A new MFCVSSEWOAPAECOR (Memory Fast Convergence Variable Step Size Exponentially Weighted Affine Projection Algorithm with Error Autocorrelation) algorithm used in smart antenna networks

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Abstract. In this paper a new adaptive algorithm used for beamforming in smart antenna systems is described [8]. The proposed algorithm is based on the APA (Affine Projection Algorithm) algorithm. Also a new method for the computation of the projection matrix using superior order projection vectors is proposed. The vectors length is higher than the projection order and they are made up of elements of the previous vector and the next sample of the input signal. The proposed algorithm is tested for the alpha-stable noise distribution present on the communication channel and the results obtained are superior to those obtained using classic algorithms like LMS (Least Mean Squares), VSSLMS (Variable Step Size LMS), RLS (Recursive Least Squares) or GAMSAPA (Gradient Adaptive Matrix Step Size APA).

Keywords: adaptive algorithms, smart antenna systems, the APA algorithm, the “over - sampling” method.

1. INTRODUCTION

Smart antennas systems are becoming one of the most interesting solutions in modern wireless communication systems today. The physical limita-

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This is called beamforming and the access technique SDMA (Space Division Multiple Access). Combined with one of the other classical access techniques like FDMA, TDMA or CDMA the technology becomes viable for integration with today’s wireless communication networks, like 3G, 4G, Wi-Fi and LTE. The most evolved smart antenna systems are the adaptive antenna systems which use complex mathematical algorithms capable of intercepting and tracking a certain signal. These algorithms have been long studied. In this paper an algorithm for which good results are obtained in environments with alpha – stable noise distributions is proposed. These distributions describe the noise in urban areas better than the Gaussian distribution [1].

The remainder of this paper is structured as follows. Section 2 is a short introduction in smart antenna systems. Section 3 presents the mathematical equations that describe the proposed algorithm. Section 4 presents the results obtained from simulations for the proposed algorithm MFCVSSEWOAPAECOR for alpha – stable noise distributions in comparison with the results obtained for the LMS, RLS, VSSLMS and GAMSAPA algorithms in the same conditions. The conclusions are stated in section 5.

2. SMART ANTENNA SYSTEM

The most comprehensive definition for a smart antenna system classifies them in three categories [2]:
- the switched beam antenna systems;
- the phased array antenna systems;
- the adaptive antenna systems.

The first two categories are easier to implement. We are talking about predefined radiation patterns for the first category and the possibility to choose between certain patterns using the received power criteria. In case of the second system we are talking about DoA (Direction of Arrival) algorithms combined with a phase delay procedure in order to steer the main lobe of the radiation pattern in a certain direction.

The third category includes the most complex of these algorithms. The adaptive antenna systems use an antenna array and a signal processing unit that is capable of pointing the main lobe in a certain desired direction and suppress interfering signals coming from other directions. This is done by multiplying the signals received by each antenna element with complex weights computed with an adaptive algorithm ran by the processing unit.

An adaptive antenna system is depicted in figure 1.

Fig. 1. Adaptive antenna system.
The functionality of a smart antenna system can be seen in this figure. The antenna array, the desired and interfering signals, the complex weights and the processing unit that runs the adaptive algorithm are depicted in the figure. The elements of the array are equally spaced. The adaptive algorithms split into two main categories, the algorithms that use a reference signal in order to converge and the adaptive algorithms that do not use a reference signal in order to converge. The algorithm proposed in this paper uses a reference signal in order to reach the adaptation stage. The adaptive antenna array works on the following principle: considering a signal source in a certain direction, in the far field, the signal that reaches the antenna array takes the shape of a front wave. The front wave first reaches one of the elements and after that it reaches the other elements one by one with a certain time delay. This time delay translates into phase difference. The adaptive algorithms process this phase difference in order to steer the main lobe in a desired direction and place nulls in the radiation pattern in the directions of interfering signals. The processing implies multiplying received signals with complex weights. These weights are computed by minimizing the adaptation error, actually the mean square error between the received signal and the reference signal. The first algorithms developed on this principle are the LMS and the RLS algorithms. The APA class of algorithms is newer and benefits of improved performance. The proposed algorithm uses the projection principle from the APA algorithm.

3. THE MFCVSSEWOAPAECOR ALGORITHM

A new adaptive algorithm developed on the backbone of the APA algorithm is proposed. The APA algorithm offers good convergence speed and good stability for the Gaussian noise distribution [3]. The convergence speed and the stability are the most important parameters that have to considered when designing an adaptive algorithm. For the proposed algorithm a speed convergence module [4] is used, based on the series acceleration method proposed by Aitken. Also the variable step size version [6] of the exponentially weighted APA algorithm [3] is used. For stability the error autocorrelation method [5] is used. The sliding window principle [7] also improves the algorithm’s speed. This principle implies the computation of the weights in two stages separated by a number of iterations equal to the size of the sliding window. The speed convergence module is inserted between these two stages. In this way a good stability is achieved along with the increase in speed of convergence. A new method of constructing the projection vectors is proposed, that works by storing sequential samples of the input signal in a buffer with a length higher than the projection order. This way the projection matrix is computed by using vectors with dimensions higher than the number of the array elements. A high redundancy is achieved by considering two consecutive vectors that differ only by one element, the next sample of the input signal. We called this “over-sampling”. The rank of the projection matrix is higher but the algorithm converges in a significant smaller number of iterations. The proposed “over-sampling” method makes the algorithm suitable for use in environments with alpha-stable noise distributions. In the following the equations of the algorithm are enounced. Scalars are described by lower case letters, vector by bold lower case letters and matrixes by bold upper case letters.

\[
d_p(k) = [d(n) \cdot d(n-1) \ldots d(n-P)]^T \quad (1)
\]

\[
X_p(k) = [x(k) \cdot x(k-1) \ldots x(k-P)]^T \quad (2)
\]

\[
e_p(k) = d_p(k) - X_p^*(k)w^*(k) \quad (3)
\]

\[
P(k) = \lambda^{-1}_p[P(k-1) - g(k)U^T(k)P(k-1)] \quad (4)
\]
The equations above describe the algorithm without the sliding window method applied. The sliding window variant implies the computations of the weights in (11) again after resolving (16) for the \(k + N\) \((N = \text{the sliding window size})\) iteration and than running the algorithm form start continuing with the \(k + 1\) iteration, next the \(k + N + 1\) and so on. In this way the functionality of the speed convergence module is constrained into a loop between two sequences of the modified APA algorithm in concordance with the proposed “over-sampling” method and the other two methods, the error autocorrelation and the variable step size.

4. RESULTS

The proposed algorithm is studied considering the alpha-stable noise distribution. This type of distribution describes the noise in urban areas better than the gaussian distribution. It is a heavy tailed distribution suited for simulating algorithms in environments where short duration, high amplitude noises are present. The equation describing the distribution is:

\[
\phi(\omega) = \exp(-\gamma |\omega|) ^ \alpha \left(1 - i \beta \text{sgn}(\omega) \tan \left(\frac{\pi \omega}{2}\right) + i \delta \omega\right)
\]

where: \(\alpha\) is the characteristic exponent restricted to values between \(0 < \alpha \leq 2\) , \(\beta \in [-1,1]\) is a parameter that tells if the distribution is shifted to the left or to the right, \(\delta\) is also a location parameter with \(-\infty < \delta < \infty\) , and \(\gamma < 0\) is the dispersion of the distribution. The dispersion parameter determines the spread of the dispersion around the location parameter.

The LMS algorithm, the RLS algorithm, the GAMSAPA algorithm and the proposed MFCVSSEWOAPAECOR algorithm are simulated for the alpha-stable noise distribution. The results show that the algorithm becomes unstable when the SNR (Signal to Noise Ratio)
A new MFCVSEWOAPAECOR algorithm used in smart antenna networks decreases. Although stable for a high SNR, the precision in combating interference is not that good and the algorithm takes a long time to reach the convergence stage. In the figures above with a green color line is depicted the direction of the desired signal source and with red color lines are depicted the directions for the sources of interference. A relatively lower value for the step size is chosen in simulations, $\mu = 0.005$. Choosing a higher value for the step size increases the speed of convergence for a higher SNR. This is not the case for a lower SNR when the algorithm becomes divergent.

Fig. 3:
- $a$ – the Radiation pattern in the Cartesian coordinates system;
- $b$ – the adaptation error for the LMS algorithm, in case of: a SNR = 20dB, a step size $\mu = 0.005$, a cycle of 100 iterations and the alpha-stable distribution parameters: $\alpha = 1.85$, $\beta = -0.7$, $\delta = -3.5$, $\gamma = 3.5$.

Fig. 4:
- $a$ – the Radiation pattern in the Cartesian coordinates system;
- $b$ – the adaptation error for the LMS algorithm, in case of: a SNR = 0dB, a step size $\mu = 0.005$, a cycle of 100 iterations and the alpha-stable distribution parameters: $\alpha = 1.85$, $\beta = -0.7$, $\delta = -3.5$, $\gamma = 3.5$. 

TELECOMUNICAȚII • Anul LV, nr. 1/2012 57
In environments with alpha-stable noise distributions, the RLS algorithm obtains better results. The RLS's speed of convergence is better than the LMS's for high or low SNR's. The RLS algorithm has though better chances than the LMS to become divergent. In many of the cases the amplitude of the secondary lobes is higher compared to the case of the LMS algorithm. The precision with which nulls are created in the radiation pattern on the direction of the interference signals is higher compared to the LMS for any SNR.

The variable step size variants of the LMS algorithm obtain better performance in case of high SNR than the standard LMS algorithm but the same problem appears as in the case of the RLS when considering a lower SNR for an alpha-stable noise distribution. These algorithms can easily become divergent.
A new MFCVSSEWOAPAECOR algorithm used in smart antenna networks

The results obtained by the GAMSAPA algorithm considering the case of gaussian noise distribution on the channel are very good. In case of the alpha – stable distribution the performance is still good for high SNR. When the SNR drops, the performance is similar with the one obtained in case of the RLS algorithm in the same conditions. The performance is improved by diminishing the value for the upper limit of the elements of the gradient adaptive matrix step size. A value for the upper limit of $\mu = 0.009$ is considered in the case depicted in figure 8. The algorithm converges slower, the nulls on the directions of the signals of interference are not so precise but the amplitudes of the secondary lobes are lower. The fact that the nulls are not so well placed in the radiation pattern...
is noticeable in figure 8 b). For a better control over the error the error autocorrelation method is used.

The algorithm retains its good performance obtained for a high SNR even for a low SNR. This makes the MFCVSSEWOAPAECOR algorithm a robust algorithm suited for use in very harsh environmental conditions.

Analyzing the results in table 1 it is obvious that the proposed algorithm converges in a small number of iterations compared to the other studied algorithms and the time in which it converges is also very short. The algorithm is of course more complex and uses more mathematical equations, but it is more stable.

Fig. 9:

*a* – the Radiation pattern in the Cartesian coordinates system; *b* – the adaptation error for the MFCVSSEWOAPAECOR algorithm, in case of: a 80 iteration cycle, a SNR = 20dB, a projection order of $P=20$, a variable step size constrained to the interval: $0.009 < \mu < 0.3$, the regularization factors: $\lambda_R = 0.9995$ and $\lambda_A = 0.05$, the regularization parameter $\Delta = 10$, the functional parameters for the computation of the optimum variable step size: $\alpha = 0.0005$, $\gamma = 0.005$, $\beta = 0.05$ and the alpha-stable distribution parameters: $\alpha = 1.85$, $\beta = –0.7$, $\delta = –3.5$, $\gamma = 3.5$.

Fig. 10:

*a* – the Radiation pattern in the Cartesian coordinates system; *b* – the adaptation error for the MFCVSSEWOAPAECOR algorithm, in case of: a 80 iteration cycle, a $\text{SNR} = 0$ dB, a projection order of $P=20$, a variable step size constrained to the interval: $0.009 < \mu < 0.3$, the regularization factors: $\lambda_R = 0.9995$ and $\lambda_A = 0.05$, the regularization parameter $\Delta = 10$, the functional parameters for the computation of the optimum variable step size: $\alpha = 0.0005$, $\gamma = 0.005$, $\beta = 0.05$ and the alpha-stable distribution parameters: $\alpha = 1.85$, $\beta = –0.7$, $\delta = –3.5$, $\gamma = 3.5$. 
A new MFCVSSEWOAPAECOR algorithm used in smart antenna networks

Table 1

<table>
<thead>
<tr>
<th>Simulated adaptive algorithm</th>
<th>MATLAB processing time for 200 iterations (in seconds)</th>
<th>MATLAB processing time for 2000 iterations (in seconds)</th>
<th>Approximately number of iterations till algorithm converges</th>
<th>MATLAB processing time till algorithm reaches convergence (in seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>0.019716</td>
<td>0.115084</td>
<td>120 to 180</td>
<td>0.006803</td>
</tr>
<tr>
<td>RLS</td>
<td>0.031764</td>
<td>0.174148</td>
<td>50 to 90</td>
<td>0.003945</td>
</tr>
<tr>
<td>GAMSAPA</td>
<td>0.075289</td>
<td>1.034581</td>
<td>40 to 60</td>
<td>0.010805</td>
</tr>
<tr>
<td>MFCVSSEWOAPAECOR</td>
<td>0.086210</td>
<td>6.257560</td>
<td>15 to 20</td>
<td>0.001872</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

Adaptive algorithms have been used for a long time in communication applications. Initially used to suppress noise along the communication channel their applicability is studied intensively nowadays for beamforming. Starting from the LMS algorithm the engineers developed new solutions in order to improve the performance of the smart antenna systems. The solutions are new faster, more reliable and more stable algorithms. Till not so long ago it was believed that the Gaussian distribution was the one of the best distributions used to model random noise when studying algorithms used in communications on platforms like MATLAB. Now the stable distributions are believed to be more reliable when we want to study the behavior of the algorithms as it describes better the conditions in real urban environments, with short, random and high amplitude interfering signals occur.

In this paper an algorithm that has a very good behavior in such an environment is proposed. The results obtained when using this algorithm are compared with the results obtained when using the LMS, the RLS and GAMSAPA algorithm. In each case the algorithm performed better.

References