THEORETICAL RESEARCHES ON THE STRESS STATE AT VIBRO-DRAWING

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Abstract
The paper contains the mathematical modelling of the stress state at vibro-drawing for the case of introducing the vibrating motion on axial direction, by means of the punch, into the drawing die. Starting from the definition of the total radial stress as sum of the radial stress for the transformation of the flange dimension and the stress due to the bending-unbending of the material on the die radius, there was emphasised the influence of introducing axial vibrations on the radial stress for the transformation of the blank's flange dimensions. The determined mathematical relations show a decrease of the stress when a dynamic motion is introduced into the drawing die.

1. GENERAL CONSIDERATIONS

Nowadays, the current industrial society faces very important problems, since the dependence on fossil fuels and on conventional technologies cannot continue forever, a fact that emphasises the explosion of technologies that is felt in the last few decades. Of course, currently one cannot know what technologies or combinations of technologies will prove to be useful and for what purposes, but it is certain that there exists a very large variety of equipment and technologies that will be applied from an economical point of view. Certainly, the nonconventional metal forming procedures that appeared in the last years represent aspects of the mentioned explosion of technologies.

Being part of the nonconventional metal forming procedures, the forming of metallic materials in the presence of sustained vibrations is in the same group as hydrostatic forming, metal forming in the superplastic domain, electro-hydraulic extrusion, forming in a magnetic field etc.

The problem of metal forming in the presence of sustained vibrations was tackled over the years by researchers from various countries: Austria, the former Soviet Union, the U.S.A., the United Kingdom, Germany and more recently the researches on vibration-assisted forming are carried out in several other countries including Romania in a more systematic manner. The tools or blanks are vibrated in a very wide range of frequencies, from a few hertz to several hundreds of thousands of hertz, with amplitudes in the range of millimeters or microns. Differences, but also similarities are determined between the forming in the presence of sustained vibrations and the conventional forming.

The current paper aims to carry out a theoretical study of the stress state in the material (blank) subjected to forming, during a drawing process unfolded in the presence of sustained vibrations introduced into the tool by means of the punch or the drawing die.

It is known that the forming of a blank by means of drawing begins with the pushing of the punch on the central part of the blank's material, thus achieving a gradual pulling of the flange in order to obtain the sidewalls. The load state is characterised by the presence of radial drawing stresses and of tangential compression stresses. The coordinates system used for the analysis of the stress state is the plane coordinates system $\sigma_\rho$ and $\sigma_\theta$, which represent, at the same time, the main normal stresses.

The stress state at the cylindrical drawing is influenced by the continuous modification of the position and dimensions of the blank's flange in view of its transformation into a side wall, by the friction between the blank and the active elements and by the material's bending at the passing over the filleted area of the die. All these influence the size of the radial tensile stress $\sigma_\rho$ that can be written as:

$$\sigma_{\rho t} = (\sigma_\rho, \sigma_\theta) e^{\log} + \sigma_\rho,$$

where: $\sigma_{\rho t}$ is the total radial stress;  
$\sigma_\rho$ - the radial stress that characterises the gradual transformation of the dimensions of the plane flange of the blank's material;
σ₁ - the radial stress stemming from the friction between the blank and the active elements;
σ₁ - the radial stress stemming from the by the material's bending at the passing over the filleted area of the die.

It is known that the maximal stress at drawing occurs when the centres of the fillet radii of the punch and die have the same ordinate.

Also, previously there has been determined the correlation function between the initial diameter of the blank D and that of the flange dᵢ, for which the value of the pressure of the pressing element is maximal, Qₘₐₓ, namely dᵢ = 0.87 D. In that moment, the ordinates of the filleting radii of the punch and the die coincide too.

2. DETERMINING THE RADIAL STRESS THAT CHARACTERISES THE GRADUAL TRANSFORMATION OF THE DIMENSIONS OF THE BLANK'S PLANE FLANGE

The effect of introducing the vibrational motion on axial direction in drawing dies has been analysed for the case in which the vibrations were introduced by means of the punch, overlaying over the static forming motion. Due to the vibrations being introduced co-linear with the static forming force, the size of the radial stress stemming from the friction between the blank and the active elements is the same as in the case of the conventional drawing. The same is valid for the size of the stress stemming from the bending of the material at the drawing over the filleted part of the die.

In the following, there will be determined the radial stress that characterises the gradual transformation of the blank's plane flange dimensions, stress influenced by the introduction of vibrations in drawing dies. It will be determined for the area of the blank's flange, but also for the filleting area, at the moment when the centres of the punch and of the die coincide. There is considered the case when the drawing takes place without any blankholding. It should be also mentioned that the moment of the occurrence of the maximal stress at drawing and vibro-drawing is the same, but the maximal value differs.

The radial stress that occurs in the flange of the drawn part is given by the relation:

\[ \sigma_r = \beta \sigma_c \ln \left( \frac{r_f}{r_0} \right), \]  

(2)

where: \( r_f \) is the radius of the flange at the considered moment;
\( r_0 \) is the radius corresponding to the circle delimiting the area of the flange from the filleting area (\( r_0 = R_f \)).

Relation (2) is valid, from an analytical point of view, also for the case of vibro-drawing. Quantitatively, the size of the stress differs because of the modification of the yield stress in the blank's material at vibro-drawing compared to conventional drawing.

In order to study the stress state in the filleting area, there is considered an annular element (fig.1) characterised by the angle \( \alpha \), \( d\alpha \), from which there is detached the area defined by the solid angle \( d\phi \). Following notations are valid:

\[ s \] is the area of the lateral side of the annular element;
\[ s_1 \] - area of the upper side of the annular element;
\[ s_2 \] - area of the lateral sides of the element e;
\[ r_p \] - fillet radius of the punch;
\[ r_d \] - fillet radius of the die.

Following the static and dynamic motions, introduced by the press ram by means of the punch, in the die there will occur a normal reaction between the blank and the die and a friction force:

\[ F_n = F_{n1} + F_{n2} \sin \omega t; \]  

(3)

\[ F_f = F_{f1} + F_{f2} \sin \omega t. \]  

(4)
The component $F_{n1}$ is given by the static force of the ram, while $F_{n2} \cdot \sin \omega t$ is its dynamic force. The friction force can be expressed simplified as $F_f = \mu_d F_n$, where $\mu_d$ is the classic dynamic friction coefficient; $\mu_d < \mu_s$.

As a consequence of the existence of the vibration motion, there occur an acceleration and an inertial force oriented on the direction of the vibration motion (vertical direction): $F_i$.

From the equation of forces on the direction $y$ (fig. 1), there results:

$$F_n \cdot dS_1 = - \left[ d(\sigma_p S) \cos \alpha + dF_i \right] 1 / [\sin \alpha + \mu_d \cos \alpha], \quad (5)$$

And from the equation of forces on the direction $x$:

$$d(\sigma_p S) \sin \alpha \left( \frac{d\phi}{2\pi} \right) + 2\sigma_0 S_2 \left( \frac{d\phi}{2} \right) + F_n \ dS_1 \left[ \mu_d \sin \alpha - \cos \alpha \right] \left( \frac{d\phi}{2\pi} \right) = 0 \quad (6)$$

By calculating the areas $S_1$ and $S_2$, there can be obtained:

$$S_1 = 2\pi (z + r) g; \quad (7)$$
$$S_2 = (r_p + g/2) g \ d\alpha, \quad (8)$$

where: $r$ is the radius of the drawn part, $r = d/2$;
$g$ - the thickness of the blank's material;
$z = (r_p + g/2) (1 - \cos \alpha)$.

Replacing (4), (6) and (7) in (5), taking into account the Huber-Mises-Hencky plasticity criterion, $\sigma_p + \sigma_{pl} = \beta\sigma_c$, and writing,

$$a = 1 + r/(r_p + g/2),$$

there can be obtained:

$$d\sigma_p/d\alpha - \mu_d \sigma_p \cos \alpha / (a - \cos \alpha) - \beta\sigma_c (\sin \alpha + \mu_d \cos \alpha) / (a - \cos \alpha) -$$
$$- (a-1) (\cos \alpha - \mu_d \sin \alpha) \left( \delta F_i / \delta \alpha \right) / 2\pi \ rg (a - \cos \alpha). \quad (9)$$

In order to determine $\delta F_i / \delta \alpha$ there is considered an infinitely small element that can be approximated as linear and that is located at the angle $\alpha$ (fig. 2).

This is on the ordinate $y = r_p \sin \alpha$. The length of the element is $dl$ and corresponds to a projection on the vertical direction equal to $dy$. For the infinitely small element there can be approximated that the angle between the vertical and the tangent is equal to $\alpha$. Consequently, $dy = dl \cos \alpha$. Along the finite element there occur the stresses $F_n$ and $(\delta F_n / \delta \alpha) \cdot dl$, the inertia force on the vertical $dF_i$, and the distributed oscillating force $F_0(y, t) \cdot dl$. $F_n$ is a function of $y$ by means of $l$, i.e. $F_n = F_n(l(y))$. 

**Fig. 1.** Stress state in a deformed part.
There can be written:
\[ \frac{dF_n}{dy} = (\delta F_n / \delta l) \frac{dl}{dy} = (\delta F_n / \delta l) \frac{1}{\cos \alpha}, \]
from where:
\[ (\delta F_n / \delta l) = (\delta F / \delta y) \cos \alpha. \] (10)

From the projections equations on Oy, using \( dy = dl \cos \alpha \) and (9), there results:
\[ (\delta F_n / \delta y) \cos \alpha + F_0 (y,t) (1/\cos^3 \alpha) = dF_i /dy. \] (11)

Expressing the effort \( F_n \) and the inertial force on the length unit \( \delta F_i / \delta y \), function of the displacement along the element \( dl \), is done with the relations:
\[ F_n = A \sigma = A E \varepsilon = A E (\delta u / \delta l) = A E (\delta u / \delta y) (\delta y / \delta l) = A E \cos \alpha (\delta u / \delta y); \] (12)
\[ \delta F_i / \delta y = dF_i /dy = \rho A \delta^2 u / \delta t^2. \] (13)

where \( A \) is the area of the cross-section;
\( \sigma \) - stress;
\( \rho \) - density of the blank's material.

By replacing (12) and (13) in (11), there can be obtained:
\[ A E (\delta^2 u / \delta y^2) + F_0 (y,t) (1/\cos^3 \alpha) = (\rho A / \cos^2 \alpha) (\delta^2 u / \delta t^2). \] (14)

and by replacing (13) in (9), there can be obtained:
\[ (d \sigma_i / d\alpha) - \mu_d \cos \alpha (a - \cos \alpha) = Q_1 (\alpha) + Q_2 (\alpha) (\delta^2 u / \delta t^2), \] (16)
where: \( Q_1 (\alpha) = - \beta \sigma_c (\sin \alpha + \mu_d \cos \alpha)/(a - \cos \alpha); \) (17)
\[ Q_2 (\alpha) = -(a-1) (\cos \alpha - \mu_d \sin \alpha) \rho A r_{pl} \cos \alpha / 2\pi r g (a - \cos \alpha). \] (18)

By integrating the differential equation (16) either by using the Bernoulli method, or through the method of constant variation, considering also \( e^x = 1 + x \) and neglecting the integrals within integrals (they have very small values), there can be obtained:
\[ \sigma_0 - \rho \sigma_c [(1 + \mu_d k (a, \alpha_i)) [\ln d_L (d_L + g) - \mu_d k(a,a_i)] - (a - 1) \rho A r_{pl} (1 + \mu_d) [1 + ak (a, \alpha_i) + \mu_d (1 - a \ln (a - 1)/a)] \delta^2 u / \delta t^2/2\pi r g, \] (19)
where: \( k (a, \alpha_i) = a_i - [2a/\sqrt{(a^2 - 1)] \arctg [(a + 1) \tan \alpha_i / \sqrt{(a^2 - 1)}]. \) (20)
From relation (19) it can be noticed that \( \sigma_p \) depends on:
- time;
- characteristics of the drawing material;
- friction coefficient between the material and the die;
- logarithm of the ratio between the flange diameter and the average diameter of the drawn part;
- the geometric parameter of the drawing, \( A \), which in turn depends on the ratio between the average drawing diameter and the radius of the die and the material's thickness. Thus, the value of the die radius must be chosen function of the material thickness and the dimension of the drawn part.

In the relation used for defining the radial stress at vibro-drawing there occurs, supplementary to the conventional drawing, the supplemental term that is deducted, thus emphasising the decreasing of the yield strength in the vibro-drawn blank's material.

Relation (19) represents the stress corresponding to the conventional drawing for the case in which function \( u \) does not depend on time. In this situation, \( \delta^2 u / \delta t^2 = 0 \) and the expression of \( \sigma_p \) becomes the one determined for the conventional drawing. It should be noticed that this expression stays valid also for the case in which \( u \) varies in time (\( u = at + b, \delta^2 u / \delta t^2 = 0 \)).

However, for obtaining the definitive expression of the yield stress, it is necessary to solve equation (14), i.e. an equation that represents the equation of a vibrating rod or one-dimensional elastic oscillations. The unfolding of the vibro-drawing process implies the determining of initial conditions and of boundary conditions. The initial conditions define the displacements and the speeds of the material's points at \( t = 0 \). In the studied case, the conditions are zero, i.e.:

\[
\begin{align*}
  u(y,0) &= 0; \quad \delta u / \delta t|_{t=0} = 0, \quad (21) \\
  u(l,t) &= 0; \quad \delta u / \delta y|_{y=0} = 0, \quad (22)
\end{align*}
\]

which means that the material starts to vibrate while being still not formed and in standstill. The boundary conditions define the manner in which the considered material element is fastened. The rod is fastened at one end and is free at the other end. Following conditions are set:

\[
\begin{align*}
  u(l,t) &= 0; \quad \delta u / \delta y|_{y=0} = 0, \quad (22)
\end{align*}
\]

where \( l = \pi r_{pl}/2 \).

Using the notations:

\[
\begin{align*}
  b^2 &= E \cos^2 \alpha / \rho; \quad (23) \\
  f(y,t) &= F_0(y,t)/\rho \, A \cos \alpha, \quad F_0(y,t) = F_0(t) \quad (the \ excitating \ forces \ do \ not \ depend \ on \ the \ ordinates), \end{align*}
\]

equation (14) takes the shape known as the equation of one-dimensional waves:

\[
\delta^2 u / \delta t^2 = b^2 \delta^2 u / \delta y^2 + f(y,t). \quad (14')
\]

The solution of problem (14'), (21), (22) is sought in the shape:

\[
\begin{align*}
  u(y,t) &= v(y,t) + w(y,t), \quad (24)
\end{align*}
\]

where \( v(y,t) \) is the solution of the problem:

\[
\begin{align*}
  \delta^2 v / \delta t^2 - b^2 \delta^2 v / \delta y^2 &= 0; \quad (25) \\
  v(y,t)|_{t=0} &= 0, \quad \delta v(y,t) / \delta t|_{t=0} = 0; \quad (26) \\
  v(y,t)|_{y=l} &= 0, \quad \delta v(y,t) / \delta y|_{y=0} = 0, \quad (27)
\end{align*}
\]
and \( w(y,t) \) is the solution of the problem:

\[
\begin{align*}
\delta^2 w/\delta t^2 - b^2 \delta^2 w/ \delta y^2 &= F_0(t)/\rho \cos \alpha; \quad (28) \\
w(y,t)|_{t=0} &= 0; \quad \delta w(y,t)/\delta t|_{t=0} = 0; \quad (29) \\
w(y,t)|_{y=0} &= 0; \quad \delta w(y,l)/\delta y|_{y=0} = 0. \quad (30)
\end{align*}
\]

For solving problem (25), (26), (27) there can be used the method of variables separation. For the beginning, there are sought particular solutions of equation (6) as:

\[
v(y,t) = Y(y) T(t), \quad (31)
\]

which, introduced into (25) leads to the differential equations:

\[
\begin{align*}
T''(t) - \lambda b^2 T(t) &= 0; \quad (32) \\
Y''(y) - \lambda Y(y) &= 0. \quad (33)
\end{align*}
\]

For equation (33) there have to be found non-trivial solutions that satisfy the conditions:

\[
Y(l) = 0, \quad Y'(l) = 0. \quad (34)
\]

The problem that needs to be solved is a Strum-Lionville problem whose eigenvalues are:

\[
\lambda_k = - (2k + 1)^2 \pi^2/4 l^2, \quad k = 0,1,2, \ldots
\]

and to which correspond eigenfunctions:

\[
Y_k(y) = C_k \cos (2k + 1) \pi y/2 l, \quad k = 0,1,2, \ldots
\]

For \( \lambda, \lambda_k \), the equation (32) has the general solution:

\[
T_k(t) = C_{1k} \cos (2k + 1) \pi b t/2 l + C_{2k} \sin (2k + 1) \pi b t/2 l,
\]

Thus the function:

\[
v_k(y,t) = (A_k \cos (2k + 1) \pi b t/2 l + B_k \sin (2k + 1) \pi b t/2 l) \cos (2k + 1) \pi y/2 l
\]

satisfies equation (25) with the boundary condition (27); \( A_k, B_k \) are variable parameters.

The solution to the problem (25), (26), (27) is sought as:

\[
v_k(y,t) = \sum_{k=0}^{\infty} (A_k \cos (2k + 1) \pi b t/2 l + B_k \sin (2k + 1) \pi b t/2 l) \cos (2k + 1) \pi y/2 l \quad (35)
\]

For this, it is necessary to determine the parameters \( A_k \) and \( B_k \) so that \( v(y,t) \) satisfies the initial conditions (24); this leads to:

\[
\sum_{k=0}^{\infty} A_k \cos (2k + 1) \pi y/2 l, \quad \sum_{k=0}^{\infty} B_k \left[ \cos (2k + 1) \pi b/2 l \right] \cos (2k + 1) \pi y/2 l = 0,,
\]

which in turn leads to \( A_k = 0, B_k = 0, k = 0,1, \ldots \).
Thus, the solution to the problem (25), (26), (27) is:
\[ v(y,t) = 0. \] (36)

The solution to problem (28), (29), (30) is sought as a series:
\[ w(y,t) = \sum_{k=0}^{\infty} T_k(t) \cos \frac{(2k + 1) \pi y}{2l}. \] (37)

Substituting (19) in (10), there results:
\[ \sum_{k=0}^{\infty} \left[ T_k''(t) + \left( (2k + 1) \frac{\pi b}{2} \cdot \frac{l}{2} \right)^2 T_k(t) \right] \cos \frac{(2k + 1) \pi y}{2l} = \frac{F_0(t)}{\rho A} \cos \alpha. \] (38)

By developing function \( f(y,t) = \frac{F_0(t)}{\rho A} \cos \alpha \) into a Fourier series of cosines, relation (38) leads to the differential equations:
\[ F_k''(t) + \left( (2k + 1) \frac{\pi b}{2} \cdot \frac{l}{2} \right)^2 T_k(t) = 4(-1)^k \frac{F_0(t)}{2k + 1} \pi \rho A \cos \alpha. \] (39)

By integrating equation (39) using the method of constant variation, there results:
\[ T_k(t) = C_1 \cos \left( \frac{(2k + 1) \pi b}{2l} \cdot \frac{l}{2} \right) + C_2 \sin \left( \frac{(2k + 1) \pi b}{2l} \cdot \frac{l}{2} \right) + \frac{8}{(2k + 1)^2} \frac{l}{(2k + 1)^2} \right) \int_0^t F_0(\tau) \sin \left( \frac{(2k + 1) \pi b}{2l}(t - \tau) \right) d\tau \] \[ \cdot \frac{l}{(2k + 1)^2} \frac{l}{2} \pi b A \cos \alpha. \] (40)

Introducing into relation (22) the initial conditions:
\[ T_k(0) = 0, \quad T_k'(0) = 0, \quad k = 0, 1, 2, \ldots, \]
there results:
\[ T_k(t) = \frac{8}{(2k + 1)^2} \frac{l}{(2k + 1)^2} \right) \int_0^t F_0(\tau) \sin \left( \frac{(2k + 1) \pi b}{2l}(t - \tau) \right) d\tau \cdot \frac{l}{(2k + 1)^2} \pi b A \cos \alpha. \] (41)

From relations (37) and (41) there results the solution of problem (28), (29), (30),
\[ w(y,t) = \sum_{k=0}^{\infty} (-1)^k \left( \int_0^t F_0(\tau) \sin \left( \frac{(2k + 1) \pi b}{2l}(t - \tau) \right) d\tau \right) \cos \frac{(2k + 1) \pi y}{2l} \] \[ / (2k + 1)^2 /\pi b A \cos \alpha. \] (42)

With these, from (24), (36), (42), one obtains the solution to problem (14') as:
\[ u(y,t) = \sum_{k=0}^{\infty} (-1)^k \left( \int_0^t F_0(\tau) \sin \left( \frac{(2k + 1) \pi b}{2l}(t - \tau) \right) d\tau \right) \cos \frac{(2k + 1) \pi y}{2l} \] \[ / (2k + 1)^2 /\pi b A \cos \alpha. \] (43)

If \( F_0(t) = F_0 \sin \omega t \), \( F_0 \) being constant, then the solution of equations (39) can be written in a simpler form as:
\[ T_k(t) = -2l R_0 8 \omega \sin (2k + 1) \pi b t/2l/(2k + 1) \pi b + R_0 \sin \omega t, \] (44)
where:
\[ R_0 = 16l^2 (-1)^k F_0 /((2k + 1) \pi \rho A \cos \alpha [(2k + 1) \pi \rho A - \omega^2], \] (45)

The solution of problem (14'), (21), (22) then has the expression:
\[ u(y,t) = \sum_{k=0}^{\infty} T_k(t) \cos (2k + 1) \pi y, \]
with \( T_k(t) \) given by (44).
REFERENCES

[7] Romanovski, V.P. Ştanţarea şi mătăria la rece (trad, din lb. ruse), Bucureşti: Editura Tehnică, 197

CERCETĂRI TEORETICE ASUPRA STĂRII DE EFORȚURI UNITARE LA VIBROAMBUTISARE

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Rezumat: Lucrarea conține modelarea matematică a stării de eforturi unitare la vibroambutisare pentru cazul introducerii mișcării vibratorii pe direcție axială, prin intermediul poansonului, în matrița de ambutisare. Pornind de la definirea efortului unitar radial total ca sumă dintre efortul unitar radial pentru transformarea dimensiunilor flanșei precum și efortul unitar datorat îndoirei – dezdoirii materialului pe raza plăcii active, a fost evidențiată influența introducerii vibrațiilor axiale asupra efortului unitar radial pentru transformarea dimensiunilor flanșei semifabricatului. Relațiile matematice stabilite oglindesc mișcătura efortului unitar odată cu introducerea mișcării dinamice în matrița de ambutisare.