MODELING OF CONSTANT VOLUME COMBUSTION OF PROPELLANTS FOR ARTILLERY WEAPONS

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Abstract: Combustion of artillery propellants is a very complex physical and chemical process which depends on a large number of parameters. This paper presents a mathematical model consisting of the Nobel-Abel equation of state, the energy conservation equation, the equation of conservation of mass and burning rate law. The system of equations associated with this mathematical model is solved numerically using a computer program. Finally it presents theoretical results obtained with this program for M30 triple-base seven perforated granular propellant, compared with experimentally results measured using closed vessel technique.

Keywords: combustion, solid propellant, closed vessel, mathematical model, computer program.

1. INTRODUCTION

During combustion the propellant gradually turns into products, mostly in the gas phase. The amount of propellant that turns into gases per unit time, called gas formation rate, depends on the nature of the powder, the shape and size of the propellant and the pressure at which combustion occurs, increasing with increasing pressure. The heat released by propellant combustion, heats resulting gases, raising their temperature. The increase of the temperature and the amount of gases released during propellant combustion at constant volume, leads to the increase of the gas pressure, which in turn increases the rate of gas formation. Therefore, the propellant combustion at constant volume is a self-accelerating non-stationary process. During combustion, a small part of the reaction heat is lost through the walls of the closed vessel. Due to the complexity of the combustion process, the evaluation of these losses is mainly experimentally.

For the combustion process modeling the following assumptions are done, which together are known as geometric combustion law [1]:

– the propellant elements forming the load are identically in terms of size and physic and chemical properties;

– the propellant elements ignition occurs instantaneously when using a special igniter;

– the combustion takes place at constant speed on all propellant elements surfaces on the normal to these surfaces (combustion takes place in parallel layers);

In addition, when assessing the amount of the heat transferred during combustion to the closed vessel walls, it is considered that it is proportional to the pressure [2].

The variables that describe the propellant combustion process versus time on the bases of these assumptions are: propellant mass, pressure and gas temperature, thickness of burned powder, combustion surface area and combustion rate. These variables can be derived analytically for simple shaped elements (sphere, cylinder, tube), or evaluated numerically for complicated shaped elements (e.g. multi-perforated).

The analytical derivation of the propellant gas pressure versus time at the constant volume is presented in detail in [1]. In this case, for the variables simplification and separation, the following approximations are made: burning surface area is constant and equal to its average value; burning rate linearly varies with the gas pressure and heat transfer losses are neglected.

Further the numerical evaluation of the variables that describe the combustion process is presented which, unlike analytical calculation does not require making any simplifying approximations and can be applied for any form of the propellant elements.

2. THE MATHEMATICAL MODEL

It is considered a closed vessel having interior volume W_0 [m^3] in which was introduced a propellant load with an initial mass of ω_0 [kg] and a special igniter (mass ω_a [kg]). All propellant

elements are instantly ignited and start to burn at time t = 0 s, after the igniter complete its combustion. During combustion inside the vessel is a two-phase mixture of gases (gas from propellant combustion predominantly plus a small amount of gas produced by the burning of the igniter and the air present inside the vessel before combustion starts) and solid unburned propellant. Burning ends at the moment $t = t_k$, when the unburned propellant mass is equal to zero.

Let consider $\omega(t)$ [kg] the unburned propellant mass at a time t, $0 < t < t_k$.

The gas mixture pressure and temperature, are P [Pa] and respectively T [K], depend on the unburned propellant mass ω [kg]. The functions $p(\omega)$ and $T(\omega)$ describing this dependence is determined by solving the system formed using Nobel-Abel equation of state and the energetic balance equation:

$$\begin{cases} p\left[W_0 - (\omega_0 - \omega(t))\alpha - \omega_a\alpha_a - \omega_{aer}\alpha_{aer} - \omega(t)/\delta\right] = \\ = \left[(\omega_0 - \omega(t))R + \omega_a R_a + \omega_{aer}R_{aer}\right]T \\ (\omega_0 - \omega(t))c_v T_f + \omega_a c_{v,a} T_{f,a} = \\ = \left[(\omega_0 - \omega(t))c_v + \omega_a c_{v,a} + \omega_{aer}c_{v,aer}\right]T + Q_{\max,c}\dfrac{p}{p_{\max,c}} \end{cases} \quad (1)$$

where: α [m^3/kg], T_f [K], c_v [J/(kg K)] and R [J/(kg K)] are covolume, adiabatic flame temperature, specific heat and propellant gas constant respectively, ω_a [kg], α_a [m^3/kg], T_f [K], $c_{v,a}$ [J/ (kg K)] and R_a [J/(kg K)] – mass, covolume adiabatic flame temperature, specific heat and igniter gas constant respectively, ω_{air} [kg], α_{air} [m^3/kg], $c_{v,air}$ [J/(kg K)] and R_{air} [J/(kg K)] – mass, covolume specific heat and air constant respectively, $Q_{max,c}$ [J] – the maximum amount of heat lost through closed vessel walls and $p_{max,c}$ [Pa] – maximum gas pressure experimentally measured in the closed vessel exactly in the conditions of the problem. The maximum amount of heat lost is calculated solving the system (1) for the end of combustion, when $\omega(t_k) = 0$ and $p = p_{max,c}$.

The thickness of the burned propellant layer e[m] depends on the unburned propellant mass $\omega(t)$ and is determined by solving the equation

$$\omega(t) = \delta\, n\, V(e), \quad (2)$$

where: δ [kg/m^3] is the propellant density, n – the number of elements forming the propellant load; $V(e)$ [m^3] – the volume of one propellant element. The analytical derivation of $e(\omega)$ function is possible for simple shaped propellant elements only. For elements with more complicated shapes (cylinder or prism with multiple perforations), equation (2) is solved numerically.

The burning surface area S[m^2] depends on the thickness of the burned propellant layer $e(\omega)$, and is calculated using the relation

$$S(e) = n\, A(e), \quad (3)$$

where $A(e)$ is the total area of one propellant element. The functions which describe the variation of the surface area and the volume of one propellant element with the thickness of the burned propellant layer e for different shaped propellant elements are given in the literature [2], [3].

After an infinitely small time dt, the unburned propellant mass inside the closed vessel will decrease by the amount

$$d\omega = \delta\, S(e)\, u(p)\, dt, \quad (4)$$

where $u(p)$ [m/s] is the propellant linear burning rate, which is defined as the speed of the combustion surface in the direction of the normal to itself. The linear burning rate dependence on the combustion pressure, called the combustion rate law, is expressed by the formula

$$u(p) = u_1 p^{\nu}, \quad (5)$$

where: u_1 [m/(Pa s)] is the characteristic burning velocity; ν [-] – the pressure exponent.

Equations (1) - (5) allow for computing the next six unknown variables: the mass of the unburned propellant $\omega(t)$, the pressure $p(\omega)$ and the temperature $T(\omega)$ of the gas mixture, the thickness of the burned propellant layer $e(\omega)$, the surface combustion area $S(e)$ and the linear burning rate $u(p)$. At the beginning of the combustion, when t = 0, these values are: $\omega(0) = \omega_0$, $p(\omega_0) = p_0$, $T(\omega_0) = T_0$, $e(\omega_0) = 0$, $S(0) = S_0$, and $u(p_0) = u_0$.

3 COMPUTER PROGRAM

A computer program has been written based on the above described mathematical model. Initial data necessarily for running the program are written in advance by the user in an input file in the order shown in Table 1. Symbols from no. 3 to 6 of Table 1 represents the propellant element sizes (L – length, D – outer diameter, d – perforations diameter and e_k – web thickness), the remaining symbols having meaning given in the description of the mathematical model.

After reading the initial data input file, the computer code calculates the following constant values: initial area, volume and mass of the propellant element, the total number of propellant elements, air mass inside the closed vessel and the maximum amount of heat lost.

The unknown variables calculation are done in the points i = 1,2, ..., at equidistant moments of time, where Δt is the time step. At every point i, the unknowns variables are determined by successive iterations until the relative error of gas pressure calculation becomes smaller than a certain value ε_r required.

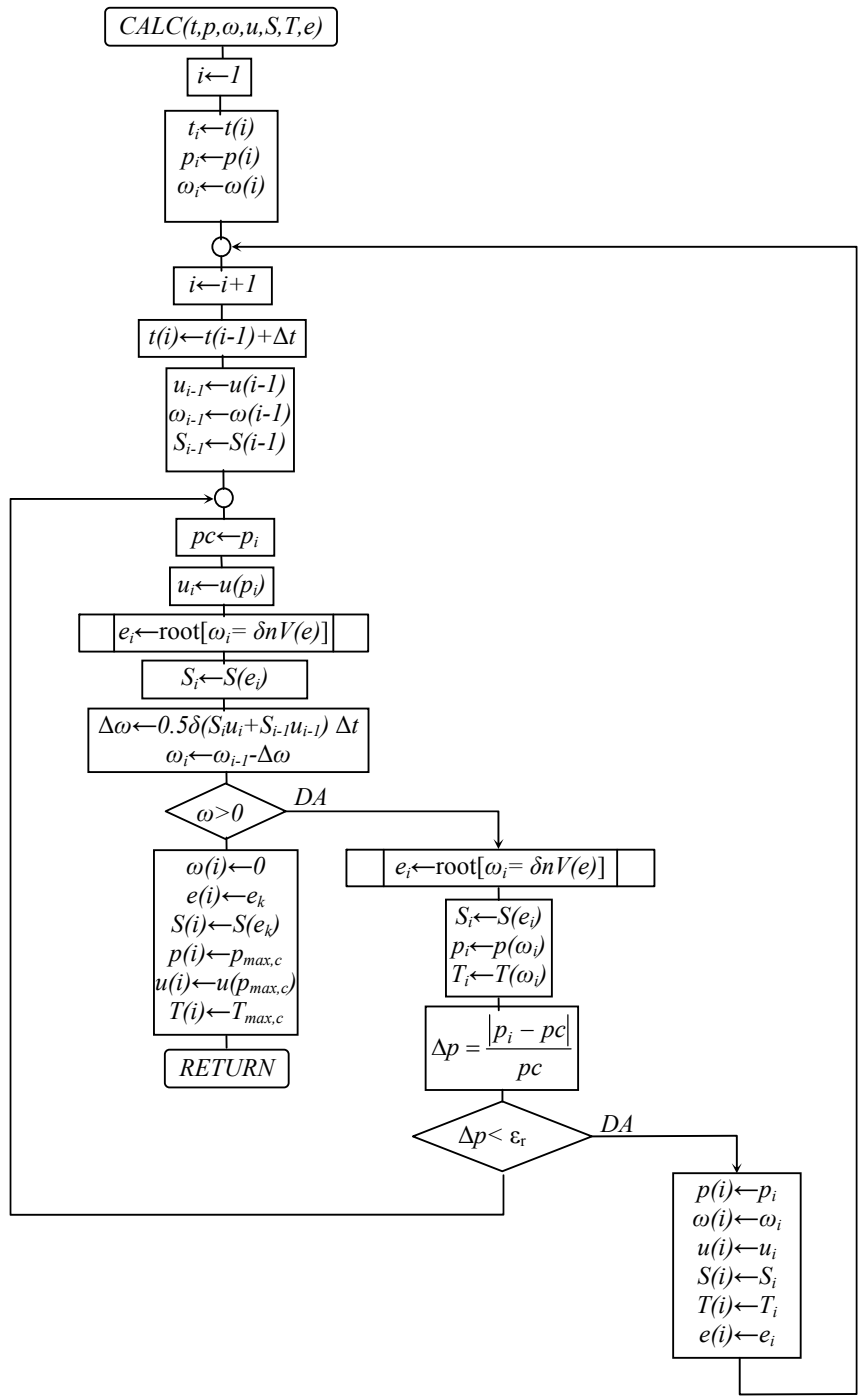

Fig. 1. CALC subroutine flowchart.

Table 1

Initial data for M30 propellant combustion modeling

No.	Symbol	MU	Value
1	W_0	cm^3	200
2	ω_0	g	40
3	L	mm	17.6
4	D	mm	6.8
5	d	mm	0.5
6	e_k	mm	0.99
7	T_f	K	3013
8	R	J/(g K)	0.358
9	c_v	J/(g K)	1.478
10	α	cm^3/g	1.048
11	δ	g/cm^3	1.66
12	u_1	cm/(s MPa)	0.464
13	ν	-	0.629

No.	Symbol	MU	Value
14	ω_a	g	1
15	$T_{f,a}$	K	2188
16	R_a	J/(g K)	0.125
17	$c_{v,a}$	J/(g K)	0.574
18	α_a	cm^3/g	0.785
19	δ_a	g/cm^3	1.75
20	R_{aer}	J/(g K)	0.288
21	$c_{v,aer}$	J/(g K)	0.719
22	α_{aer}	cm^3/g	0.98
23	δ_{aer}	g/cm^3	0.0012
24	Δt	s	0.0001
25	ε_r	-	10^{-5}
26	$p_{max,c}$	MPa	255.6

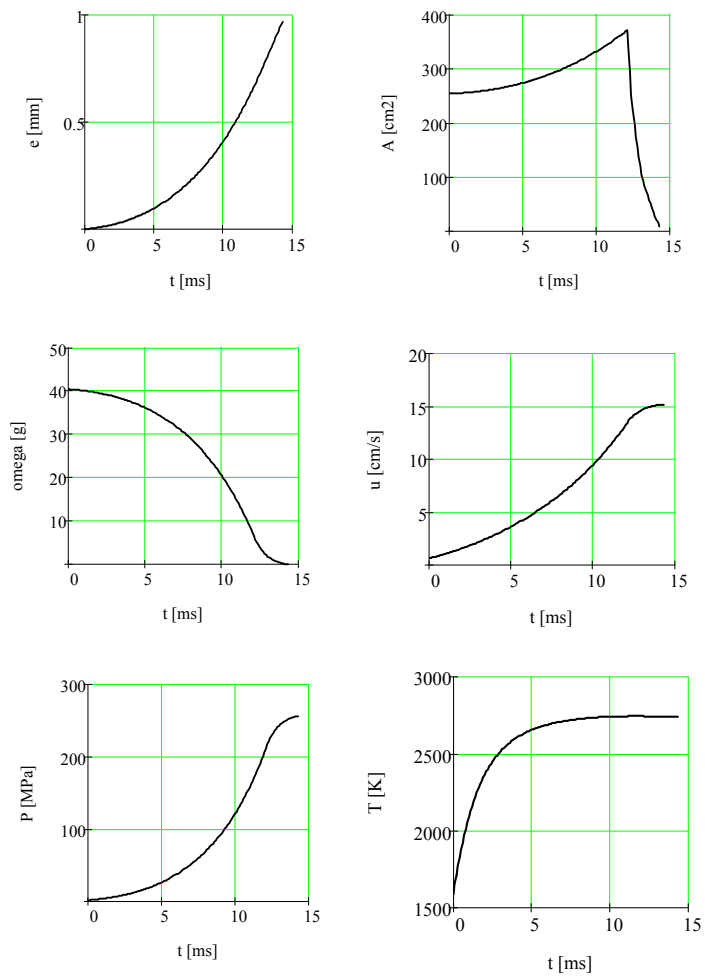

Fig. 2. The variation of the values which describe the M30 propellant combustion versus time at the 0,2 g/cm^3 loading density.

After reaching the required precision, the unknown variables are assigned to indexed variables (vectors), and then proceed to the next calculation. Integration stops when the unburned propellant mass inside the closed vessel is less than or equal to zero. These calculations are performed using the subroutine CALC, whose flowchart is shown in figure 1.

Finally, the vectors containing the values t, ω, u, S, T and E are transferred to the main program, where they are written to an output file for printing or graphics.

3. RESULTS AND CONCLUSIONS

In order to verify the mathematical model and for computing program validation, one experimental firing using M30 triple-base propellant was performed at 0.2 g/cm^3 loading density, in the experimental installation with 200 cm^3 closed vessel [4]. The series of experimental data was processed automatically by the computer program [5], resulting the propellant gases pressure variation versus time. The propellant was completely burned after $t_{k,c}$ = 18.7 ms, when into the closed vessel the pressure reached the maximum value $p_{max,c}$ = = 255.6 MPa. The initial data used for M30 propellant burning modeling are summarized in Table 1.

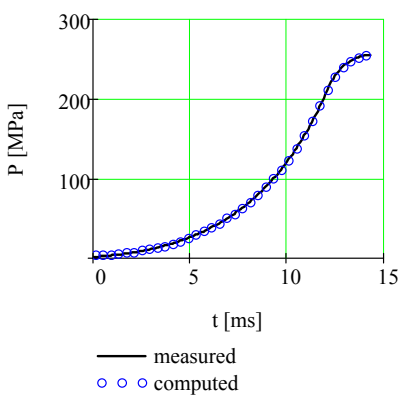

Fig. 3. The M30 propellant gas pressure versus time at the 0,2 g/cm^3 loading density.

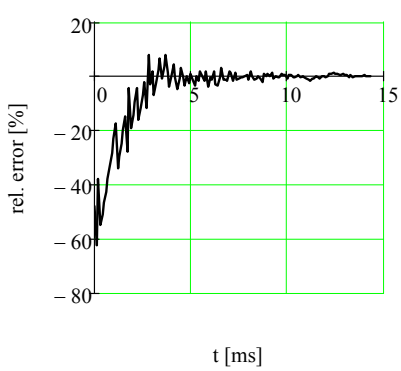

Fig. 4. The variation of relative error of calculated gas pressure versus time.

The following are the results based on these initial data. The maximum amount of the heat lost through the walls of the bomb is $Q_{max,\,c}$ = 11.52 kJ, that is 6.2% of the total heat released during the combustion. The values which described the M30 propellant burning versus time are shown graphically in Figure. 2. The M30 propellant thermo chemical constants were calculated using Corner method [6] and those of black propellant used for igniter were taken from [7]. The M30 propellant burning rate law constants have been determined from the experimental data for the pressure range of 60-190 MPa.

The propellant completely burns after t_k = 14.4 ms, with 4.3 ms less than the time experimentally measured. In order to compare the calculated gas pressure curve with the measured one, the last was translated with this difference, so that the maximum pressure points on the two curves should coincide. From the Figure 3 it is observed that the calculated pressure is similar to the measured pressure. To assess the accuracy of the calculation, the relative error of the pressure versus time was computed. From the Figure 4 it is seen that this error is less than 2% for the times periods greater than 7.5 ms (namely at the pressures greater than 60 MPa, for which the combustion law coefficients were calculated).

In order to obtain good results at lower pressures, it is necessary to modify the computer program in such way to allow the burning rate law defining on two or three pressure sub-ranges.

The presented mathematical model may be useful for development of new interior ballistic models of artillery guns.

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