

DETERMINATION OF THE DIPOLE MAGNETIC FIELD FOR RFID STRUCTURES

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REZUMAT. Studiul teoretic și practic al proceselor de interacțiune *reader-tag* reprezintă un subiect actual de interes al comunității științifice, începând cu fizicienii interesați de domeniul RF, continuând cu inginerii proiectanți de echipamente și sisteme RFID și cercetătorii în comunicațiile analogice și digitale de date, până la orice specialist într-unul din domeniile științifice moderne ce apelează la tehnologia RFID. Scopul articolului constă în ridicarea unor modele de câmp apropiat și îndepărtat, utile în proiectarea structurilor RFID, folosind tehnicile de abordare specifice potențialului vector și scalar.

Cuvinte cheie: system, RFID, electromagnetic field, potential vector, model

ABSTRACT. Theoretical and practical study of reader-tag interaction processes is a current topic of interest of the scientific community, from physicists interested in the RF, continuing with equipment and system design engineers and researchers RFID in analog and digital data communications, to any specialist in the areas of modern science that uses RFID technology. The purpose of article is lifting near and far field models, useful in designing RFID structures, using techniques of specific approach by vector and scalar potential.

Keywords: system, RFID, electromagnetic field, potential vector, model

1. INTRODUCTION

The theoretical and practical study of interaction processes RFID tag reader is a current topic of interest to the scientific community, from physicists interested in the RF, continuing with equipment and system design engineers and researchers RFID into analog and digital data communications, to any specialist in the areas of modern science that uses RFID technology.

If experiments provide data analysis for researchers, models created by experts in the field underlying the procedures for processing such data, a synthesis that provides formal mechanisms and mapping techniques to approach the analysis and/or synthesis of specific modern technologies.

A reader-tag interaction process implies a RFID tag identifier attached to an entity of identification and a tag reader. Depending on the passive or active tag, the entire identification system asks the researcher to use specific techniques to address specific issues raised by the case.

In what follows, we will present a theoretical approach of an RFID reader-tag system and will model the magnetic field structure from electromagnetic field generated, over a clear set of systemic conditions. The stated purpose of the study will be to lift a model for a well-defined situation, so that on this basis it will

be possible to achieve behavioral assumptions of the system when we make changing parameters.

2. BASICS

In any electromagnetic phenomenon model we begin, either implicitly or explicitly, from Maxwell's electrodynamics equations. The set of equations valid in the vacuum space (or with good approximation in the air) can be written [1]:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \quad (2.1)$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \quad (2.2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.3)$$

$$\nabla \times \mathbf{B} = (1/c^2)(\mathbf{j} / \epsilon_0 + \partial \mathbf{E} / \partial t) \quad (2.4)$$

Used notations have the following meanings:

- ∇ is the *nabla* operator (Hamilton's operator);
- $\nabla \cdot$ is the *divergence* operator;
- $\nabla \times$ is the *curl* operator;
- \mathbf{E} is the electric intensity vector;
- \mathbf{B} is the magnetic induction vector;
- \mathbf{j} is the current density vector;

- ρ is the electric charge density and this is considered source of electric field;
- ϵ_0 is the permittivity of the electric field for vacuum;
- c is the constant of light speed in vacuum.

Maxwell's four equations provide a local description, dependent of a point (spatial coordinates). Although we could use a global formalism, the present approach will appeal to the local equations for technical interpretation and easy results obtaining.

First, we'll do a brief characterization of the applicative framework of this set of four differential equations with partial derivatives, noting initially that they were systematized in theoretical and practical results of preceding researchers of Maxwell.

Maxwell's major contributions have consisted, as mentioned above, in the systematization and formalization of laws discovered by leading scientists such as Ampere, Faraday, Biot, Savart, Lenz, Gauss etc., and in the discovery of new laws such as displacement current effect (which appear in equation (2.4)). Finally, Maxwell made the theoretical discovery of the mechanism of occurrence and propagation of electromagnetic waves, which led to the establishment of the theoretical speed of propagation of the electromagnetic waves. This proved to be identical with the propagation speed of light in vacuum, and this suggested for the first time in history of science that light is only a particular form of electromagnetic wave.

Equation (2.1) identifies the sources of electric field with electric charges of the substance loaded. This law is a remarkable mathematical result known as Gauss theorem. It turns out that the Gauss theorem is the generalization of Coulomb's law. If these charges are canceled (talking electrically neutral substance) we notice that divergence for electric field strength becomes zero, with all the consequences arising from this. Equation (2.1) ensures the existence of the electric monopoly.

Equation (2.2) indicates that it is possible for an electric field to appear when the magnetic field varies in time. This result is a direct consequence of the observations made by Faraday into the electromagnetic induction theory.

Equation (2.3) can be interpreted as Gauss theorem of magnetism. It suggests the inexistence of the magnetic monopoly and special status of the magnetic field relative to the electric field. The theory of relativity has the merit of showing that what is commonly called magnetic field is actually just a relativistic effect of moving an electric field of charged

particles, highlighted by various observers placed in different reference systems (inertial or non-inertial).

Finally, equation (2.4) is a summary of the results of Ampère's observations plus an assumption made by the existence of so-called Maxwell's displacement current (the rigorous should say displacement current density).

It turns out that the set of Maxwell equations provide the formal frame of a non-contradiction and completeness description. Moreover, the Maxwell equations of classical electrodynamics show a very important feature: relativistic invariance. This shows that electrodynamics, unlike Newtonian mechanics, is not affected by Lorentz transformations in the transition from one inertial system to another.

The set of Maxwell equations can be reduced to two pairs of distinct equations in some special cases. Thus, if we accept the absence of a magnetic field in the considered space, from the four remain only the following two equations:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \quad (2.5)$$

$$\nabla \times \mathbf{E} = \mathbf{0} \quad (2.6)$$

The field of study in this case is the electrostatics.

Similarly, considering the lack of an electric field in the considered space, we get:

$$\nabla \cdot \mathbf{B} = 0 \quad (2.7)$$

$$\nabla \times \mathbf{B} = \mathbf{j} / c^2 / \epsilon_0 \quad (2.8)$$

Obviously, we are now in the magnetostatics area.

By separating the two formal representation of electrical and magnetically phenomena, we get a mathematical and physical simplicity of their approach, but loses the capability to analyze phenomena arising from the interaction of the two types of fields. In many cases, for some researchers, one descent of the complete formalism (electromagnetic) into the one simplified formalisms (electrostatic or magnetostatic) is preferred. Such an approach will be preferred below. Before doing so, however, we present another conclusion which can be deduced from Maxwell equations, a conclusion - as we shall see later - which allows us to simplify notations and suggests a more general approach to modeling reader-tag interaction into the area of study that interests us.

We will start from equation (2.6), for which we apply the curl operator for both members. We obtain:

$$\nabla \times (\nabla \times \mathbf{E}) = - \frac{\partial}{\partial t} (\nabla \times \mathbf{B}) \quad (2.9)$$

We can show that [2]:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \quad (2.10)$$

The first term of the right member is null because in vacuum the field's divergence has a null value. In consequence from (2.9) and (2.10) we obtain:

$$\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) = \nabla^2 \mathbf{E} \quad (2.11)$$

From (2.4) result:

$$c^2(\partial/\partial t) (\nabla \times \mathbf{B}) = \partial^2 \mathbf{E}/\partial t^2 \quad (2.12)$$

We have:

$$\nabla^2 \mathbf{E} = \frac{1}{c^2} (\partial^2 \mathbf{E}/\partial t^2) \quad (2.13)$$

Equation (2.13) is the three-dimensional wave equation, in the case of electromagnetic waves. It is found that these waves propagate with speed c , the speed of light in vacuum.

Maxwell built his equation (2.4) of the form:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \partial \mathbf{E}/\partial t \quad (2.14)$$

If we compare (2.4) with (2.14), we will be able to observe:

$$c^2 = 1/\mu_0/\epsilon_0 \quad (2.15)$$

where μ_0 is the magnetic permeability of vacuum.

The right side was replaced with the left side after Maxwell's equations system was developed.

As observed, the presence of the constant means that relativistic effects are involved into the interpretation of magnetic field.

3. SCALAR AND VECTOR POTENTIALS

According to equation (2.6), a rotor with null value involves the existence of one *rot(div)* operator:

$$\nabla \times \mathbf{E} = \mathbf{0} \Rightarrow \nabla \times (\nabla \phi) = \mathbf{0} \quad (3.1)$$

with \mathbf{E} being the electric field intensity and ϕ the scalar potential.

From (3.1) we obtained:

$$\mathbf{E} = -\nabla \phi \quad (3.2)$$

After integration (3.2) lead to:

$$\phi(P) = - \int_{P_0}^P \mathbf{E} \cdot d\mathbf{s} \quad (3.3)$$

The reference P_0 point is considered at the infinite. One equivalent expression for (3.3) is:

$$\phi(P) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho dV}{r} \quad (3.4)$$

where ρ stands for volumetric charge density which generated the potential. The potential in point P can be written as:

$$\phi(P) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \mathbf{e}}{r^2} \quad (3.5)$$

with \mathbf{p} as the electrical dipolar momentum and \mathbf{e} unit vector for the position vector for point P .

We mention that the dipolar electrical momentum is defined as:

$$\mathbf{p} = \sum (q_i \cdot \mathbf{d}_i) \quad (3.6)$$

One interesting observation is that:

$$\nabla \times (\nabla \phi) = \nabla \times (\nabla \phi') = \mathbf{0} \quad (3.7)$$

and this lead to:

$$\phi' = \phi + C \quad (3.8)$$

where C is a scalar constant. The result of the scalar potential is not determined in an unique mode but defined as a C scalar constant.

We know that the divergence of a curl is always zero. In consequence, in conformity with (2.3) equation, we can write:

$$\nabla \cdot \mathbf{B} = \nabla \cdot (\nabla \times \mathbf{A}) = \mathbf{0} \quad (3.9)$$

The \mathbf{A} is named the vector potential. In this case, the magnetic field's induction is:

$$\mathbf{B} = \nabla \times \mathbf{A} = \mathbf{0} \quad (3.10)$$

On components, equation (3.10) can be write:

$$B_x = (\nabla \times \mathbf{A})_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \quad (3.11.1)$$

$$B_y = (\nabla \times \mathbf{A})_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \quad (3.11.2)$$

$$B_z = (\nabla \times A)_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \quad (3.11.3)$$

But, just like the scalar potential, the vector potential is not uniquely defined. If we begin with (3.10), we can write:

$$B = \nabla \times A' = \nabla \times A \quad (3.12)$$

Result:

$$\nabla \times A' - \nabla \times A = \nabla \times (A' - A) = \mathbf{0} \quad (3.13)$$

In conclusion:

$$A' = A + \nabla \Psi \quad (3.14)$$

This shows that the vector potential is not uniquely defined, also. It is defined by a vector constant, noted as $\nabla \Psi$.

4. FARAWAY MAGNETIC FIELD CREATED BY A RECTANGLE FRAME

Consider a rectangle wire placed in a rectangular coordinate with axes system with particular orientation (Figure 1).

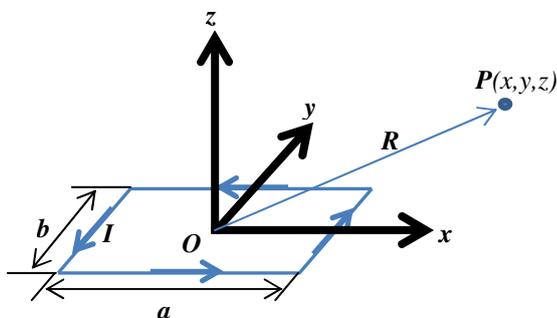


Fig. 1. Current rectangle wire

It notes that there is not current in the Oz direction, so that component A_z is zero. But there is current in the Ox direction along the two sides of length a and same in the Oy direction along the sides of length b . The corresponding components of vector potential are A_x and A_y concerned. Consider them in turn.

Since the current density is uniformly distributed, the A_x component is analogue to the electrostatic potential provided by two linear conductor segments with opponent charged, for example negative on the

upper segment and positive on the lower. In point P we can consider a dipolar potential:

$$\Phi = \frac{1}{4\pi\epsilon_0} \frac{p \cdot e}{R^2} \quad (4.1)$$

where p notes the dipolar momentum for charge distribution and e is the unity position vector for point P . The dipolar momentum is defined as product between the total charge from one segment and the distance between the two segments:

$$p = \lambda ab \quad (4.2)$$

Here, λ linear charge density.

We can observe that the dipolar momentum is oriented in the sense of $-Oy$ semi-axis, so that the cosine for angle between the direction of p and the direction of e is $-y/R$. We have:

$$\Phi = -\frac{1}{4\pi\epsilon_0} \frac{\lambda aby}{R^3} \quad (4.3)$$

The relation (2.1) can be written:

$$\nabla \cdot \nabla \Phi = -\rho/\epsilon_0$$

$$\nabla^2 \Phi = -\rho/\epsilon_0 \quad (*)$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

If we define A so that divergence to be zero, the relation above becomes:

$$\nabla \times B = -\nabla^2 A = j/c^2/\epsilon_0$$

or:

$$\nabla^2 A = -j/c^2/\epsilon_0 \quad (**)$$

Solution for (*) has the form (3.4). So, it is natural that the solution for (**) has the same form:

$$A(P) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{j dv}{r} \quad (***)$$

Now, we obtain A_x component by switching (4.3) λ with I/c^2 :

$$A_x = -\frac{1}{4\pi\epsilon_0 c^2} \frac{I aby}{R^3} \quad (4.4)$$

In same way, we find the component of vector potential on the Oy axe:

$$\mathbf{A}_y = \frac{1}{4\pi\epsilon_0 c^2} \frac{Iabx}{R^3} \quad (4.5)$$

Evidently:

$$\mathbf{A}_z = 0 \quad (4.6)$$

If we look up the last three equations we conclude that the vector potential, at large distance from wire frame, is configured as circular lines around the Oz axis, with orientation given by the sense of I in the wire frame. The \mathbf{A} vector magnitude is proportional with Iab (product called *magnetic dipole moment* or simply *magnetic moment*):

$$\mu = Iab \quad (4.7)$$

We are led to one theorem which affirms that the vector potential for a wire-frame, little and plane, with any form, is:

$$\mu = IS \quad (4.8)$$

with S the wire-frame value of surface.

The μ vector is normal to wire-frame plane.

The vector results are:

$$\mathbf{A} = \frac{1}{4\pi\epsilon_0 c^2} \frac{\mu \times \mathbf{R}}{R^3} = \frac{1}{4\pi\epsilon_0 c^2} \frac{\mu \times \mathbf{e}}{R^2} \quad (4.9)$$

The \mathbf{B} compute is made by the relations (4.9), (3.11.1), (3.11.2), (3.11.3) and lead to:

$$\mathbf{B}_x = -\frac{\mu}{4\pi\epsilon_0 c^2} \frac{\partial}{\partial z} \frac{x}{R^3} = \frac{\mu}{4\pi\epsilon_0 c^2} \frac{3xz}{R^5} \quad (4.10)$$

$$\mathbf{B}_y = -\frac{\mu}{4\pi\epsilon_0 c^2} \frac{\partial}{\partial z} \frac{y}{R^3} = \frac{\mu}{4\pi\epsilon_0 c^2} \frac{3yz}{R^5} \quad (4.11)$$

$$\begin{aligned} \mathbf{B}_z &= \frac{\mu}{4\pi\epsilon_0 c^2} \left(\frac{\partial}{\partial x} \frac{x}{R^3} + \frac{\partial}{\partial y} \frac{y}{R^3} \right) = \\ &= -\frac{\mu}{4\pi\epsilon_0 c^2} \left(\frac{1}{R^3} - \frac{3z^2}{R^5} \right) \end{aligned} \quad (4.12)$$

The equations set noted (4.10)÷(4.12) provide the magnetic induction components in points with x, y, z coordinates, and has as field source one plane rectangle wire-frame localized in the xOy plane. The utility of

those equations is proved at large distance (large in terms with geometrical dimensions of the wire-frame).

Next, we'll make some correction to the (4.10)÷(4.12) equations, so that we can obtain the best model for the near field.

5. THE NEAR MAGNETIC FIELD OF THE RECTANGLE FRAME

We start from the magnetic field computed around a long wire conductor (figure 2).

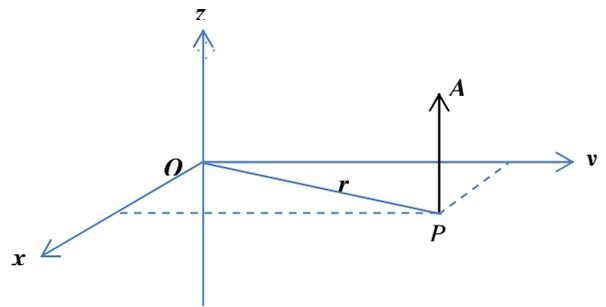


Fig. 2. Long conductor current-carrying

In this configuration, the \mathbf{j} current density vector has only non-zero z component. The value for that is provided by next relation:

$$\mathbf{j}_z = \frac{I}{\pi\rho^2} \quad (5.1)$$

and is defined into the conductor.

Because the x and the y components are nulls, the result:

$$\mathbf{A}_x = 0 \quad (5.2.1)$$

$$\mathbf{A}_y = 0 \quad (5.2.2)$$

We'll compute the \mathbf{A}_z component. For that we'll use the \emptyset potential for one conductor wire which has a uniform charge density $\rho = \frac{j_x}{c^2}$. For points from the next area of the conductor (to ensure conditions of near field) we can consider the conductor as one infinite length. In this case, the potential is:

$$\phi = -\frac{\sigma}{2\pi\epsilon_0} \ln r \quad (5.3)$$

with $r = \sqrt{x^2 + y^2}$. The linear charge density was be noted σ .

In this conditions, we have:

$$A_x = -\frac{\pi\rho^2 j_z}{2\pi\epsilon_0 c^2} \ln r \quad (5.4)$$

Since $\pi\rho^2 j_x = I$, the relation (5.4) is wrote:

$$A_x = -\frac{I}{2\pi\epsilon_0 c^2} \ln r \quad (5.5)$$

Using (3.11.1) and (3.11.2), we obtain:

$$B_x = -\frac{I}{2\pi\epsilon_0 c^2} \frac{y}{r^2} \quad (5.6)$$

$$B_y = \frac{I}{2\pi\epsilon_0 c^2} \frac{x}{r^2} \quad (5.7)$$

$$B_z = 0 \quad (5.8)$$

The value of the \mathbf{B} vector is:

$$\mathbf{B} = \frac{I}{2\pi\epsilon_0 c^2 r} \quad (5.9)$$

The magnetic field lines look like concentric circles in the Oxy plane, with the center on the conductor axe.

Considering the prismatic volume built on the wire-frame contour, we'll compute the magnetic field in points from its sections with parallel planes with wire-frame plane.

The current from all four segments of the frame generate four vectors \mathbf{B}_k in the P point. These vectors are oriented in same direction. The sum of these vectors conduct to a \mathbf{B} result vector. Its direction is normal for a wire-frame plane.

The s_k distances are involved in (5.9) formula on the r variable position. The four values are:

$$s_1 = \sqrt{z^2 + \left(\frac{a}{2} + y\right)^2} \quad (5.10.1)$$

$$s_2 = \sqrt{z^2 + \left(\frac{b}{2} + x\right)^2} \quad (5.10.2)$$

$$s_3 = \sqrt{z^2 + \left(\frac{a}{2} - y\right)^2} \quad (5.10.3)$$

$$s_4 = \sqrt{z^2 + \left(\frac{b}{2} - x\right)^2} \quad (5.10.4)$$

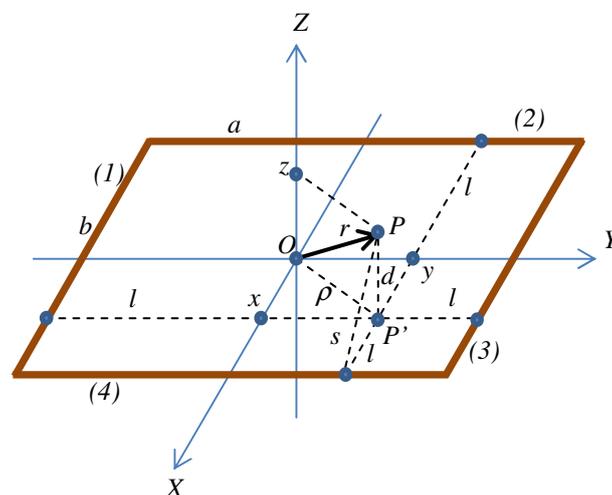


Fig. 3. Defining P points to calculate field

The magnetic induction in the point P is:

$$\mathbf{B} = \frac{I}{2\pi\epsilon_0 c^2} \left[\frac{1}{\sqrt{z^2 + \left(\frac{a}{2} + y\right)^2}} + \frac{1}{\sqrt{z^2 + \left(\frac{b}{2} + x\right)^2}} + \frac{1}{\sqrt{z^2 + \left(\frac{a}{2} - y\right)^2}} + \frac{1}{\sqrt{z^2 + \left(\frac{b}{2} - x\right)^2}} \right] \quad (5.11)$$

In the origin of the reference system we find:

$$\mathbf{B} = \frac{2I}{\pi\epsilon_0 c^2} \frac{a+b}{ab} \quad (5.12)$$

At the z distance from origin, on the Oz axe, the magnetic induction becomes:

$$\mathbf{B} = \frac{I}{\pi\epsilon_0 c^2} \frac{\sqrt{z^2 + \left(\frac{a}{2}\right)^2} + \sqrt{z^2 + \left(\frac{b}{2}\right)^2}}{\sqrt{\left(z^2 + \left(\frac{a}{2}\right)^2\right)\left(z^2 + \left(\frac{b}{2}\right)^2\right)}} \quad (5.13)$$

We can observe that the magnetic induction vectors are computed exactly in the wire-frame plane, but when the z coordinates increase we notice that the errors in computing increase also. In conclusion, the model is one near field kind, precise enough for the $z \approx \sqrt{a \cdot b}$ distance.

6. SIMULATIONS

The following example is build on the below data:

- $a = 20\text{cm}$
- $b = 10\text{cm}$
- $z = 10\text{cm}$
- $I = 100\text{mA}$

The wire-frame is positioned into the xOy plane. The model used is of near field and it uses the relation (5.11).

The mesh field diagram is represented in term with xOy plane, at $z=10\text{cm}$ distance. This diagram is showed in figure 4.

The colors from the diagram correspond with the cotes of z variable. For higher cotes there are smaller color frequencies.

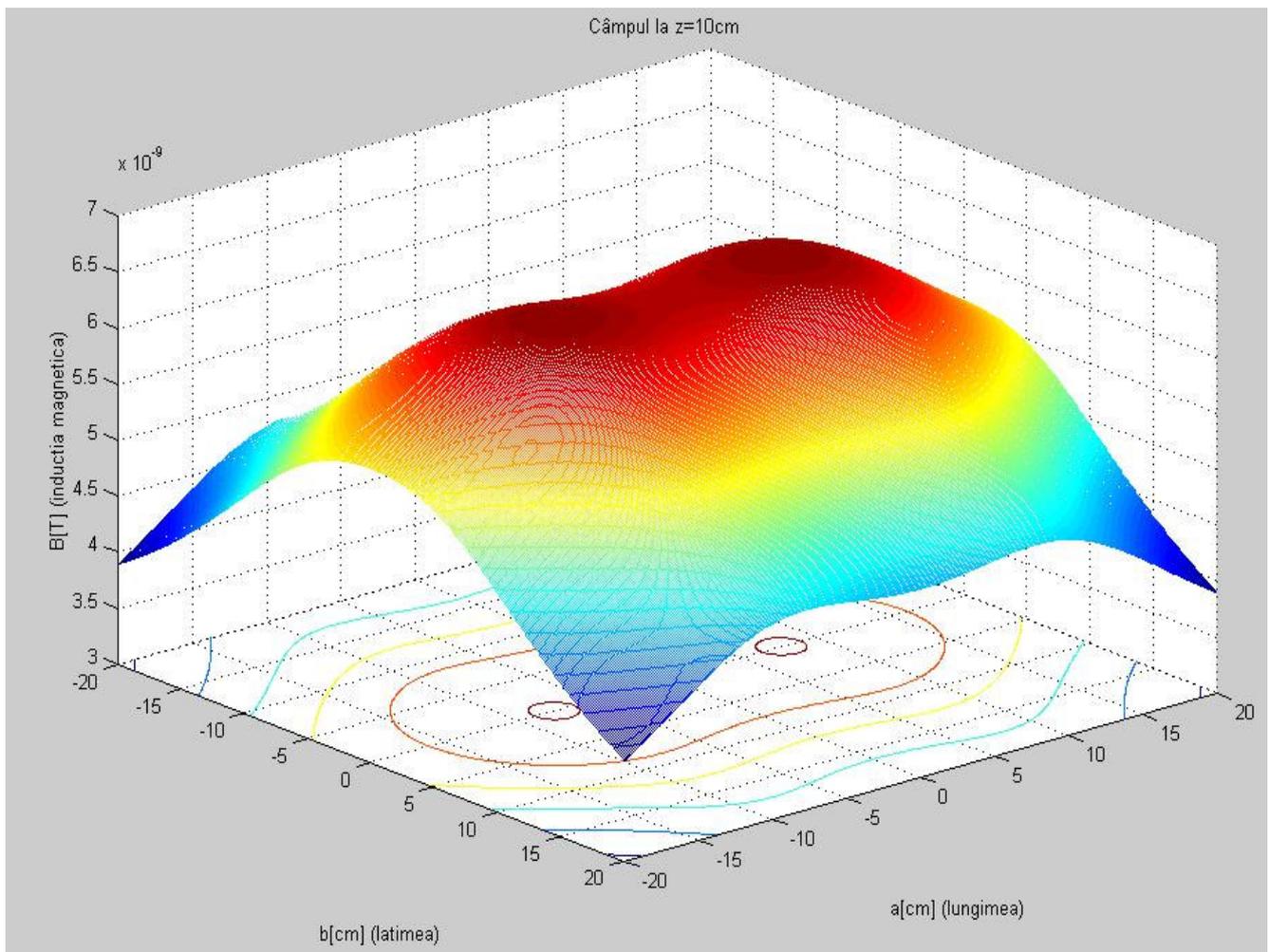


Fig. 4. Meshgrid diagram for B field at $z=10\text{cm}$

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