r	–	radial coordinate, m
Re	–	Reynolds number, $\rho D_h u_m/\mu$
T	–	temperature, K
ΔT	–	temperature difference, K
u	–	axial velocity, m/s
x	–	axial coordinate, m
v	–	velocity in your direction, m/s

Greek Symbols

α	thermal diffusivity, m^2/s
γ	heat capacity ratio, c_p/c_v
ζ	dimensionless x
η	dimensionless r
θ	dimensionless temperature
λ	molecular mean free path, m
μ	dynamic viscosity, kg/m.s
ν	kinematic viscosity, m^2/s
ρ	density, kg/ m^3

1. INTRODUCTION

With the increase of integrated circuit density and power dissipation of microelectronic devices, it is becoming more necessary to employ effective cooling devices and cooling methods to maintain the operating temperature of electronic components at a safe level. New thermal control methods have become mandatory as existing heat flux levels exceed 100 W/cm^2. These trends in thermal packaging are discussed by Bar-Cohen (1992) [1] and in Cooling of Electronics System by Kakac et al. (eds. 1992) [2].

There are basically two ways of modeling a flow field in microchannels. Either as the fluid really is-a collection of molecules-or as a continuum where the matter is assumed continuous and indefinitely divisible. The former modeling is subdivided into deterministic methods and probabilistic ones, while in the latter approach the velocity, density, pressure, etc., are defined at every point in space and time, and conservation of mass, energy and momentum lead to a set of nonlinear partial differential equations (Navier-Stokes).

Knudsen number is proportional to (length of mean free path) / (characteristic dimension) and is used in momentum and mass transfer in general and very low pressure gas flow calculations in particular. It is normally defined in the following form [3]:

$$\mathrm{Kn} = \frac{\lambda}{D} \tag{1.1}$$

where λ is the length of mean free path of the molecules. D is the characteristic dimension. Generally the traditional continuum approach is valid, albeit with modified boundary conditions as long as Kn < 0.1

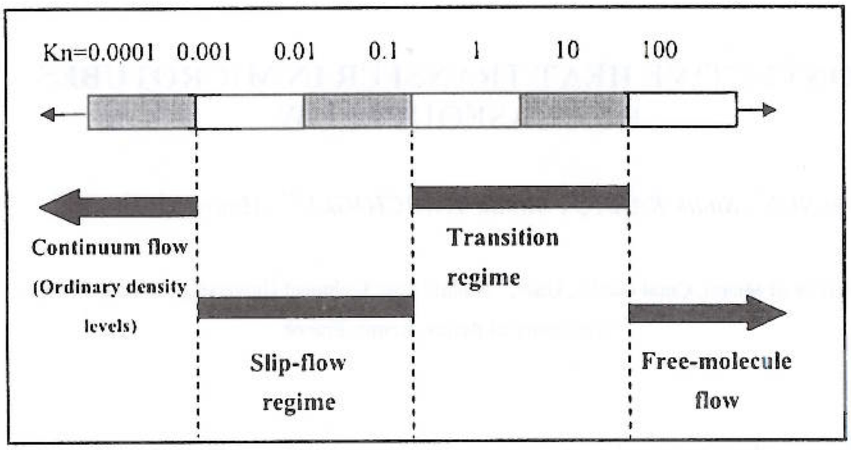

Fig. 1. Knudsen number regimes.

From Fig. 1, we know that for the small values ($Kn \leq 10^{-3}$), the fluid is considered to be continuum, while for large values ($Kn \geq 10$), the fluid is considered to be a free molecular flow. For $10^{-3} < Kn < 10^{-1}$ is the near continuum region.

Brinkman number is the ratio of heat generated by viscous dissipation to heat transferred by conduction over the cross-section which is used in energy equation considering temperature jump on the wall in general and very low pressure gas in particular. It is normally defined in the following form:

$$Br = \frac{\mu u_m^2}{k\Delta T} \qquad (1.2)$$

where μ is dynamic viscosity of flow in microtubes, u_m is sectionally-averaged velocity of the flow in microtubes, k is the axially local and sectionally-averaged thermal conductivity coefficient, and ΔT is axially local wall-flow temperature difference($T_0 - T_s$).

The Brinkman number also emerges from the dimensionless general energy equation as the product of Eckert and Prandtl numbers. The negative values of Br means that the fluid is being cooled. It measures the relative importance of viscous heating (work done against viscous shear) to fluid conduction along the microchannel dimension.

Gaseous flow in microchannels was experimentally analyzed by Shih et al. (1996) [5] with helium and nitrogen as the working fluids. Mass flow rate and pressure distribution along the channels were measured. Helium results agreed well with the result of a theoretical analysis using slip flow conditions, however there were deviations between theoretical and experimental results for nitrogen.

Tunc and Bayazitoglu (2000 and 2001) [6, 7] studied the convective heat transfer for steady state, laminar, hydrodynamically developed flow in microtubes with uniform temperature and uniform heat flux boundary conditions by the use of the integral transform technique. Temperature jump condition at the wall and viscous heating within the medium are included. The solution method is verified for the cases where viscous heating is neglected. For uniform temperature case, with a given Brinkman number, at specified axial lengths, the viscous effects are presented for the developing range, reaching the fully developed Nusselt number. The effect of viscous heating is investigated for both of the cases where the fluid is being heated or cooled. Prandtl number analysis has shown that, as we increase the Prandtl number the temperature jump effect diminishes which gives a rise to the Nusselt number.

In another paper Tunc and Bayazitoglu (2001) [8] considered heat transfer by convection in a rectangular microchannel is investigated. The flow is assumed to be fully developed both thermally and hydrodynamically. The H2-type boundary condition, constant axial and peripheral heat flux, is applied at the walls of the channel. Since the velocity profile for a rectangular channel is not known under the slip flow conditions, the momentum equation is first solved for velocity. The resulting velocity profile is then substituted into the energy equation. The integral transform technique is applied twice, once for velocity and once for temperature. The results show a similar behavior to previous studies on circular microtubes.

2. ANALYSIS

2.1. Constant wall temperature

Considering the flow in microtubes with viscous dissipation, experiments have shown that the fluid flow and heat transfer characteristics of microtubes deviate from well-known macroscale correlations. As the channel size is reduced, wall effects on fluid flow and heat transfer are considerable. Molecular mean free path and channel size will have values on the same order of magnitude, allowing molecular structure to affect heat transfer. Collisions of gas molecules with the flow boundary will occur more often than those between molecules. And then macrochannel boundary conditions are different from the typical ones (no temperature difference between the wall and the flow and the relative velocity along the wall is zero). There exist a slip velocity along the wall and temperature jump between wall and fluid.

We analyze two-dimensional heat transfer in microtubes with constant fluid properties. Since the fluid next to wall has a temperature finitely different from the wall temperature, slip temperature should be used for the first boundary condition. The second boundary condition is the centerline symmetry and uniform temperature at the channel entrance.

Slip velocity: $$u_s = -\frac{2-F_m}{F_m}\lambda\left(\frac{du}{dr}\right)_{r=R} \tag{2.1}$$

In this relation, u_s is the slip velocity, λ the molecular mean free path, and F_m is the momentum accommodation coefficient. It is a function of the interaction between gas molecules and the surface. If the surface is absolutely smooth and reflect the molecules specularly, the tangential momentum will not change. Therefore F_m will be equal to zero. Molecules can also be reflected diffusely, which results in $F_m = 1$. This means that all the tangential momentum is lost at the wall [6]. Diffuse reflection results from the penetration of the molecules into interstices in the surface where multiple strikes occur before the molecule leaves and from the surface roughness. Accommodation coefficient may be significantly different from one for light atoms and closer to one for heavy atoms.

The fully developed velocity profile for slip flow is derived from the momentum equation using the slip velocity as.

$$u = \frac{2u_m\left[1-\left(\frac{r}{R}\right)^2+4K_n\right]}{1+8K_n} \tag{2.2}$$

where u_m is the mean velocity.

Temperature jump:

$$T_s - T_w = -\frac{2-F_t}{F_t}\frac{2\gamma}{(\gamma+1)}\frac{\lambda}{\Pr}\frac{\partial T}{\partial r} \tag{2.3}$$

where T_s is the temperature of the flow at the wall, T_w the wall temperature, and F_t is the thermal accommodation coefficient is defined as $F_t = \frac{(E_a - E_t)}{(E_a - E_w)}$ where E_a is the energy of the approaching stream, E_t is the energy carried by the molecules leaving the surface and $E_w\sqrt{b^2-4ac}$ is the energy of the molecules leaving the surface at the wall temperature. F_t can be defined as the fraction of molecules reflected by the wall that accommodated their energy to the wall temperature if we have a rough surface or if we have a phase change.

Energy equation under the fully developed velocity, with constant fluid properties including the heat dissipation can be written as

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\nu}{c_p}\left(\frac{du}{dr}\right)^2 \tag{2.4}$$

$$\text{at } r = R \qquad T = T_s \tag{2.4a}$$

$$\text{at } r = 0 \qquad \frac{\partial T}{\partial r} = 0 \tag{2.4b}$$

$$\text{at } x = 0 \qquad T = T_0 \tag{2.4c}$$

The energy equation (2.4) can be made dimensionless by defining the following dimensionless numbers:

$$\theta = \frac{T - T_s}{T_0 - T_s}, \quad \eta = \frac{r}{R}, \quad \varsigma = \frac{x}{L},$$

$$u^* = \frac{u}{u_m} = \frac{2\left(1-\eta^2+4K_n\right)}{1+8K_n} \tag{2.5}$$

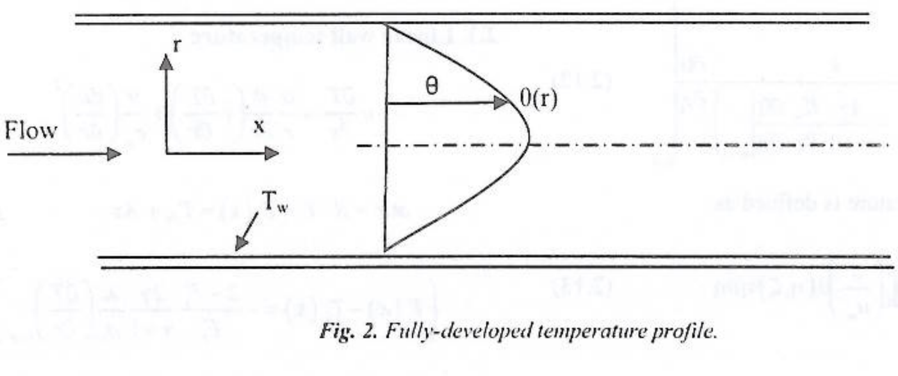

Fig. 2. Fully-developed temperature profile.

Then, we obtain the following dimensionless form of the energy equation (2.4):

$$\frac{\mathrm{Gz}\left(1-\eta^2+4K_n\right)}{2\left(1+8K_n\right)}\frac{\partial\theta}{\partial\zeta}=\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right)+\frac{16\mathrm{Br}}{\left(1+8K_n\right)^2}\eta^2 \tag{2.6}$$

$$\eta=1 \qquad \theta=0 \tag{2.6a}$$

$$\eta=0 \qquad \frac{\partial\theta}{\partial\eta}=0 \tag{2.6b}$$

$$\zeta=0 \qquad \theta=1 \tag{2.6c}$$

where Gz is the Graetz number defined as $\mathrm{Gz}=\dfrac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}$, and Br is the Brinkman number defined as $\mathrm{Br}=\dfrac{\mu u_m^2}{k\Delta T}$, where $\Delta T=T_0-T_s$ is the temperature difference between the fluid inlet and at the wall of the tube.

Heat transfer from the fluid to the wall by convection is written as

$$q_x=h_x\left(T_b-T_w\right). \tag{2.7}$$

Heat flux at the wall can also be written using Fourier's law

$$q_x=-k\left.\frac{\partial T}{\partial r}\right|_{r=R} \tag{2.8}$$

$$h_x=-\frac{k}{\left(T_b-T_w\right)}\left.\frac{\partial T}{\partial r}\right|_{r=R} \tag{2.9}$$

$$h_x=-\frac{(k/R)}{\left\{\left(T_b-T_s\right)/\left(T_o-T_s\right)\right\}-\left\{\left(T_w-T_s\right)/\left(T_o-T_s\right)\right\}}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=1} \tag{2.10}$$

$$\frac{T_s-T_w}{T_o-T_s}=-\frac{4\gamma}{\gamma+1}\frac{K_n}{\mathrm{Pr}}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=1} \tag{2.11}$$

$$h_x=-\left.\frac{k}{R\left(\theta_b-\dfrac{4\gamma}{\gamma+1}\dfrac{K_n}{\mathrm{Pr}}\left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1}\right)}\frac{\partial\theta}{\partial\eta}\right|_{\eta=1} \tag{2.12}$$

where the bulk temperature is defined as

$$\theta_b=2\int_0^1\left(\frac{u}{u_m}\right)\theta(\eta,\zeta)\,\eta\,\mathrm{d}\eta. \tag{2.13}$$

The Nusselt number is obtained as follows [12]:

$$\mathrm{Nu}_x=\frac{h_xD}{k}=-\frac{2\left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1}}{\left(\theta_b-\dfrac{4\gamma}{\gamma+1}\dfrac{K_n}{\mathrm{Pr}}\left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1}\right)} \tag{2.14}$$

2.2. Constant heat flux

$$u\frac{\partial T}{\partial x}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\nu}{c_p}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2 \tag{2.15}$$

$$\text{at } r=R \qquad k\frac{\partial T}{\partial r}=q'' \tag{2.15a}$$

$$\text{at } r=0 \qquad \frac{\partial T}{\partial r}=0 \tag{2.15b}$$

$$\text{at } x=0 \qquad T=T_0 \tag{2.15c}$$

The energy equation (2.4) can be made dimensionless by defining $\theta=\dfrac{T-T_0}{q''R/k}$.

$$\frac{\mathrm{Gz}\left(1-\eta^2+4K_n\right)}{2\left(1+8K_n\right)}\frac{\partial\theta}{\partial\zeta}=\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right)+\frac{32\mathrm{Br}}{\left(1+8K_n\right)^2}\eta^2 \tag{2.16}$$

$$\eta=1 \qquad \frac{\partial\theta}{\partial\eta}=1 \tag{2.16a}$$

$$\eta=0 \qquad \frac{\partial\theta}{\partial\eta}=0 \tag{2.16b}$$

$$\zeta=0 \qquad \theta=0 \tag{2.16c}$$

where Br is defined as $\mathrm{Br}=\dfrac{\mu u_m^2}{q''D}$.

Similarly, we can derive an expression for the local Nusselt number as above. [12]

$$\mathrm{Nu}_x=\frac{h_xD}{k}=\frac{2}{\theta_s+\dfrac{4\gamma}{\gamma+1}\dfrac{K_n}{\mathrm{Pr}}-\theta_b} \tag{2.17}$$

2.3. Linear wall temperature

$$u\frac{\partial T}{\partial x}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\nu}{c_p}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)^2 \tag{2.18}$$

$$\text{at } r=R \quad T=T_s(x)=T_{s0}+Ax \tag{2.18a}$$

$$\left(T_s(x)-T_w(x)=-\frac{2-F_t}{F_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{p_r}\left(\frac{\partial T}{\partial r}\right)_{r=R}\right)$$

at $r=0$ $\quad \dfrac{\partial T}{\partial r}=0$ (2.18b)

at $x=0$ $\quad T=T_0$ (2.18c)

where A is the coefficient, $T_{s0}=T_s\big|_{x=0}$.

Introducing $\theta=\dfrac{T-T_{s0}}{T_0-T_{s0}}$, the dimensionless form of the energy equation is as follows:

$$\frac{\mathrm{Gz}\left(1-\eta^2+4K_n\right)}{2\left(1+8K_n\right)}\frac{\partial\theta}{\partial\varsigma}=\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right)+\frac{16\mathrm{Br}}{\left(1+8K_n\right)^2}\eta^2 \tag{2.19}$$

$\eta=1$ $\quad \theta=\bar{A}\varsigma$ (2.19a)

$\eta=0$ $\quad \dfrac{\partial\theta}{\partial\eta}=0$ (2.19b)

$\zeta=0$ $\quad \theta=1$ (2.19c)

where $\bar{A}=\dfrac{AL}{T_0-T_{s0}}$, $\mathrm{Br}=\dfrac{\mu u_m^2}{k\Delta T}$, where $\Delta T=T_0-T_{s0}$ is the temperature difference between the fluid at tube entrance and at the wall when x = 0.

Similarly, the local Nusselt number considering the temperature jump can be driven. [13]

$$\mathrm{Nu}_x=\frac{h_xD}{k}=-\frac{2\left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1}}{\left(\theta_b-\dfrac{4\gamma}{\gamma+1}\dfrac{K_n}{\mathrm{Pr}}\left.\dfrac{\partial\theta}{\partial\eta}\right|_{\eta=1}-\bar{A}\varsigma\right)} \tag{2.20}$$

3. NUMERICAL METHOD

With the development of high-speed digital computers, numerical techniques have been developed and extended to handle almost any problem of any degree of complexity. The finite volume method [10,11] is employed to solve above partial differential equations (Fig. 3).

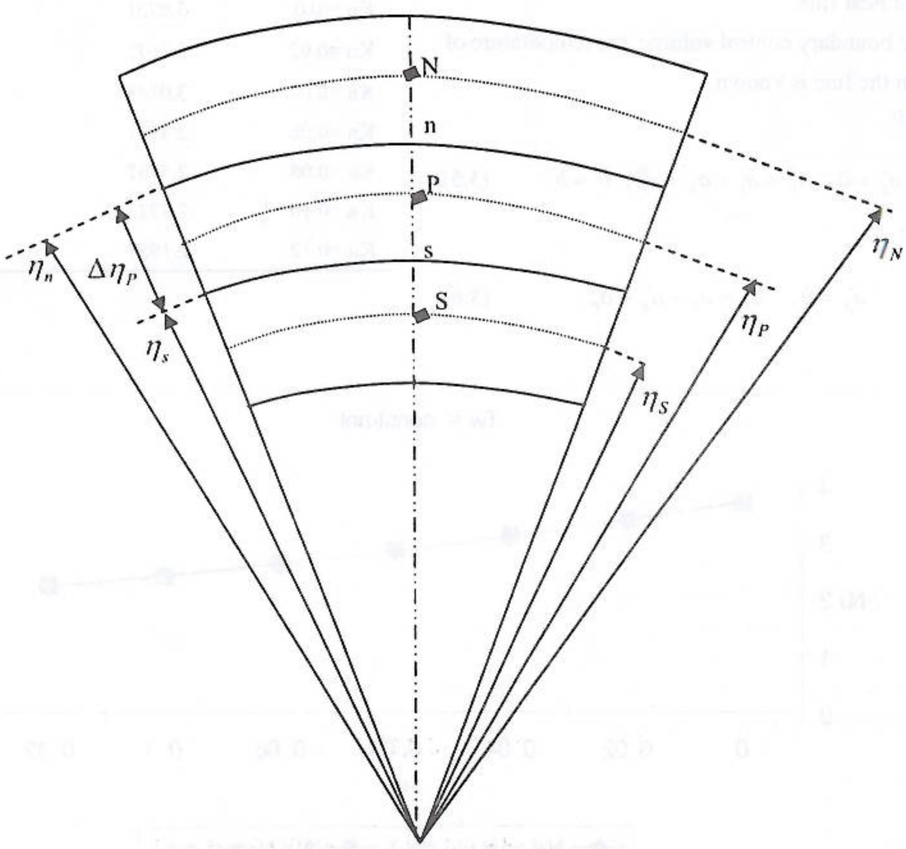

Fig. 3. Control volume.

$$\Delta V_p = \frac{\eta_n + \eta_s}{2}\Delta\eta_p, \qquad \eta_p = \frac{1}{2}(\eta_n + \eta_s),$$

$$\eta_s = \eta_p - \frac{1}{2}\Delta\eta_p, \qquad \eta_n = \eta_p + \frac{1}{2}\Delta\eta_p \qquad (3.1)$$

Integrating above partial differential equations in selected control volume, we obtain:

$$a_P\theta_P = a_N\theta_N + a_S\theta_S + b \qquad (3.2)$$

where

$$a_N = \frac{\eta_n}{\eta_N - \eta_P}, \quad a_S = \frac{\eta_s}{\eta_P - \eta_S},$$

$$a_P^0 = \frac{\frac{\mathrm{Gz}}{2(1+8\mathrm{Kn})}\left(1 + 4\mathrm{Kn} - \frac{\eta_n^2 + \eta_s^2}{2}\right)}{\Delta\zeta}\Delta V \qquad (3.3)$$

$$a_P = a_N + a_S + a_P^0 \qquad (3.4)$$

$b = S_c\Delta V$, where $S_c = \frac{16\mathrm{Br}}{(1+8\mathrm{Kn})^2}\frac{\eta_n^2 + \eta_s^2}{2} + \frac{a_P^0\theta_P^0}{\Delta V}$ for constant wall temperature and linear wall temperature boundary conditions. $S_c = \frac{32\mathrm{Br}}{(1+8\mathrm{Kn})^2}\frac{\eta_n^2 + \eta_s^2}{2} + \frac{a_P^0\theta_P^0}{\Delta V}$ for constant heat flux.

For the boundary control volume, the temperature of the node on the line is known.

when $\eta = 0$

$$a_S' = 0, \quad a_P' = a_S' + a_N + a_P^0, \quad b' = b \qquad (3.5)$$

when $\eta = 1$

$$a_N' = 0, \quad a_P' = a_S + a_N' + a_P^0 \qquad (3.6)$$

$b' = b + a_N\,\theta|_{\eta=1} = b$ for constant wall temperature

$$(3.6a)$$

$b' = b + a_N\,\theta|_{\eta=1} = b + a_N\bar{A}\varsigma_P$ for linear wall temperature

$$(3.6b)$$

$b' = b + \eta\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=1} = b + 1$ for constant heat flux $\qquad (3.6c)$

4. RESULTS AND CONCLUSIONS

The effects of the Knudsen number, the Brinkman number and the Prandtl number on heat transfer under the different boundary conditions are presented.

The analytical and numerical results are compared in Tables 1 and 2, Figs. 4. and 5, the tendency and value are matched very well.

***Table 1.** Developed Conditions, Laminar Flow Nusselt Values*

(T_W=C, Pr=0.6)

Br = 0	Nu_T(analytical)	Nu_T(Numerical)
Kn = 0.0	3.6751	3.6566
Kn =0.02	3.3675	3.3527
Kn =0.04	3.0745	3.0627
Kn =0.06	2.8101	2.8006
Kn =0.08	2.5767	2.5689
Kn =0.10	2.3723	2.3659
Kn =0.12	2.1937	2.1882

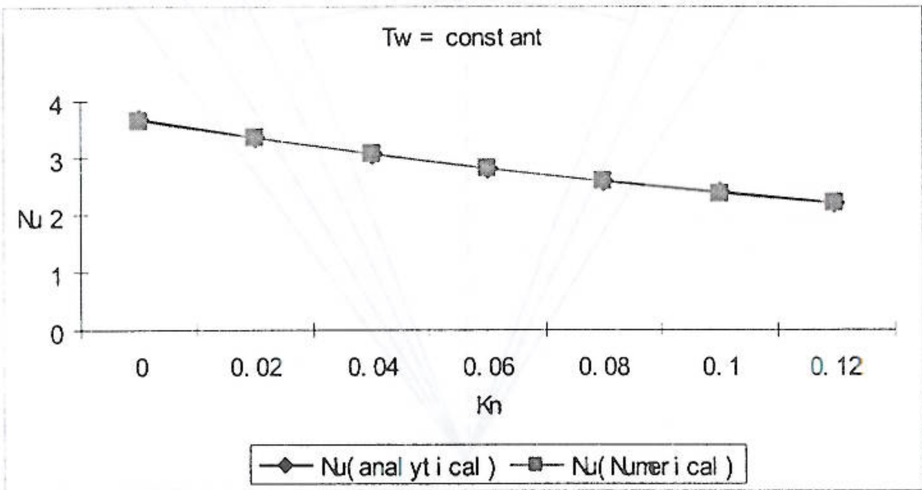

***Fig. 4.** Developed Conditions, Laminar Flow Nusselt Values* (T_w=C, Pr=0.6).

Table 2. Developed Conditions, Laminar Flow Nusselt Values (q_w=C, Pr=0.6)

Br = 0	Nu_q(analytical)	Nu_q(Numerical)
Kn = 0.0	4.3627	4.3649
Kn = 0.02	3.9801	4.0205
Kn = 0.04	3.5984	3.6548
Kn = 0.06	3.2519	3.3126
Kn = 0.08	2.9487	3.0081
Kn = 0.10	2.6868	2.7425
Kn = 0.12	2.4613	2.5125

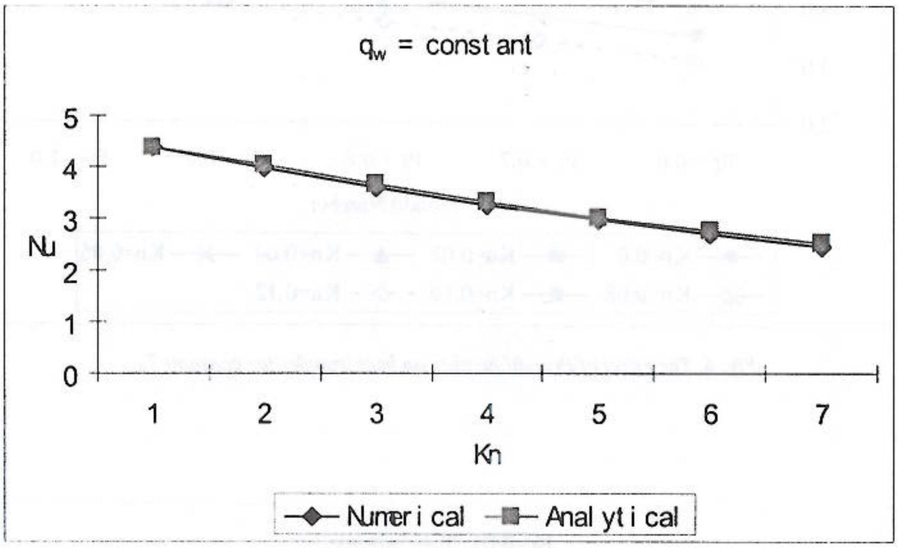

Fig. 5. Developed Conditions, Laminar Flow Nusselt Values (q_w=C, Pr=0.6).

Table 3. The fully developed Nusselt number values, Br = 0.01 constant T_w

Br = 0.01	Pr = 0.6	Pr = 0.7	Pr = 0.8	Pr = 0.9	Pr = 1.0
Kn = 0.00	9.5985	9.5985	9.5985	9.5985	9.5985
Kn = 0.02	7.1327	7.427	7.6642	7.8594	8.0229
Kn = 0.04	5.6525	6.0313	6.3505	6.6231	6.8587
Kn = 0.06	4.6708	5.0651	5.4075	5.7076	5.9727
Kn = 0.08	3.9744	4.3594	4.701	5.006	5.2802
Kn = 0.10	3.4558	3.8227	4.1535	4.4532	4.7261
Kn = 0.12	3.0552	3.4016	3.7177	4.0074	4.2738

The results are shown in Table 3 and Figs. 6, 7 for uniform wall temperature boundary conditions with Br = 0.01. The fully developed Nusselt number decreases as Kn increases. For the no-slip condition Nu = 9.5985, while it drops down to 3.4016 for Kn = 0.12 under Pr = 0.7, a decrease of 64.5%. This is due to the fact that the temperature jump reduces heat transfer. As Kn increases, the temperature jump also increases.

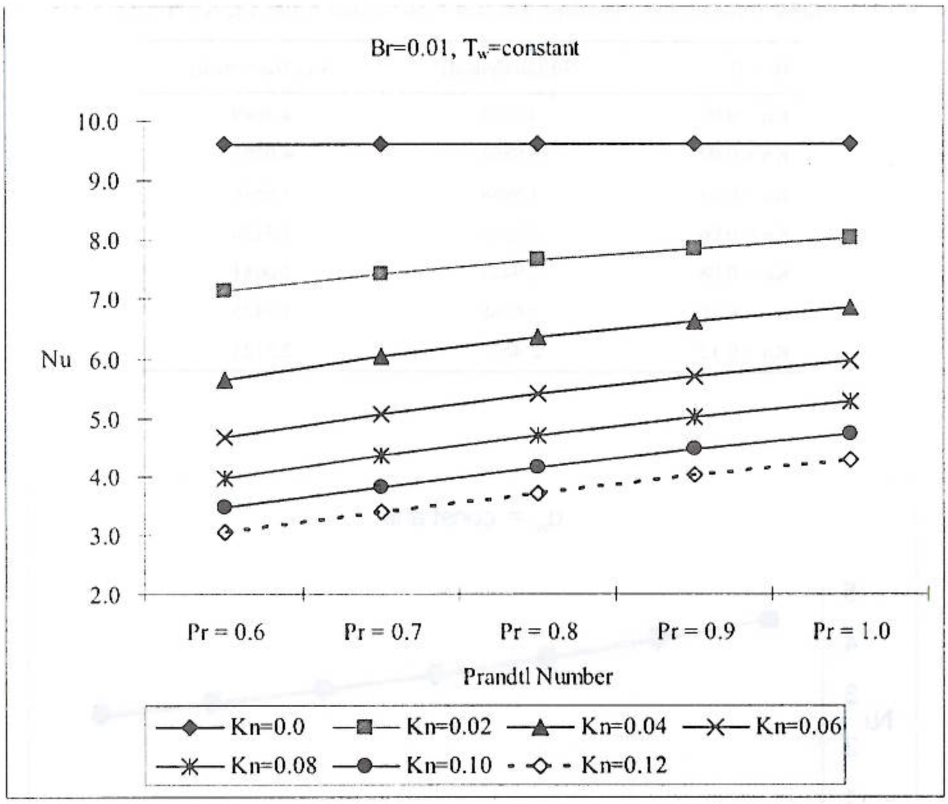

Fig. 6. The effect of Prandtl number on heat transfer for constant T_w.

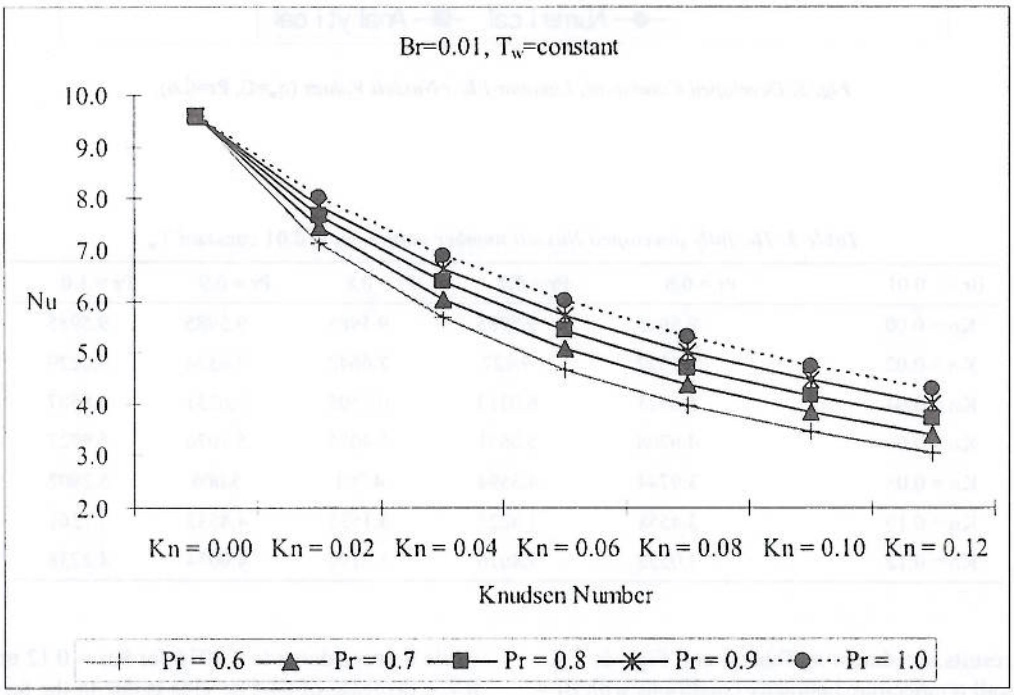

Fig. 7. The effect of Knudsen number on heat transfer for constant T_w.

In Fig. 6, the fully developed Nusselt number increases as Pr increases. This is due to the fact that the temperature jump reduces heat transfer. As Pr increases, the temperature jump decreases. Therefore, the denominator of Eq. (2.14) takes smaller values.

The variation of the fully developed Nusselt number with Kn for these two cases, with and without viscous heating, is given in Table 4 and Fig. 8. It is seen from the figure that, for the uniform temperature boundary condition at the wall, Kn number increase has more influence with the presence of viscous dissipation. Also, the Nusselt number has larger value for the case with viscous heating.

The results are shown in Table 5 and Figs. 9, 10 for uniform heat flux boundary conditions with Br = −0.01. The fully developed Nusselt number decreases as Kn increases. For the no-slip condition Nu = 4.5650, while it drops down to 2.7468 for Kn = 0.12 under Pr = 0.7, a decrease of 39.8%. This is due to the fact that the temperature jump reduces heat transfer. As Kn increases, the temperature jump also increases.

In Fig. 9, the fully developed Nusselt number increases as Pr increases. This is due to the fact that the temperature jump reduces heat transfer. As Pr increases, the temperature jump decreases. Therefore, the denominator of Eq. (2.17) takes smaller values.

***Table 4.** The fully developed Nusselt number values,* Pr = 0.7, constant T_w

Pr = 0.7	Br = 0.00	Br = 0.01
Kn = 0.00	3.6566	9.5985
Kn = 0.02	3.4163	7.4270
Kn = 0.04	3.1706	6.0313
Kn = 0.06	2.9377	5.0651
Kn = 0.08	2.7244	4.3594
Kn = 0.10	2.5323	3.8227
Kn = 0.12	2.3604	3.4016

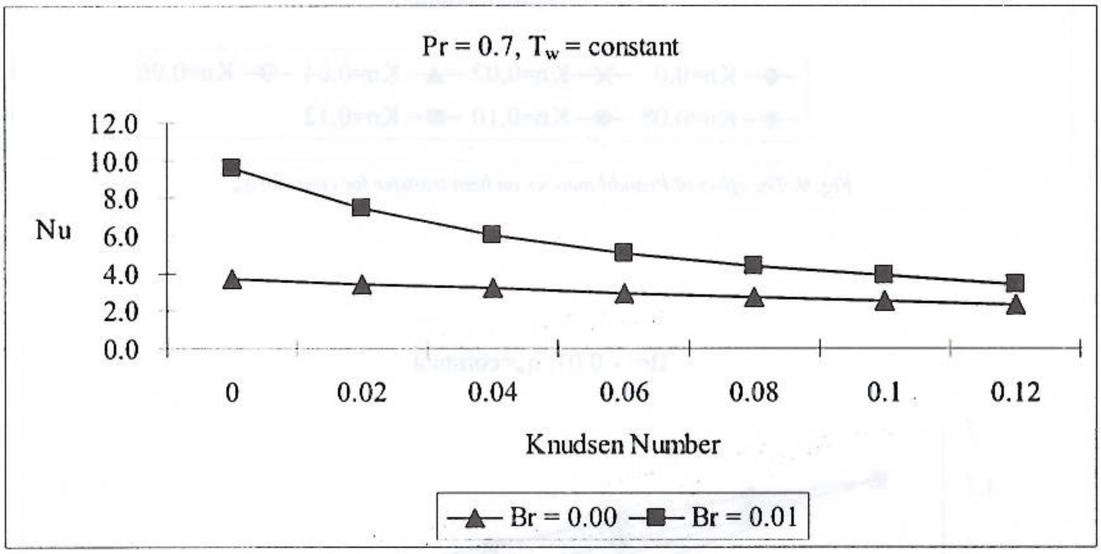

***Fig. 8.** The effect of Brinkman number on heat transfer for constant T_w.*

***Table 5.** The fully developed Nusselt number values,* Br = − 0.01, constant q_w

Br = - 0.01	Pr = 0.6	Pr = 0.7	Pr = 0.8	Pr = 0.9	Pr = 1.0
Kn = 0.00	4.5640	4.5640	4.5640	4.5640	4.5640
Kn = 0.02	4.1278	4.2212	4.2953	4.3565	4.409
Kn = 0.04	3.7154	3.8695	3.9948	4.0995	4.1894
Kn = 0.06	3.3484	3.5395	3.6985	3.8338	3.9511
Kn = 0.08	3.0302	3.2419	3.4218	3.5772	3.7137
Kn = 0.10	2.7567	2.9784	3.1701	3.3381	3.4871
Kn = 0.12	2.5219	2.7468	2.9441	3.1191	3.2759

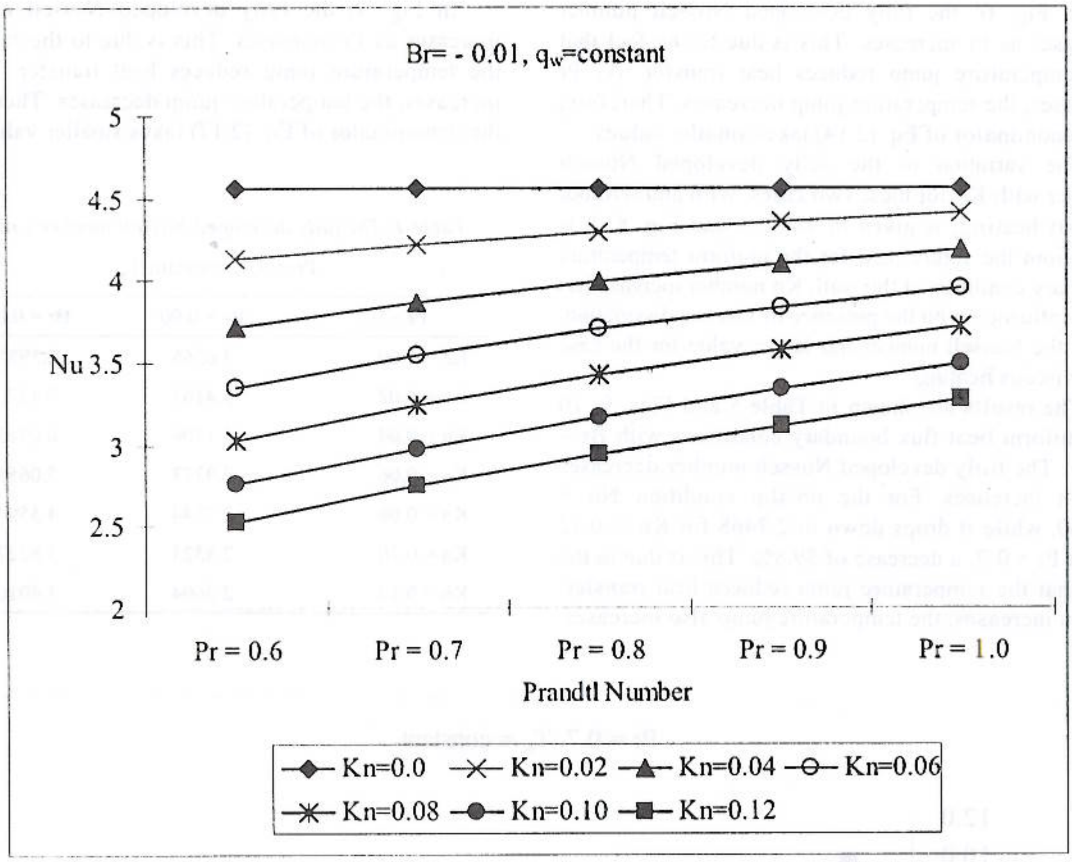

Fig. 9. The effect of Prandtl number on heat transfer for constant q_w.

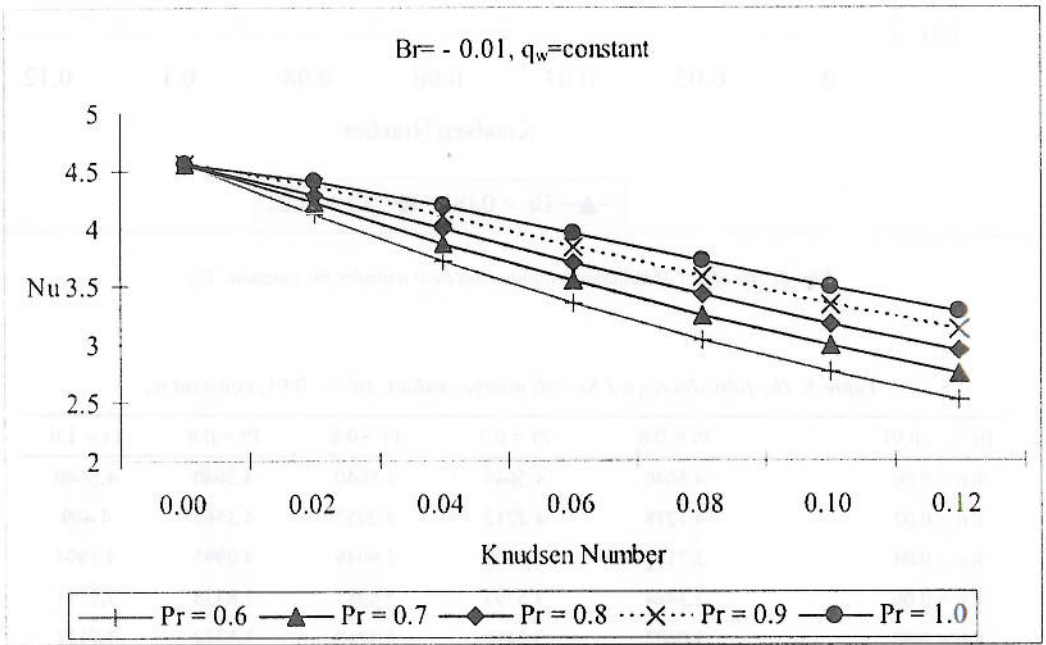

Fig. 10. The effect of Knudsen number on heat transfer for constant q_w.

Table 6, Fig. 11 show the effect of positive or negative Br values ($Br = \pm 0.01$) on heat transfer. From the definition of Br, for this type of boundary condition, a negative Br means that the fluid is being cooled. Therefore, the Nusselt number takes higher values for $Br < 0$ and lower values for $Br > 0$.

Table 6. *The fully developed Nusselt number values*, Pr =0.7, constant q at the wall

Pr = 0.7	Br = 0.00	Br = 0.01	Br = - 0.01
Kn = 0.00	4.3649	4.1825	4.5640
Kn = 0.02	4.1088	4.0022	4.2212
Kn = 0.04	3.8036	3.7398	3.8695
Kn = 0.06	3.4992	3.4598	3.5395
Kn = 0.08	3.2163	3.1912	3.2419
Kn = 0.10	2.9616	2.945	2.9784
Kn = 0.12	2.7354	2.7242	2.7468

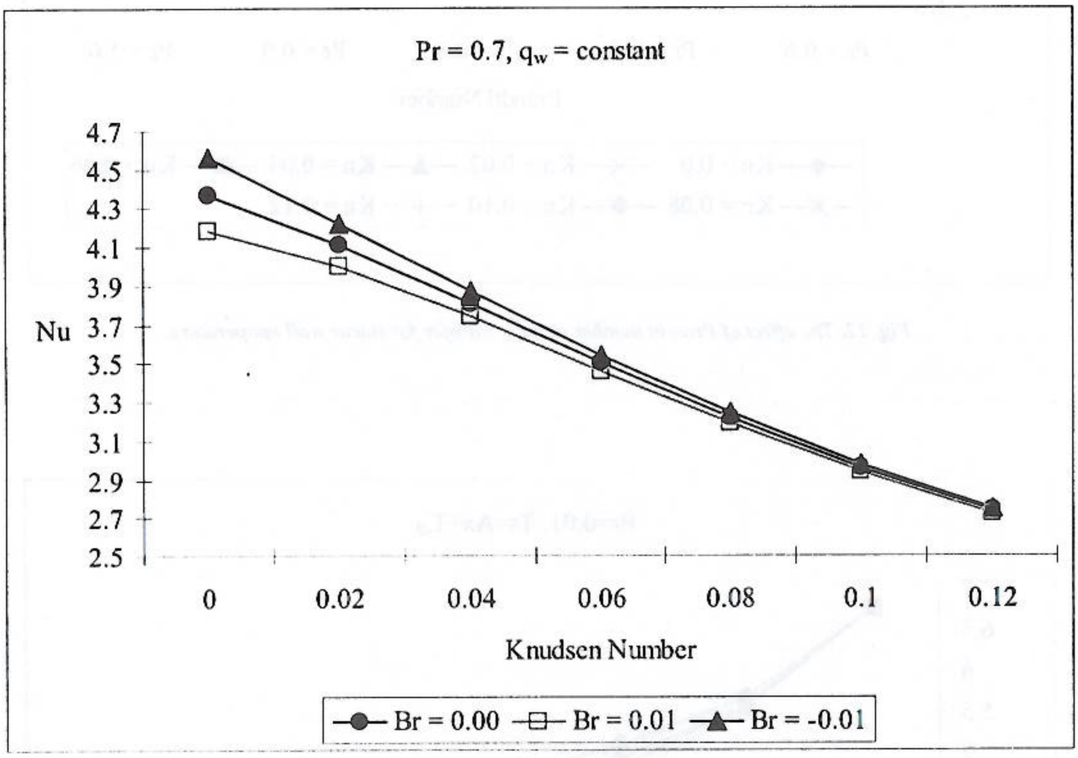

Fig. 11. *The effect of Brinkman number on heat transfer for constant q_w.*

Table 7. *Nusselt number values*, Br=0.01, $T_s = Ax + T_{s0}$ ($\bar{A} = -1.0$)at the wall, $\bar{A} = \dfrac{AL}{T_0 - T_{s0}}$

Br = 0.01	Pr = 0.6	Pr = 0.7	Pr = 0.8	Pr = 0.9	Pr = 1.0
Kn = 0.00	6.77	6.77	6.77	6.77	6.77
Kn = 0.02	5.5177	5.5479	5.5527	5.5943	5.6646
Kn = 0.04	4.6587	4.8019	4.9025	5.0043	5.0759
Kn = 0.06	4.0306	4.2312	4.3861	4.5077	4.6045
Kn = 0.08	3.5499	3.7791	3.9643	4.1163	4.2425
Kn = 0.10	3.1694	3.4111	3.6127	3.7828	3.9277
Kn = 0.12	2.8602	3.1055	3.3148	3.4951	3.6516

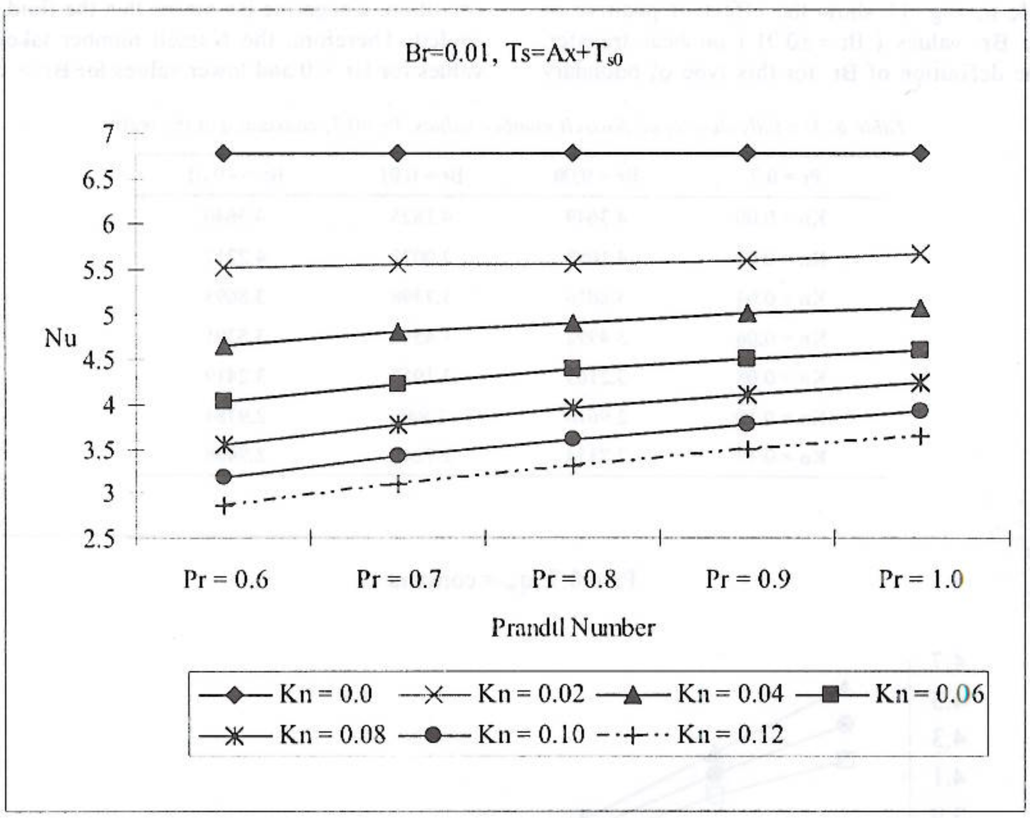

Fig. 12. The effect of Prandtl number on heat transfer for linear wall temperature.

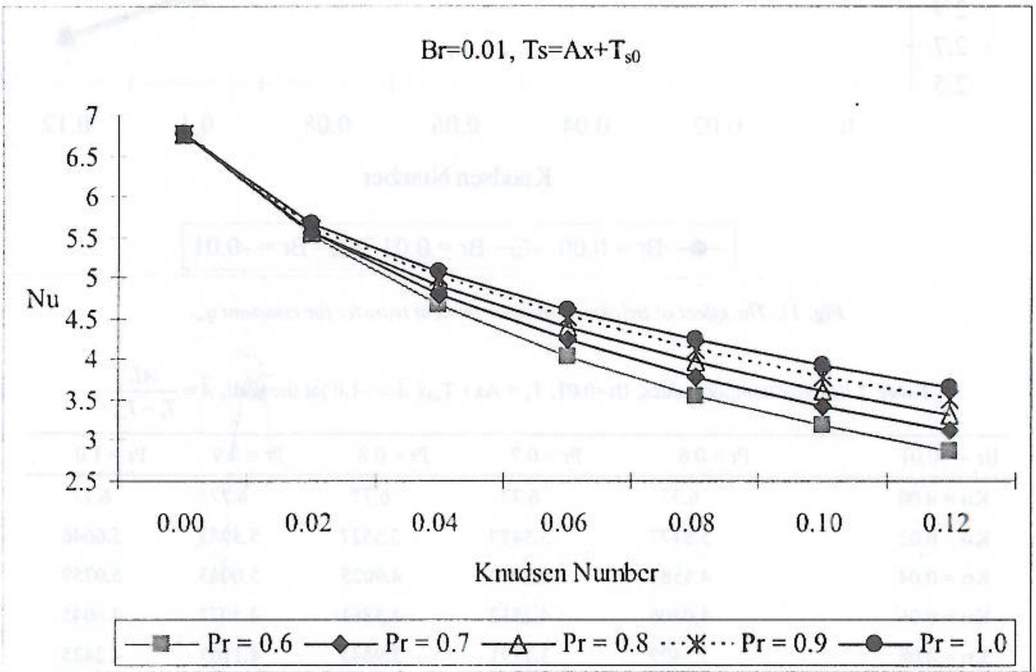

Fig. 13. The effect of Knudsen number on heat transfer for linear wall temperature.

The results are shown in Table 7 and Figs. 12, 13 for linear wall temperature boundary conditions with temperature at Br = 0.01. The fully developed Nusselt number decreases as Kn increases. For the no-slip condition Nu = 6.77, while it drops down to 3.1055 for Kn = 0.12 under Pr = 0.7, a decrease of 54.1%. This is due to the fact that the temperature jump reduces heat transfer. As Kn increases, the temperature jump also increases.

In Fig. 12, the fully developed Nusselt number increases as Pr increases. This is due to the fact that the temperature jump reduces heat transfer. As Pr increases, the temperature jump decreases. Therefore, the denominator of Eq. (2.20) takes smaller values.

In Fig. 14, the fully developed Nusselt number increases as Brinkman number increases. Due to the definition of Br, the positive value means the flow is cooled. The fluid temperature becomes closer to the wall temperature as it flows through the channel. When we include viscous heating, fluid temperature takes higher values, which increases the temperature difference between the fluid and the wall and thus heat transfer.

The results considering temperature jump are shown in Table 8 and Fig 15 at Br = 0 for cases $\bar{A}$ =0, -1. The temperature difference between the wall and the fluid is increased, thus, the heat transfer is increased too. In fact, after reaching the developed region, linear wall temperature has the same effects as uniform heat flux does.

In Fig. 16, we show the effect of temperature jump on the Nusselt number clearly. The solid and dashed lines represent the results from the present study for constant heat flux and constant temperature boundary conditions, respectively. When the temperature jump condition is not considered, in other words, only the velocity slip condition is taken into account, the Nusselt number increases with increasing Kn, which implies that the velocity slip and temperature jump have opposite effects on the Nusselt number.

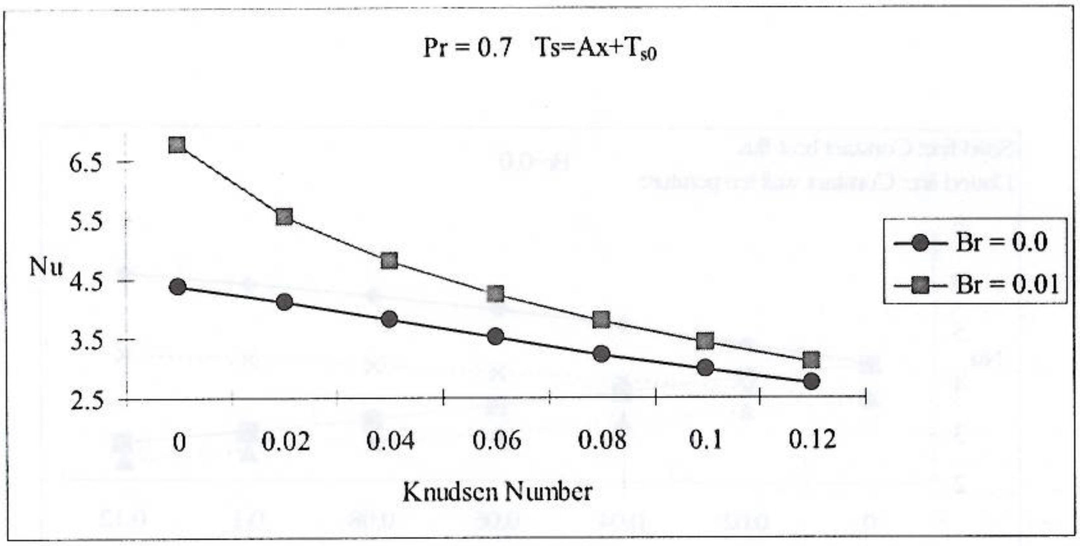

Fig. 14. The effect of Brinkman number on heat transfer for linear wall temperature.

Table 8. Nusselt number values ($T_s = Ax + T_{s0}$, Pr=0.7, $\bar{A} = -1, 0$), $\bar{A} = \dfrac{AL}{T_0 - T_{s0}}$

Br = 0	$Nu_T(0)$	$Nu_T(-1)$
Kn = 0.0	3.6566	4.3654
Kn = 0.02	3.4163	4.1125
Kn = 0.04	3.1706	3.8043
Kn = 0.06	2.9377	3.5009
Kn = 0.08	2.7244	3.2296
Kn = 0.10	2.5323	2.9691
Kn = 0.12	2.3604	2.7391

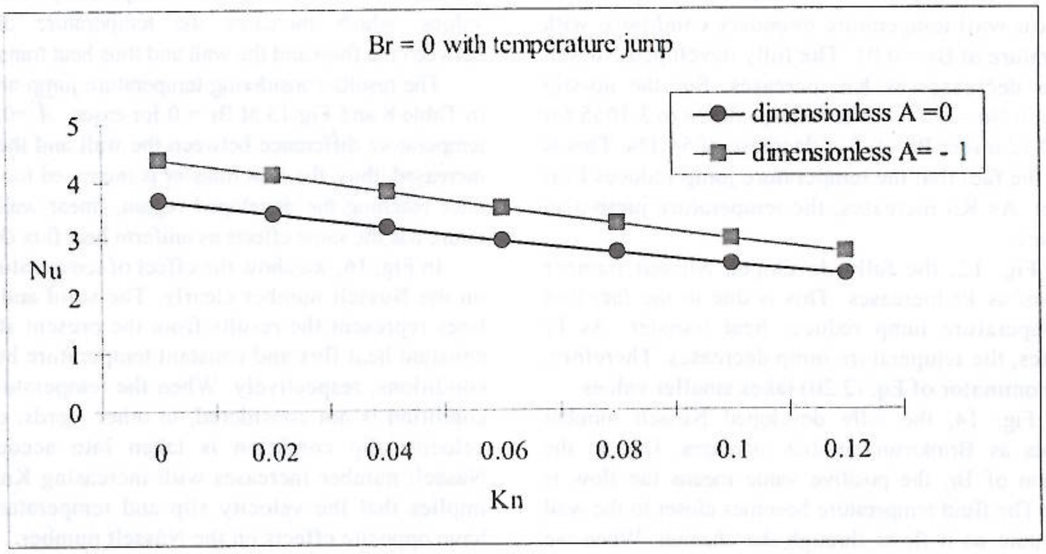

Fig. 15. The effect of $\bar{A}$ on heat transfer.

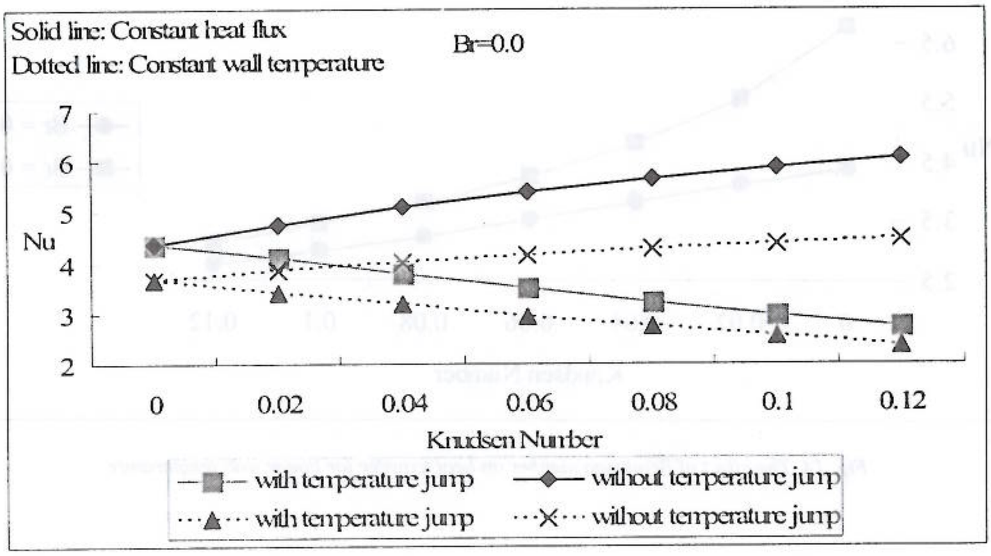

Fig. 16. The effect of temperature jump on heat transfer.

In Table 9, we show the Nusselt number values in the thermally developing range. For both of the cases, as Kn increases, the Nusselt number decreases due to the increasing temperature jump. We note here that, the decrease is greater and reaching developed region is later when we consider viscous dissipation. The Nusselt number reaches the developed value as if there is no viscous heating.

In Table 10, the Nusselt number values in the thermally developing range are shown. For all the cases, as Kn increases, the Nusselt number decreases due to the increasing temperature jump.

In Table 11, the system first reaches the fully developed condition as if there is no viscous heating. Then, at some point, Nu makes a jump to its final value. As Br increases, the jump occurs at a shorter distance from the entrance. Since the wall temperature is constant, they all converge to the same fully developed value. This effect can be explained by the same reason we mentioned before.

Table 9. *Variation of the Nusselt number with Kn for uniform T_w.*

	$\frac{x/D}{RePr}$	0.001	0.006	0.01	0.06	0.1	0.6	1
Br = 0.0	Kn = 0.0	6.1614	3.9092	3.7152	3.6566	3.6566	3.6566	3.6566
	Kn = 0.04	4.7628	3.3319	3.2052	3.1706	3.1706	3.1706	3.1706
	Kn = 0.08	3.7591	2.8308	2.7461	2.7244	2.7244	2.7244	2.7244
	Kn = 0.12	3.0676	2.4339	2.3749	2.3604	2.3604	2.3604	2.3604
Br = 0.01	Kn = 0.0	6.1658	3.9181	3.7275	4.0997	7.2072	9.5985	9.5985
	Kn = 0.04	4.764	3.3352	3.21	3.4154	5.1649	6.0313	6.0313
	Kn = 0.08	3.7595	2.8322	2.7482	2.8656	3.9352	4.3594	4.3594
	Kn = 0.12	3.0677	2.4346	2.3759	2.4464	3.1516	3.4016	3.4016

Table 10. *Nusselt number with the effect of viscous heating at the entrance for uniform q_w.*

	$\frac{x/D}{RePr}$	0.001	0.006	0.01	0.06	0.1	0.6	1
Br= 0.0	Kn = 0.0	7.5884	4.8272	4.5196	4.3649	4.3649	4.3649	4.3649
	Kn = 0.04	5.8036	4.0975	3.8872	3.8036	3.8036	3.8036	3.8036
Br= 0.01	Kn = 0.0	7.3531	4.6346	4.3336	4.1825	4.1825	4.1825	4.1825
	Kn = 0.04	5.7474	4.0335	3.8231	3.7398	3.7398	3.7398	3.7398
Br=-0.01	Kn = 0.0	7.8392	5.0365	4.7223	4.564	4.564	4.564	4.564
	Kn = 0.04	5.8608	4.1635	3.9535	3.8695	3.8695	3.8695	3.8695

Table 11. *Nusselt number with the effect of viscous heating at the entrance for uniform T_w.*

$\frac{x/D}{RePr}$	0.001	0.006	0.01	0.06	0.1	0.6	1
Br = 0.0	4.7628	3.3319	3.2052	3.1706	3.1706	3.1706	3.1706
Br = 0.001	4.764	3.3352	3.21	3.4154	5.1649	6.0313	6.0313
Br = 0.006	4.7697	3.3515	3.2338	4.2009	5.8386	6.0313	6.0313
Br = 0.01	4.7743	3.3645	3.2527	4.5568	5.9128	6.0313	6.0313
Br = 0.015	4.78	3.3807	3.276	4.8462	5.9514	6.0313	6.0313

Table 12. *Nusselt number with the effect of viscous heating, uniform q_w.*

$\frac{x/D}{RePr}$	0.001	0.006	0.01	0.06	0.1	0.6	1
Br = - 0.01	5.8608	4.1635	3.9535	3.8695	3.8695	3.8695	3.8695
Br = 0.0	5.8036	4.0975	3.8872	3.8036	3.8036	3.8036	3.8036
Br = 0.001	5.7979	4.091	3.8807	3.7971	3.7971	3.7971	3.7971
Br = 0.006	5.7698	4.0589	3.8485	3.765	3.765	3.765	3.765
Br = 0.01	5.7474	4.0335	3.8231	3.7398	3.7398	3.7398	3.7398

Since the definition of the Brinkman number is different for the uniform heat flux boundary condition case, a positive Br means that the heat is transferred to the fluid from the wall as opposed to the uniform temperature case. The fully developed Nusselt numbers with and without viscous dissipation are given in Tables 13-15.

Table 13. The fully developed Nusselt number values, Br = 0, constant T at the wall.

Br = 0.00	Pr = 0.6	Pr = 0.7	Pr = 0.8	Pr = 0.9	Pr = 1.0
Kn = 0.00	3.6566	3.6566	3.6566	3.6566	3.6566
Kn = 0.02	3.3527	3.4163	3.4657	3.5050	3.5372
Kn = 0.04	3.0627	3.1706	3.2567	3.3269	3.3853
Kn = 0.06	2.8006	2.9377	3.0497	3.1429	3.2216
Kn = 0.08	2.5689	2.7244	2.8540	2.9636	3.0576
Kn = 0.10	2.3659	2.5323	2.6733	2.7944	2.8994
Kn = 0.12	2.1882	2.3604	2.5084	2.6371	2.7499

Table 14. The fully developed Nusselt number values, Br =0, constant q at the wall

Br = 0.00	Pr = 0.6	Pr = 0.7	Pr = 0.8	Pr = 0.9	Pr = 1.0
Kn = 0.00	4.3649	4.3649	4.3649	4.3649	4.3649
Kn = 0.02	4.0205	4.1088	4.1788	4.2367	4.2865
Kn = 0.04	3.6548	3.8036	3.9243	4.0253	4.1118
Kn = 0.06	3.3126	3.4992	3.6544	3.7863	3.9006
Kn = 0.08	3.0081	3.2163	3.3932	3.5459	3.6798
Kn = 0.10	2.7425	2.9616	3.151	3.3168	3.4638
Kn = 0.12	2.5125	2.7354	2.9309	3.1043	3.2595

Table 15. The fully developed Nusselt number values, Br = 0.01, constant q at the wall

Br = 0.01	Pr = 0.6	Pr = 0.7	Pr = 0.8	Pr = 0.9	Pr = 1.0
Kn = 0.00	4.1825	4.1825	4.1825	4.1825	4.1825
Kn = 0.02	3.9186	4.0022	4.0684	4.1233	4.1705
Kn = 0.04	3.5962	3.7398	3.8563	3.9536	4.0371
Kn = 0.06	3.2776	3.4598	3.6113	3.7399	3.8514
Kn = 0.08	2.9863	3.1912	3.3651	3.5151	3.6466
Kn = 0.10	2.7285	2.945	3.1321	3.2957	3.4408
Kn = 0.12	2.5031	2.7242	2.9179	3.0896	3.2432

5. CONCLUDING REMARKS

In general, for above two boundary conditions, velocity slip and temperature jump affect the heat transfer in opposite ways: a large slip on the wall will increase the convection along the surface. On the other hand, a large temperature jump will decrease the heat transfer by reducing the temperature gradient at the wall. Therefore, neglecting temperature jump will result in the overestimation of the heat transfer coefficient.

Prandtl number is important, since it directly influences the magnitude of the temperature jump.

For linear wall temperature boundary conditions with considering temperature jump, the Nusselt number decreases as Kn increases and increases as Pr increases.

Nusselt number takes higher values for Br > 0 and lower values for Br < 0. It is due to the fact that positive Br means the fluid is cooled.

On the other hand, for linear wall temperature boundary conditions without considering temperature jump, as Kn increases, the fully developed Nusselt number increases. The Nusselt number takes higher values for Br >0 and lower values for Br < 0.

For the linear wall temperature boundary condition, the Nusselt number approached the value with uniform heat flux under the same conditions.

Prandtl number, the increase of which increases heat transfer, also plays an important part in this case with considering temperature jump.

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(*Continuare din pag. 4*)

În consecință, ceea ce se impune în momentul de față sunt următoarele direcții prioritare de cercetare:

a) Analiza critică și pe cât posibil obiectivă (nepărtinitoare, nepartizană) a celor trei direcții, menționate mai înainte, cu ilustrarea concretă a realizărilor și limitelor acestor metode de abordare a proceselor și interacțiunilor de neechilibru (ireversibile) din ciclurile mașinilor termice.b) Ilustrarea cât se poate de clară a contribuțiilor fundamentale la dezvoltarea termodinamicii ireversibile a acestor metode, a corelațiilor dintre aceste contribuții fundamentale și a posibilităților de unificare a lor. Se impune mai întâi o unificare a două câte două dintre aceste metode, și în perspectivă, ca obiectiv pretențios și ambițios (dar extrem de promițător și necesar în etapa actuală), de reunirea tuturor celor trei metode într-o *metodă unică și generală de abordare globală a proceselor și interacțiunilor de neechilibru* pentru toate tipurile de cicluri termodinamice ale mașinilor și instalațiilor termice (motoare, refrigeratoare, pompe de căldură ș.a.).

c) Particularizarea acestor *metode avansate*, prin aplicarea lor la cicluri și mașini concrete (ciclul Carnot, ciclul Otto, ciclul Stirling, ciclul Brayton, ciclul Ericson, ciclul Rankine, ciclurile refrigeratoarelor și pompelor de căldură), pentru a elucida și ilustra aspectele diferite pe de o parte și a celor similare pe de altă parte, care apar în aplicarea concretă a acestor metode.

d) Validarea *metodelor avansate* prin comparația performanțelor calculate cu performanțele mașinilor reale, pentru ca aceste metode să devină instrumente concrete și puternice de optimizare și proiectare în mâna cercetătorilor și a proiectanților de mașini termice cu performanțe avansate, desigur în vederea economisirii de energie (de fapt a economisirii de exergie).

e) Stabilirea rolului și locului *metodei entropice, metodei exergetice* și a *metodei directe* în acest tablou al *„**Metodelor avansate globale**"* de apreciere, evaluare, identificare, cuantificare, minimizare a pierderilor prin ireversibilitate din mașinile și instalațiile termice și optimizarea acestora.

Aceste direcții prioritare de cercetare în cadrul ***Termodinamicii ireversibile*** pot constitui obiective majore ce trebuie să fie în centrul atenției, în prezent și în viitorul apropiat, pentru *Școala de termodinamică românească*, care a avut și are contribuții remarcabile în acest domeniu.

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