# THEORETICAL RESEARCH ON THE MODELING OF CHARACTERISTICS OF COMPOSITE MATERIALS

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**Abstract**: the two components of the composite material, i.e., the matrix and the wire, take the requests differently compared with their rigidity. The way the force applied is taken over by composite components and their structural deformities depend on the connection that exists between the phases of the material. Matrix composite shaft and a linear elastic behaviour can be modelled by springs assembled in series or parallel.
**Keywords** :modeling, composite, beam.

## 1. INTRODUCTION

The two components of the composite material, i.e., the matrix and the wire, take the requests differently compared with their rigidity.

The way the force applied is taken over by composite components and their structural deformities depend on the connection that exists between the phases of

Matrix composite shaft and a linear elastic behavior can be modelled by springs assembled in series or parallel.

Figure 1 shows the schema modeling using Springs assembled in series and parallel.

I. The Assembly in parallel
Equality forces movements

II. The Assembly in sumary
Equality forces movements series

**Fig. 1.** Modeling methods using Springs assembled in series and parallel.

## 2. ELASTIC CHARACTERISTICS OF COMPOSITE UNIDIRECTIONAL REINFORCED BLADES

In Figure 2 shows the arrangement of parallel yarns in cylindrical or paralelipipedica matrix

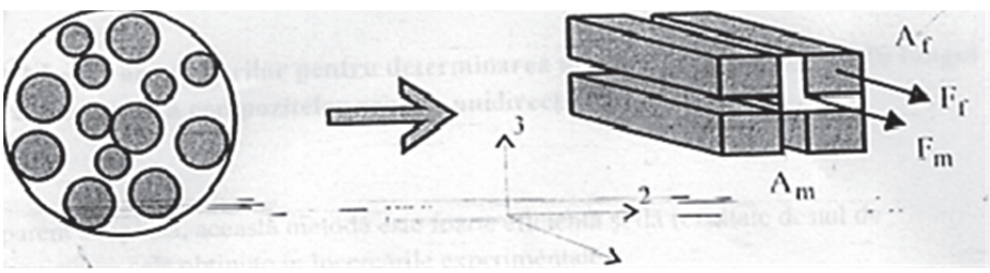


**Fig. 2.** Arrangement of parallel yarns in cylindrical or parallel arrays.

Volumetric fractions of reinforcement (fiber, yarn) and the matrix are

$$V_f = \frac{A_f}{A_f + A_m} \tag{1}$$

$$V_m = (1 - V_f) = \frac{A_m}{A_f + A_m} \tag{2}$$

Reinforcement and matrix composite, follow Hooke's law

– for fittings $\sigma_f = E_f\, \varepsilon_f$ (3)

– for the matrix $\sigma_m = E_m\, \varepsilon_m$ (4)

– for composite $\sigma_1 = E_1 \varepsilon_1$ (5)

The basic law to the Eurocentric expanse

$$\sigma = \frac{N}{A} = \frac{F}{A} \tag{6}$$

results: $$\sigma_1 = \frac{F_f + F_m}{A_f + A_m} = \frac{\sigma_f A_f + \sigma_m A_m}{A_f + A_m} = V_f \sigma_f + V_m \sigma_m = V_f E_f \varepsilon_f + V_m E_m \varepsilon_m \tag{7}$$

It introduces the condition of compatibility of deformations,

$$\varepsilon_f = \varepsilon_m = \varepsilon_1 \tag{8}$$

You get: $$\sigma_1 = E_1 \varepsilon_1 = [V_f E_f + V_m E_m]\varepsilon_1 \tag{9}$$

where results: $$E_1 = V_f E_f + V_m E_m \tag{10}$$

## 3. THE MODULE OF ELASTICITY PERPENDICULAR TO THE DIRECTION OF THE LONGITUDINAL WIRES, E2

The model used is an average of the deformations ponderata reinforcement elements and arrays

$$\varepsilon_2 = \frac{\sum_i (\varepsilon_i\, l_i)}{\sum_i (l_i)} \tag{11}$$

$$\varepsilon_2 = \frac{a}{a+b}\varepsilon_{f,2} + \frac{b}{a+b}\varepsilon_{m,2} = V_f\, \varepsilon_{f,2} + V_m\, \varepsilon_{m,2} = \left[\frac{V_f}{E_{f,2}} + \frac{V_m}{E_m}\right]\sigma_2 \tag{12}$$

Because $$\varepsilon_2 = \frac{\sigma_2}{E_2} \tag{13}$$

result $$\frac{1}{E_2} = \frac{V_f}{E_{f,2}} + \frac{V_m}{E_m} \tag{14}$$

where $$E_2 = \frac{E_{f,2} E_m}{V_f E_m + V_m E_{f,2}} \tag{15}$$

The law of mixtures for determination of longitudinal elasticity module perpendicular to the direction of the armature, E2, do not give satisfactory results when compared with the experimental results. Its expression can be considered as a lower bound of the modululuIde longitudinal elasticity perpendicular to the direction of wires.

If it is considered that the force is directed after Division 3, longitudinal elasticity module according to direction 2 will be

$$E_2 = \frac{\sum_i E_i l_i}{\sum_i l_i} = V_{f2} E_{f2} + V_m E_m \tag{16}$$

What is the upper limit of scattering of longitudinal elasticity module perpendicular to the direction of the armature. It was found that the experimental results are closer to the lower than the upper one.

In Figure 3 it shows longitudinal elasticity module

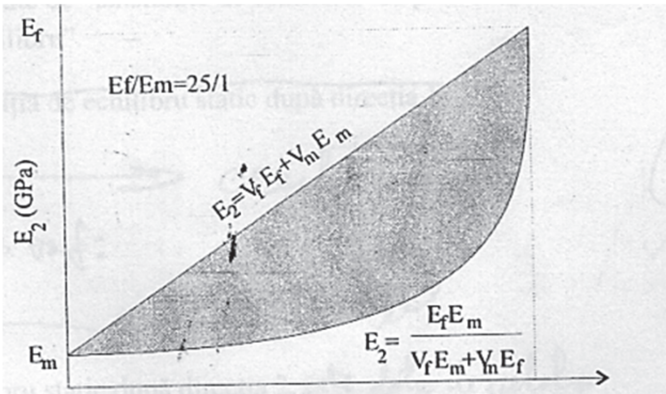


**Fig. 3**. Longitudinal elasticity module.

Ekvall enhances the expression of longitudinal elasticity module perpendicular to the direction of the armature starting Take the lower limit and taking into account the limitation of the strains between the reinforcement and the matrix on the diagonal axis direction. If the loading is applied only on the direction of 2, (perpendicular to the axis of the frame), transverse reinforcement and contractions arrays are different.

Because the matrix and reinforcement strips are bonded to one another, in the direction of 1 will get a pair of the same size but forces of meanings to the contrary, which is different from the contractions will oppose reinforcement and matrix (Fig. 4)

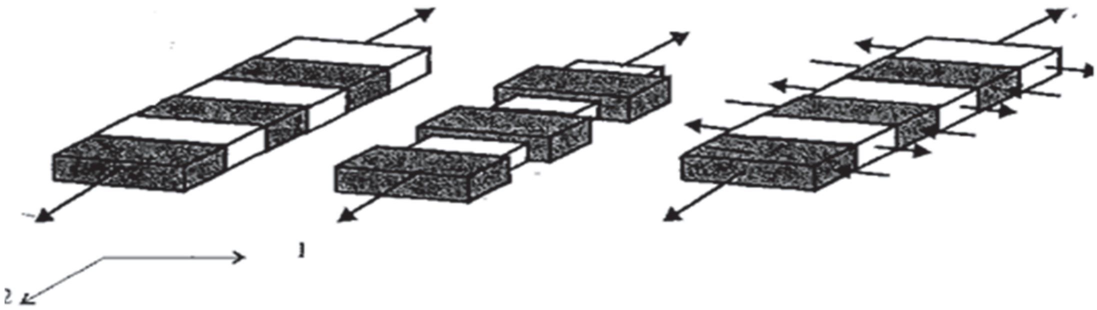


**Fig. 4**. Contrary forces that oppose the contractions of the reinforcement and matrix.

The direction of the overlap between two deformations of deformations of $\sigma_2$ and voltage products generated by "balance tensions" on the right direction 1. Shall be determined in such a "steady tensions".

$$V_f\sigma_{f,1}=-V_m\sigma_{m,1} \tag{17}$$

Kinematic condition will be:

$$\varepsilon_{f,1}=\varepsilon_{m,1}=\varepsilon_1 \tag{18}$$

The condition. static equilibrium after the direction 2 is given by the relationship:

$$\sigma_{f,2}=\sigma_{m,2}=\sigma_2 \tag{19}$$

and the voltage relationships global deformation response of frame-(anisotropic) and matrix (isotroph) are:

$$\varepsilon_{f,1}=\frac{\sigma_{f,1}}{E_{f,1}}-V_{f,12}\frac{\sigma_{f,2}}{E_{f,2}} \tag{20}$$

$$\varepsilon_{m,1} = \frac{\sigma_{m,1} - \nu_{m,12}\sigma_{m,2}}{E_m} = \frac{\sigma_{m,1} - \nu_m \sigma_{m,2}}{E_m} \tag{21}$$

Steady tension reinforcement and matrix of the direction it will be:

$$\sigma_{m,1} = V_f \sigma_2 K \tag{22}$$

$$\sigma_{f,1} = -V_m \sigma_2 K \tag{23}$$

where:

$$K = \frac{\nu_m E_{f,1} - \nu_{f,21} E_m}{V_f E_{f,1} + V_m E_m} \tag{24}$$

Allowable deformation direction 2 due to load on this direction is:

$$\begin{aligned} \varepsilon_2 = V_f \varepsilon_{f,2} + V_m \varepsilon_{m,2} = V_f \left( \frac{\sigma_2}{E_{f,2}} - \nu_{f,21} \frac{\sigma_{f,1}}{E_{f,1}} \right) + V_m \frac{\sigma_2 - \nu_m \sigma_{m,1}}{E_m} = \\ = V_f \left( \frac{\sigma_2}{E_{f,2}} - \nu_{f,21} V_m \frac{K\sigma_2}{E_{f,1}} \right) + V_m \frac{\sigma_2 - \nu_m V_f \sigma_2}{E_m} \end{aligned} \tag{25}$$

$$\frac{\varepsilon_2}{\sigma_2} = \frac{1}{E_2} = \frac{V_f}{E_{f,2}} + \frac{V_m}{E_m} + V_f V_m K \left( \frac{\nu_{f,21}}{E_{f,1}} - \frac{\nu_m}{E_m} \right) \tag{26}$$

$$\frac{1}{E_2} = \frac{V_f}{E_{f,2}} + \frac{V_m}{E_m} + V_f V_m \frac{\left( E_m \nu_{f,21} - E_{f,21} \nu_m \right)^2}{E_{f,1} E_m \left[ V_f E_{f,21} + V_m E_m \right]} \tag{27}$$

The first two terms coincide with those of the law of mixtures, while the last term that represents the influence of constraints after the direction 1, is roughly equal to 10% for glass-epoxy resin composite ($V_t$ = 50%), the difference of 30% compared to experimental results is unacceptably high.

## 4. THE BOX MODEL

In the previous case the rectangular bar reinforcement and matrix foşţ _ considered the line (or column) the results are not satisfactory. Another way to approach the problem is to represent a cross section through a lot of composite elements, each element consisting of a frame with square section placed in a parallelepiped array drill (Fig. 5).

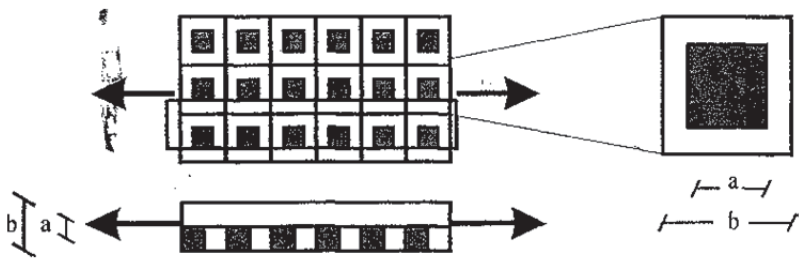


**Fig. 5.** Box Model.

Thus it isolates a strip made up of two different areas (fig. 5) in the base area of reinforcement and matrix elements stacked in alternating pattern of Fig. 4 and the upper area, consisting of two layers of arrays. The rigidity "of composites" will be:

$$\frac{1}{E_c} = \frac{a/b}{E_{f,2}} + \frac{1 - a/b}{E_m} = \frac{\sqrt{V_f}}{E_{f,2}} + \frac{1 - \sqrt{V_f}}{E_m} \tag{28}$$

It is considered that the matrix and "composite" are related to (cooperate), and therefore the deformations after the direction 2 are equal. Modulus of elasticity perpendicular to the direction of the armature will be:

$$E_2 = \frac{a}{b} E_c + (1 - \frac{a}{b}) E_m = \sqrt{V_f} E_c + (1 - \sqrt{V_f}) E_m \tag{29}$$

Introducing phrase of Ec, I get:

$$E_2 = \frac{V_f + 1 - \sqrt{V_f} + (1 - \sqrt{V_f}) \dfrac{E_m}{E_{f,2}}}{1 - \sqrt{V_f}(1 - \dfrac{E_m}{E_{f,2}})} E_m \tag{30}$$

The results obtained by applying this formula are nearly 30 percent higher than those in the first approximation, without taking into account different from the contractions and the collector array.

In composites with rigid wires, the array of fire in the direction of load is a dominant factor of rigidity. The wires usually have a circular or square cross-section (Annular).

Influence of the distance between the wires can be demonstrated as follows: square cross-section of the wires is replaced by a regular polygon that is much closer to circular section (Fig. 6)

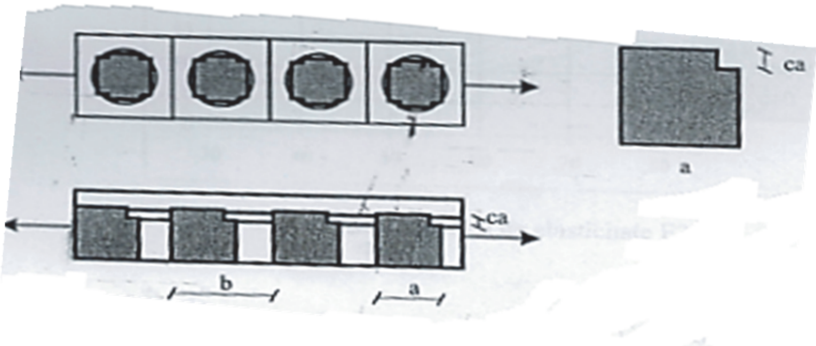


**Fig. 6.** Square transverse Section of the yarn in the tab through a polygon that is approaching the circular section.

Apply the process having three different layers: one for arrays and two "composite". Transverse elasticity module in the direction of the armature will be

$$E_2 = \left\{1 - \sqrt{V_f}\left[1 + \frac{c^2}{2(1+c)}\right]\right\} E_m + \sqrt{V_f}\left(1 - \sqrt{V_f}\right) E_{c2} + \sqrt{V_f}\left[1 + \frac{c}{2(1+c)}\right] E_{c1} c \tag{31}$$

and

$$\frac{1}{E_{c2}} = \sqrt{V_f}\left[1 + \frac{c^2}{2(1+c)}\right]\frac{1}{E_f} + \left\{1 - \sqrt{V_f}\left[1 + \frac{c^2}{2(1+c)}\right]\right\}\frac{1}{E_m} \tag{32}$$

and

$$\frac{1}{E_{c1}} = \sqrt{V_f}(1-c)\frac{1}{E_f} + \left[1 - \sqrt{V_f}(1-c)\right]\frac{1}{E_m} \tag{33}$$

where $c$ is the size of the tapping.

Figure 7 shows the resiliency model E2.

Figure 7 shows the effect of this “corner nick”. When the volumetric fraction of the collector head is small, the influence of the “nick” is minor, but for a fraction of the high volumetric, nick’s influence is big enough. This demonstrates that changes, considered a shallow as insignificant, leading to considerable influences of transverse elasticity module, at least for the volumetric fractions of posing. Among other arguments, it turns out that shaping wires with square section do not represent actual composites.

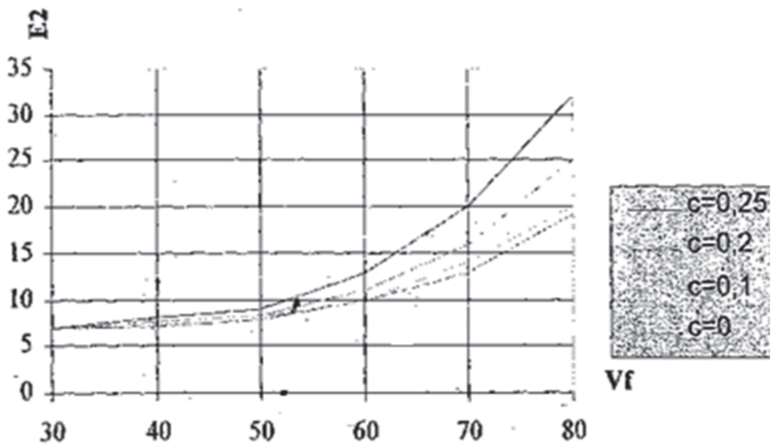


**Fig. 7.** Model of elasticity E2.

Figure 3 compares the longitudinal elasticity module perpendicular to the direction of the armature for the various models presented. In the model with cross-section of armature notched it was considered $c = 0{,}2$.

## 5. CYLINDRICAL MODEL

If the wires are shaped with circular section, for a fraction of the bulk of Vf = 0,70, the space between the wires represent 5.9% of armâturii diameter; for the regular hexagon (volumetric fraction of reinforcement being the same), the distance between the wires is 13.8%. replacement of an arrangement with the other represents an alteration of the space between the wires by a factor greater than 2. In composites of real spaces between threads are distributed statistically the greatest effort to simulate the composition of composite wires was done by Zvi Hashin based on the work of Hill. In Figure 8 shows the cylindrical model

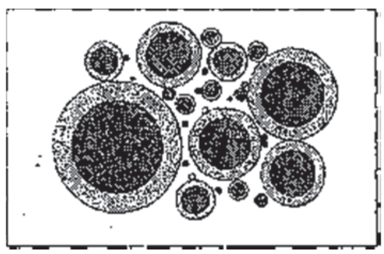

**Fig. 8.** Cylinder Model.

Composite material was modeled through a statistical elements, each consisting of a cylindrical valve inserted into a hollow cylinder-matrix.

The report "reinforcement/aria aria matrix" correlates with the volume fraction of reinforcement in composite. These items are very different in cylindrical dimensions and arranged so that, ideally, there remain gaps. The expression module of elasticity perpendicular to the direction of the armature is:

$$E_{22} = \frac{4G_{23}K_{23}}{K_{23} + G_{23} + \dfrac{4\nu_{21}^{2}G_{23}K_{23}}{E_1}} \tag{34}$$

## 6. CONCLUSIONS

The two components of the composite material, i.e., the matrix and the wire, take the requests differently compared with their rigidity.

The way the force applied is taken over by composite components and their structural deformities depend on the connection that exists between the phases of the material.

Matrix composite shaft and a linear elastic behaviour can be modelled by springs assembled in series or parallel.

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# CERCETĂRI TEORETICE PRIVIND MODELAREA UNOR CARACTERISTICI ALE MATERIALELOR COMPOZITE

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**Rezumat:** Cele două componente ale materialului compozit, respectiv, matricea şi firul, preiau solicitările în mod diferit, în raport cu rigiditatea lor. Modul în care forța aplicată este preluată de componentele compozitului și deformațiile lor structurale depind de legătura care există între fazele materialului. Matricea și firul compozitului, având o comportare liniar elastică, pot fi modelate prin metoda resorturilor asamblate în serie sau paralel.