MATHEMATICAL MODEL BASED OF TRANSFER FUNCTIONS FOR DYNAMIC OPERATIONS OF SUPERCHARGED ENGINE

Salvadore Mugurel BURCIU

University “Dunarea de Jos” of Galati, Romania,

Rezumat: Lucrarea prezinta modelul matematic pentru comportarea dinamica a motorului diesel supraalimentat cu turbocompresor cu rotatie libera. Modelul matematic se bazeaza pe cunoasterea caracteristicilor subsistemelor principale, asa cum sunt motorul propriu-zis, turbocompresorul, galeria de admitere si evacuare, instalația de injectie. Caracteristicile acestor subsisteme sunt cunoscute pentru regimurile stationare de functionare si in apropierea acestora, dupa datele experimentale, pe baza determinarilor pe stand. Modelul matematic este exprimat de ecuația diferențială de ordinul doi care exprima comportarea dinamica a întregului sistem. Sunt stabilita functiile de transfer, care pot fi folosite pentru reglarea automată a întregului sistem.

Cuvinte cheie: motor diesel supraalimentat, functii de transfer, functionare in regim dinamic, regimuri nestationare de functionare.

Abstract: The paper deals with the differential equation expressing the dynamic operations of turbocharged internal combustion engines using supercharger with free rotation units. Mathematical model is based on knowledge of the characteristics of subsystems, such as engine itself, turbocharger, exhaust and intake manifold and the injection system to stationary regimes. Transfer functions are determinated, functions that are used for achieving and adjusting automatic control system which controls the operation of the whole system (engine) under consideration.

Keywords: supercharged diesel engine, transfer functions, dynamic regime, unsteady working conditions.

1. INTRODUCTION

The paper presents the mathematical model for calculating transfer function for the whole supercharged internal combustion engine, with free rotation supercharger units. Mathematical model is based on differential equations and the transfer functions established, [8], [3], [4], [5], [6], [7], [2] for dynamic operations of subsytems, as they are exhaust and intake manifold, injection system, engine itself, turbocharger [1], as part of internal combustion engines, as shown in figure 1. On the basis of these equations results the dynamical behavior of internal combustion engine. Using transfer functions can build automatic control system (controller) and make its award with engine, such as existing automatic governor for revolution.

2. DYNAMIC REGIME OF DIESEL SUPERCHARGED ENGINE

We consider the following diagram (Fig. 1) with the subsystems [8], [1], parts of the supercharged internal combustion engine, with free rotation turbocharger. In these diagram, subsystems are: K - turbocompressor; S. Inj. - Fuel injection system; TG - gas turbine; Diesel - diesel engine itself; C.AD - intake pipe; C.EV - exhaust pipe; \( \omega \) - angular speed of engine; \( \omega_{TK} \) - angular speed of turbocharger; \( \omega_{inj} \) - angular speed of fuel injection pump; \( c_{cycle} \) - cycle fuel consumption for a cycle; \( h_{K,T} \) - the position of adjustment actuator of turbocompressor respectively turbine;

![Fig.1 Diagram for supercharged engine subsystems](image-url)
The mass flow of air admitted into the compressor is denoted by $m_{ad,K}$, the mass flow of air entering the intake manifold by $m_K$, the mass flow of air entering the engine cylinders by $m_{K,cil}$, the actuator position of the injection pump for a diesel engine by $h$, the exhaust gas mass flow entering the exhaust manifold by $m_{g0}$, and the exhaust gas mass flow entering the turbine by $m_T$. The exhaust gas mass flow coming out in the environment is denoted by $m^*_g0$.

Using the differential equations established in [3], [4], [5], [6], [8], [2], which shape the dynamical operations of constitutive parts near the steady working conditions, it's possible to achieve the differential equation for the whole supercharged internal combustion engine and therefore the dynamical behavior.

According to [8], [3], [4], [5], [6], [7] can write:

--- for Diesel engine itself:

$$T_{eng} \frac{d(\Delta \omega^*_T)}{dt} + K_{eng} (\Delta \omega^*_T) = \Delta C^*_cyl (s) + \theta_{eng} \Delta P^*_K - \theta_{eng} \Delta h^*_s \Delta h^*_s$$

(1)

after use Laplace transformation:

$$\Delta C^*_cyl (s) = \frac{\Delta \omega^*_T}{c_{cycle0}} \Delta h^*_s (s)$$

(1')

--- for injection system:

$$K_{ap \cdot inj} \Delta C^*_cyl (s) = \Delta h^*_s (s) + \theta_{ap \cdot inj} \Delta \omega \Delta \omega$$

(2)

after use Laplace transformation:

$$K_{ap \cdot inj} \Delta C^*_cyl (s) = \Delta h^*_s (s) + \theta_{ap \cdot inj} \Delta \omega \Delta \omega$$

(2')

--- for turbocharger:

$$T_{TK} \frac{d(\Delta \omega^*_{TK})}{dt} + K_{TK} (\Delta \omega^*_{TK}) = \Delta P^*_T + \theta_{TK \Delta \omega^*_cyc} \Delta C^*_cyl (s) - \theta_{TK \Delta p^*_K} \Delta P^*_K$$

(3)

after use Laplace transformation:

$$T_{TK} + K_{TK} \Delta \omega^*_T (s) = \Delta h^*_T + \theta_{TK \Delta \omega^*_cyc} \Delta C^*_cyl (s) - \theta_{TK \Delta p^*_K} \Delta P^*_K$$

(4)

--- for intake manifold:

$$T_{c.ad} \frac{d(\Delta P^*_K)}{dt} + K_{c.ad} (\Delta P^*_K) = \Delta \omega^*_TK - \theta_{c.ad} \Delta \omega \Delta \omega - \theta_{c.ad} \Delta T^*_K \Delta T^*_K$$

(5)

after use Laplace transformation:

$$T_{c.ad} \frac{d(\Delta P^*_K)}{dt} + K_{c.ad} (\Delta P^*_K) = \Delta \omega^*_TK - \theta_{c.ad} \Delta \omega \Delta \omega - \theta_{c.ad} \Delta T^*_K \Delta T^*_K$$

(6)

Where:

$$\omega = \omega_0 + \Delta \omega; \Delta \omega = \frac{\Delta \omega}{\omega_0}$$

(7)

$$\Delta C^*_cyc = \frac{\Delta C^*_cyl}{c_{cycle0}} ; \Delta \omega = \frac{\Delta P^*_K}{P_{cyl}}$$

(8)

$$\omega_0 = \text{angular speed for steady working conditions}; c_{cycle0} = \text{fuel consumption for steady working conditions}; h_0 = \text{adjustment devices at consumer for steady working conditions}; p_{cyl} = \text{pressure furnished by compressor for steady working conditions}; p_{cyl} = \text{gas pressure at the entrance of gas turbine for steady working conditions};$$

Where:

$$T_{eng} = \frac{J_{eng} \cdot \omega_0}{\left(\frac{\partial M_{te}}{\partial C_{cycle}}\right)_{\omega_0 \cdot p_{cyl}}} \cdot c_{cycle0}$$

(9)

$$\theta_{eng \Delta p} = \frac{\partial M_{te}}{\partial p_{cyl}} \cdot p_{cyl}$$

(10)

$$K_{eng} = \frac{F_{st \cdot eng} \cdot \omega_0}{\left(\frac{\partial M_{te}}{\partial C_{cycle}}\right)_{\omega_0 \cdot p_{cyl}}} \cdot c_{cycle0}$$

(11)
\[
\theta_{\text{eng}} = \left( \frac{\partial M_{teR}}{\partial h_s} \right)_{\omega_0} \cdot h_{s0}
\]
\[
T_{TK} = \left( \frac{J_{TK}}{\omega_0} \right)_{TK0} \cdot P_{TK0}
\]
\[
K_{TK} = \left( \frac{\partial M_{TG}}{\partial P_T} \right)_{TK0} \cdot P_{TK0}
\]
\[
\theta_{TK_{\Delta p}} = \left( \frac{\partial M_{TG}}{\partial P_T} \right)_{h_T0} \cdot P_{TK0}
\]
\[
\theta_{TK_{\Delta t}} = \left( \frac{\partial M_{TG}}{\partial h_T} \right)_{P_T0} \cdot P_{TK0}
\]
\[
\theta_{TK_{\Delta h_k}} = \left( \frac{\partial M_{k}}{\partial h_k} \right)_{0} \cdot h_{K0}
\]
\[
F_{st_{\text{eng}}} = \left( \frac{\partial M_{teR}}{\partial \omega_0} \right)_{h_0} - \left( \frac{\partial M_{te}}{\partial \omega_0} \right)_{P_{TK0}}
\]
\[
F_{st_{TK}} = \left( \frac{\partial M_{k}}{\partial \omega_0} \right)_{\omega_0} - \left( \frac{\partial M_{TG}}{\partial \omega_0} \right)_{P_{TK0}}
\]
\[
K_{ap_{\text{inj}}} = \left( \frac{c_{\text{cycle}}}{\partial c_{\text{cycle}}} \right)_{h_0} \cdot h_0
\]
\[
\theta_{ap_{\text{inj}}_{\Delta \omega}} = \left( \frac{c_{\text{cycle}}}{\partial h} \right)_{\omega_0} \cdot h_0
\]
\[
\theta_{TK_{\Delta c_{\text{cycle}}}} = \left( \frac{\partial M_{TG}}{\partial c_{\text{cycle}}} \right)_{\omega_0} \cdot c_{\text{cycle}} \cdot P_{TK0}
\]
\[
K_{c_{\text{ad}}} = \left( \frac{\partial m_{k}}{\partial \omega_0} \right)_{P_{TK0}} \cdot P_{TK0}
\]
\[
\theta_{c_{\text{ad}}_{\Delta \omega}} = \left( \frac{\partial m_{k}}{\partial \omega_0} \right)_{ \omega_0} \cdot P_{TK0}
\]

\[
\frac{\partial m_{k,cil}}{\partial h_k} \cdot h_{K0}
\]

\[
\theta_{c_{\text{ad}}_{\Delta h_k}} = \left( \frac{\partial m_{k,cil}}{\partial h_k} \right)_{h_{K0}}
\]

\[
\theta_{c_{\text{ad}}_{\Delta T_k}} = \left( \frac{\partial m_{k,cil}}{\partial T_k} \right)_{T_{K0}}
\]

\[
T_{c_{\text{ad}}} = \frac{V_{c_{\text{ad}}} \cdot P_{TK0}}{n_{TK} \cdot P_k \left( \frac{\partial m_{k}}{\partial \omega_0} \right)_{TK0}}
\]

\[
F_{st_{\text{c}}} = \left( \frac{\partial m_{k,cil}}{\partial \omega_0} \right)_{\omega_0} - \left( \frac{\partial m_{k}}{\partial \omega_0} \right)_{P_{TK0}}
\]

\[
K_{c_{\text{ev}}} = \left( \frac{\partial m_{g}}{\partial \omega_0} \right)_{\omega_{TK0}} \cdot P_{TK0}
\]

\[
\theta_{c_{\text{ev}_{\Delta c_{\text{cycle}}}}} = \left( \frac{\partial m_{g}}{\partial c_{\text{cycle}}} \right)_{c_{\text{cycle}}0}
\]

\[
\theta_{c_{\text{ev}_{\Delta T_{k}}}} = \left( \frac{\partial m_{g}}{\partial T_k} \right)_{h_{T0}}
\]
\( J_{TK} \) (kg m^2) - mechanical moment of inertia of the rotating mechanical turbocharger components, reduced to its revolution axis;  
\( \omega_{TK} \) (s^-1) - common angular speed of the compressor and turbine;  
\( \omega \) (s^-1) - angular engine speed;  
\( J_{eng} \) (kg m^2) - mechanical moment of inertia of the mobile mechanical engine, reduced to its revolution axis;  
\( M_{te} \) (Nm) - torque of engine;  
\( M_{te} R \) (Nm) - torque to drive, the couple must overcome the brake resistance brake;  
\( M_{tcg} \) (Nm) - brake torque of compressor;  
\( M_{ttg} \) (Nm) - compressor shaft torque at stationary running;  
\( m_k \) (kg/s) - compressor mass flow rate;  
\( m_T \) (kg/s) - gas turbine mass flow rate;  
\( h \) = the position of adjustment devices for injection pump;  
\( h_T \) - actuator turbine control position;  
\( h_k \) - actuator compressor control position;  
\( c_{cycle,0} \) (kg fuel/cycle) - fuel consumption on cycle at stationary running;  
\( p_T \) (N/m^2) - gas pressure in the turbine entry;  
\( p_K \) (N/m^2) - fluid pressure at the compressor exit at stationary running;  
\( T_T \) (K) - temperature at the exit of the compressor;  
\( T_{ff} \) (K) - gas temperature at the entrance of gas turbine;  
\( c_{cycle} \) (kg fuel/cycle) - fuel consumption on cycle;  
\( \omega_{TK0} \) (s^-1) - common speed of the compressor and turbine at stationary running;  
\( M_{tgg} \) (Nm) - turbine shaft torque at stationary running;  
\( M_{k0} \) (Nm) - compressor shaft torque at stationary running;  
\( p_{t0} \) (N/m^2) - gas pressure in the turbine entry at stationary running;  
\( p_{K0} \) (N/m^2) - fluid pressure at the compressor exit at stationary running;  
\( \Delta \omega^* = 0, \Delta \omega_{TK}^* = 0 \) respectively (1',2',3',4',5') \( \Delta H_T' = 0, \Delta H_K' = 0 \) and results equations:

\[
\begin{align*}
\Delta T_{eng}(s) + \Delta C_{cycle}(s) - \Delta H_{cycle}(s) &= \theta_{c,ev}(s) - \Delta P_{k}(s) - \Delta P_{K}(s) \\
\Delta M_{te(g)}(s) - \Delta M_{te}(s) &= -\theta_{c,ev}(s) - \Delta H_{cycle}(s) \\
\Delta M_{te(g)}(s) - \Delta M_{te}(s) &= -\theta_{c,ev}(s) - \Delta H_{cycle}(s)
\end{align*}
\]

If we want to obtain the differential equations of the internal combustion engines without the adequate adjustment devices for T and K, it is necessary to solve the particular equations system resulting from the general equations 35, 36, 37, 38, 39.

Figures 2, 3, 4, 5, 6 show the structural diagrams for the turbocharger engine itself, the fuel injection system, the intake manifold, the exhaust manifold and the turbocharger. In these diagrams the transfer functions are set out in rectangles, the input signals on the left and the output signals on the right. The index number added to some input signals represents the number of the structural diagram where that signal comes out.

We consider the independent parameters \( \Delta \omega^* \) and \( \Delta \omega_{TK}^* \) [8], and the unknown quantities \( \Delta \omega^*, \Delta C_{cycle}, \Delta P_{k}, \Delta P_{T}, \Delta \omega_{TK} \).

For the internal combustion engine as the subject of automatic adjustment abiding by the revolution, \( \Delta \omega^* \) is considered to be the parameter which must be pursued in time:
\[ \Delta \Omega^*(s) = \left( \Delta \Omega \right)^*; \Delta \mathcal{A}_{cycle}^*(s) = \left( \Delta \mathcal{A}_{cycle} \right)^*/\Delta \]

\[ \Delta P_K^*(s) = \left( \Delta P_K \right)^*/\Delta \]

\[ \Delta \Omega^*_{TK}(s) = -\left( \Delta \mathcal{O}_TK \right)^*/\Delta \]

Where:

\[ \Delta = T_{eng,s_2}^2 \cdot s^2 + T_{eng,s_1} \cdot s + K_{eng,s} \cdot s + T_{act\_inj\_pump} \cdot s + \theta_{act\_inj\_pump} \cdot \Delta \mathcal{H}^*(s) \]

\[ T_{eng,s_2} = T_{eng} \cdot T_{TK} \cdot K_{ap\_inj} \cdot K_{cad} \cdot K_{cev} \] (42)

The differential equation written in the operational form, where the rotation is considered as the object of automatic adjustment, is as follows:

\[ \Delta \cdot \Delta \Omega^*(s) = \Delta \Omega^* \] (43)

Or:

\[ \left( T_{eng,s_2}^2 \cdot s^2 + T_{eng,s_1} \cdot s + K_{eng,s} \right) \cdot \Delta \Omega^* = (T_{act\_inj\_pump}^2 \cdot s + \theta_{act\_inj\_pump} \cdot \Delta \mathcal{H}^*(s) - (T_s \cdot s + \theta) \cdot \Delta \mathcal{H}^*(s) \]

(44)

It therefore leads to:

\[ \Delta \Omega^*(s) = \left( \frac{T_{act\_inj\_pump}^2 \cdot s^2 + K_{eng,s} \cdot s + s + s + \theta_{act\_inj\_pump}^2 \cdot s + \theta_{act\_inj\_pump} \cdot \Delta \mathcal{H}^*(s)}{(T_s \cdot s + \theta)} \right) \cdot \Delta \mathcal{H}^*(s) \]

(45)

where:

\[ H_{engine}\Delta h(s) = \text{the transfer function determined by the actuator of the injection pump;} \]

\[ H_{engine}\Delta h(s) = \text{the transfer function determined by the load engine;} \]

If Laplace \(^1\) is applied to the differential equation (44) written above, the result is the differential equation expressing the dynamic engine operation in relation to time (equation 46), for the supercharged internal combustion engine without adjustment actuators for the turbine and compressor.
Taking into account the different charge schemes and mathematical models of the engine subsystems, the result yields different mathematical models represented by differential equations up to grade 5. Based on these models, the dynamic engine operations of supercharged engines can be obtained.

3. CONCLUSION

On the basis of the mathematical model of the dynamic regimes of the supercharged engine and the transfer functions established, the following conclusions can be specified:

-- Transfer functions would allow the achievement of an automatic system control located on the engine, which can control all dynamic and stationary regimes and can ensure the working of the engine on optimum regimes with minimum possible fuel consumption.

-- Using an automatic system control for engine, the system will ensure quality indices for all the dynamic regimes of the whole engine and a high system performance on stationary regimes of operation, with minimum errors for speed and acceleration. The automatic system control will ensure performance for the whole engine and all its subsystems, during transitory regimes of operation.

REFERENCES


[8]. Burciu M.S, Thesis Ph.D., Contributions to the study of nonstationary processes in the internal combustion engine overloading turbo-compressors, University of Galati, Faculty of Mechanical Engineering, Department of Internal Combustion Engines, 2000.