An Investigation of the OFDM Channel Estimation

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Rezumat. *OFDM este o schemă de modulație digitală ce livrează sistemelor de comunicații de bandă largă volume mari de date și imunitate la interferența intersimbol și fading. Estimatorul de canal OFDM și egalizarea sunt probleme critice în dezvoltarea sistemului OFDM. Estimatorul de canal se bazează pe ipoteza sub-benzilor independente. Scopul acestui material este investigarea exhaustivă a schemelor de estimație pentru a decide asupra profilului statistic al canalului.*

Cuvinte cheie: *modulație digitală, zgomot, fading, multiplexare cu divizare ortogonală de frecvență, estimarea canalului.*

Abstract: *orthogonal frequency division multiplexing (OFDM) is a digital modulation scheme which serve broadband communications systems with high volume of data and robust immunity to intersymbol interference and fading. Channel estimation and equalization issue are critical for the design of the OFDM system. The channel estimation relies on the subchannels correlations even if the receiver considers the subchannels independently. It is the purpose of the present paper to investigate estimation schemes in order to decide the appropriate channel statistics.*

Keywords: *digital modulation, noise, fading, orthogonal frequency division multiplexing, channel estimation.*

1. INTRODUCTION

A wireless communications channel is a medium enabling the information carriers to travel from the transmitters to the receivers under the pressure of disturbances and noise. Neither the perturbations nor the frequency characteristics of a band-limited channel are, usually, known a priori. The wireless communications channel could change its physical properties during the time so as its frequency characteristics are time-variant. Accordingly, the signals travelling the channel are distorted by multipath fading and corrupted by noise. Fading, a multiplicative effect which reduces the signal power arriving at the receiver, can be stated as the convolution of the transmitted signal with the channel impulse response. Since the electronic noise (at the receiver) and the transmitted signal are independently (statistically uncorellated) and the noise does not experience fading, the fading degrades the signal-to-noise ratio and decreases the channel capacity. The multipath signals originates from splitting mechanisms such as reflection, refraction, diffraction, and scattering caused by obstacles in the propagation path of the wireless channel. If digital transmissions are employed, the multipath signals are reaching the receiver at different delays and different magnitudes. Both the delay and the magnitude are randomly distributed variables.

In a frequency-selective (dispersive) multipath channel (as a matter of fact the channel is frequency and time and even space selective), each multipath

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signal (which is a copy of the transmitted signal) is spread out by dispersion, with different delays and different amplitudes, causing the received signal get distorted by intersymbol interference [1]. If uncompensated, the intersymbol interference produces high error rates [2].

Multicarrier modulation schemes, including OFDM techniques, are providing high data rate transmisions with robustness against frequency selective fading and narrow band interference [3]. A multicarrier modulation system with a large number of subcarriers is expected to increase the link reliability because a very few subchannels could fail while experiencing fading. The damaged subchannels could be restored by using error correction coding [4].

OFDM is used in digital audio broadcasting systems, digital video broadcasting systems, digital subscriber line standards, wireless LAN standards (IEEE 802.11), and wireless broadband access standards (IEEE 802.16).

The channel estimators for wireless OFDM are regarding the arrangement of the pilot information and the design of a low complexity and high accuracy channel tracking [5]. The one dimensional channel estimations are block-type pilot channel estimation and comb-type pilot channel estimation [6]-[10].

The block-type pilot arrangement uses the Least Square estimator, the Minimum Mean Square Error estimator, and the Modified Minimum Mean Square Error estimator. The comb-type pilot arrangement uses the Least Square estimator, the Maximum Likelihood estimator, and the Parametric Channel Modelling-Based estimator [5].

2. LEAST SQUARE CHANNEL ESTIMATION

The Least Square estimator is the minimum square distance between the received signal and the original signal.

A schematic of a generic noise-corrupted communications system is presented in Figure 1.

The signal $s(t)$ is transmitted to a frequency selective multipath fading channel with the complex impulse response $h(t)$. The signal $s(t)$ is resulting from coding, interleaving and modulating (data mapping) binary information data. Also, pilot (known) symbols are inserted into OFDM symbols after modulation. The modulated data are undertaking Inverse Fast Fourier Transform (IFFT). Finally, a cyclic prefix, usually longer than the maximum expected delay spread is added at the beginning of each OFDM symbol in order to remove intersymbol interference and intercarrier interference in the multipath fading channel.

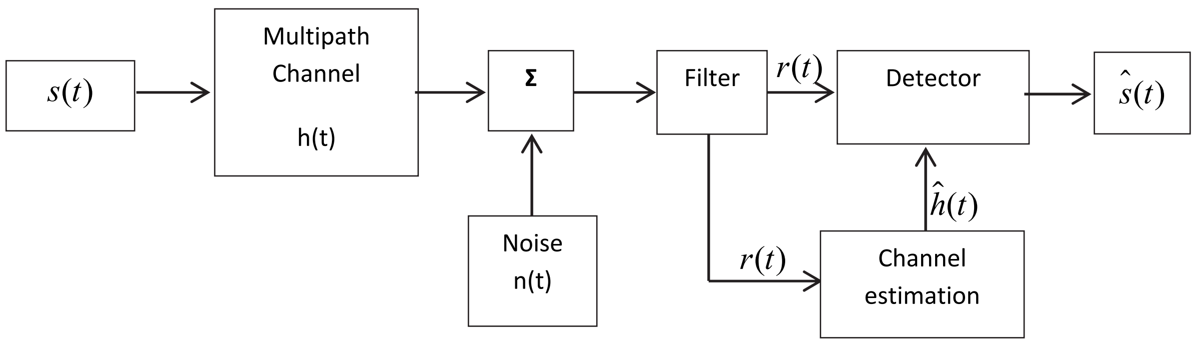

Figure 1. Model of a simple noise-corrupted communications system with channel estimation.

As OFDM scheme is splitting the broadband channel into narrowband subchannels, one sub-channel for each subcarrier, every subchannel is stationary, slow-fading and quasi-flat frequency responsively. More, the subchannels are independent each other.

The impulse response of the multipath channel is (no Doppler frequency shift):

$$h(t) = \sum_{k=1}^{L} \alpha_k \delta(t - \tau_k) \qquad (1)$$

where $\alpha_k = \alpha_k(t)$ is the complex impulse response of the k^{th} propagation path in the channel, $\tau_k = \tau_k(t)$ is the delay time of the k^{th} path, L is the number of paths, $\delta(t)$ is the Dirac function.

Let $X = [X_k]_{k=\overline{0,N-1}}$ and $R = [R_k]_{k=\overline{0,N-1}}$ stand the input data matrix of IDFT (Inverse Discrete time Fourier Transform) at the transmitter and the output data matrix of DFT (Discrete time Fourier Transform) at the receiver, respectively. Let $h = [h_k]_{k=\overline{0,N-1}}$ and $n = [n_k]_{k=\overline{0,N-1}}$ be the sampled channel impulse response matrix and the sampled noise matrix, respectively.

It can be proved that a block-type pilot channel model is [2], [3]:

$$\overline{R} = \overline{X}H + N \qquad (2)$$

where $\overline{R} = R^T$, $\overline{X} = diagX^T$, $N = Fn^T$, $H = Fh^T$; $(\bullet)^T$ is the transpose matrix of the $(\bullet)$ matrix.

$$F = \begin{pmatrix} w_N^{0,0} & \cdots & w_N^{0,N-1} \\ \vdots & \ddots & \vdots \\ w_N^{N-1,0} & \cdots & w_N^{N-1,N-1} \end{pmatrix} \qquad (3)$$

$$w_N^{i,k} = \left(\frac{1}{\sqrt{N}}\right)^{-j2\pi ik/N},\ i,k = \overline{0,N-1}$$

Equation (2) shows out that an OFDM transmission is sending data over parallel channels.

LS estimation is $\min\left\{\left(\overline{R}^T - (\overline{X}H)^T\right)^* \left(\overline{R} - \overline{X}H\right)\right\}$,

where $(\bullet)^*$ denotes the complex conjugate matrix of the $(\bullet)$ matrix.

The LS estimator of the channel, H_{LS}, for a block-type pilot channel is given by [6]:

$$H_{LS} = \overline{X}^{-1}\overline{R} = \left[(X_k / Y_k)\right]^T,\ k = \overline{0,N-1} \qquad (4)$$

The computation of LS estimator in (4) is of a low complexity but it suffers from high mean errors [5].

The comb-type pilot estimation scheme interleaves periodically pilot symbols and OFDM data symbols. The receiver learns the pilot locations $P = [P_k]^T$, $k = \overline{0,N_p - 1}$, where N_p is the number of the pilot symbols, and the pilot values $X^p = \left[X_k^p\right]^T$, $k = \overline{0,N_p - 1}$. Also, the receiver knows the received signal R.

The LS estimator of the comb-type pilot channel is proved to be [5]:

$$\widehat{H}_{LS}^p = \left[R(P_0)/X_0^p, R(P_1)/X_1^p, \ldots, R\left(P_{N_p-1}\right)/X_{N_p-1}^p\right]^T \qquad (5)$$

The channel estimation H (of length N) infers from the pilot symbols estimator $\widehat{H}_{LS}^p$ (of length N_p) and received signals R. Estimation methods (which we will not present herein) are 1D interpolation, the maximum likelihood estimator, and the parametric channel modeling-base estimator [5], [8]-[10].

3. MINIMUM MEAN-SQUARE ERROR CHANNEL ESTIMATION

The Minimum Mean-Square Error (MMSE) is a second-order statistics estimate, namely the conditional variance $\sigma_{Y|X}^2$ of a conditional density function $f_{Y|X}(y \mid x)$ [11].

The Minimum Mean-Square Error estimator for a block-type pilot arrangement channel is [5]:

$$\widehat{H}_{MMSE} = F\widehat{h}_{MMSE} = \\ = C_{HH}\left(C_{HH} + \sigma_n^2\left(XX^{T^*}\right)^{-1}\right)^{-1}\widehat{H}_{LS} \quad (6)$$

where $C_{HH} = E\left\{HH^{T^*}\right\} = FC_{hh}F^{T^*}$, C_{hh} is the auto-covariance matrix of the h^T matrix, $\sigma_n^2 = E\left\{|n|^2\right\}$ is the noise variance.

The Minimum Mean-Square Error estimator is performing much better than the Least Square estimator, at the cost of a high computation complexity [5].

4. CONCLUSIONS

A free-error (in terms of intersymbol interference and inter-carrier interference) coherent detection of a received OFDM signal is improved if proper multipath channel estimation schemes are performed. Block-type pilot arrangement and/or comb-type pilot arrangement are employed in order to estimate the fading channel. Block-type pilot channel estimation fits a slow fading channel, while the comb-type pilot channel estimation tailors fast fading channel.

Least Square and Minimum Mean-Square Error channel estimators are described for both block-type pilot and comb-type pilot arrangements.

The LS estimator is the best interpolation method for the comb-type pilot channel estimation. The Minimum Mean-Square Error estimator is much more appropriate to describe a block-type pilot channel estimation, at the cost of a high complexity computation.

Acronyms

Acronym	Signification
DFT	Discrete time Fourier Transform
IDFT	Inverse Discrete time Fourier Transform
IEEE	Institute of Electrical and Electronic Engineers
IFFT	Inverse Fast Fourier Transform
LAN	Local Area Network
MMSE	Minimum Mean-Square Error
OFDM	Orthogonal Frequency Division Multiplexing

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