CONSIDERATIONS ON THE MACHINE INTERFERENCE PROBLEM. A CASE STUDY

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REZUMAT. Lucrarea tratează o problemă de interferenţă a maşinilor de o importanţă deosebită în organizarea producţiei din industria textilă. Mai precis, este analizată o problemă de predicţie privind randamentul maşinilor şi gradul de ocupare a muncitorului de deservire, în funcţie de numărul de maşini alocate, pe baza unor modele Markov. Problema studiată se referă la cazul în care maşinile deservite de muncitor nu sunt identice şi, prin urmare, ratele medii de întrerupere accidentală şi intensităţile medii de remediere sunt diferite. Studiul evaluează acurateţea rezultatelor obţinute pe baza unor modele Markov reduse, care operează cu valori medii. Rezultatele analitice au fost validate prin comparare cu valorile obţinute prin simulare numerică.

Cuvinte cheie: Interferenţa maşinilor, modele de predicţie, lanţuri Markov, reţele Petri stohastice.

ABSTRACT. This paper deals with a machine interference problem with a significant impact on process management in medium-to-large textile factories. More exactly, a prediction problem regarding the efficiency of the machines, depending to the number of the machines allocated to the service worker, is treated in this paper. The problem we consider is related to the case where the processes are not identical, and consequently, the mean yarn breakage rates and/or the mean remedying rates differ from a machine to the other in a great extent. The analytical approach is based on the Markov chains. The accuracy of the models based on reduced Markov chains is evaluated. All analytical results presented in this work were validated by simulation.

Keywords: Machine interference problem, prediction models, Markov chains, stochastically Petri nets.

1. INTRODUCTION

In the textile processes, such as those of weaving and spinning, random stops occur, especially due to the yarn breakages. In that case, a service worker is required to intervene for remedying the broken yarn and restart the stopped process.

The efficiency of the machines must be as high as possible. For the workers is necessary to ensure an appropriate loading, but an overloading should be avoided.

When the number of the machines that are down is higher than the number of service workers, as no worker is available, a waiting time appears before the remedying process to begin. This time is also called interference time.

The phenomenon of interference affects the efficiency of the machines and must be reduced as much as possible. For this purpose, the service capacity must be increased, but, in that case, the loading of the workers is lower.

An important optimization problem in the production management is to find a solution able to balance the efficiency of the machines and the loading of the workers.

For this purpose, a prediction problem must be solved first regarding to the machine efficiency and the worker loading, depending to the number of the machines allocated to the worker. This is also called the machines interference problem.

The machine interference problem is usually treated for the case that occurs more frequently in practice, where the machines carry out similar processes [1]. For this simpler case, analytical results for evaluating the machine's efficiency and the loading of the worker, depending to the number of the machines allocated to the worker, are available (see for example, [1], [2], [4]).

These analytical results have been obtained under the hypothesis that all the random variables included by the stochastic model have a negative-exponential distribution law. Note that, the yarn breakages in the textile processes form Poissonian fluxes in the most cases.

The machine interference problem becomes much more complicated when the worker serves different textile processes, for which the yarn breakage rates or the remedying time of a broken yarn differ significantly from a machine to the other. Just this case is treated in this paper.

When the machines ensure to some extent yarn breakage tolerance (see, for example [3]), the prediction problem is more difficult to solve and this case is not covered by this study.

2. THE PROBLEM PRESENTATION

Let us consider an automatic machine, affected by accidentally breaks specific to the process that the machine achieves (an weaving or a spinning machine in our case).

Take λ be the mean rate of these random breakages. On the average, the worker can remedy μ machines affected by broken yarns in the unit of time.

The stochastic model for this process affected by accidentally breaks includes two primary random variables, namely:

- The Running Time (RT) of the machine until an accidentally break occurs;
- The Time to Repair (TR) a machine affected by an accidental break.

For these two random variables the distribution law must be known. Note that, the mean values for RT and TR are $\frac{1}{\lambda}$ and $\frac{1}{\mu}$, respectively.

In case the machine is down, the worker starts immediately to remedy the broken yarn (i.e., the interference phenomenon does not occur in this case). The efficiency of the machine is given by (2.1).

$$EF = \frac{\mu}{\lambda + \mu} \tag{2.1}$$

The prediction problem is that, based on this primary stochastic model, to be able to estimate the machine's efficiency and the worker loading, when to the worker are allocated for remedying many machines, with different values for the parameters λ and μ. Take S be the number of the machines allocated to the worker.

To solve this prediction problem, analytical methods based on Markov chains or other elements of the theory of queues can be applied [6], [7]. Also, very important are the simulation techniques based on the formalism of the stochastic Petri nets [5], [6].

The models of Markov chains can be applied when all the primary random variables have negative-exponential distribution laws. Let us consider this particular case.

The analytical method implies first to identify the states of the Markov chain (take n their number), and then to build a matrix of the transition rates between states,

$$A = \left[a_{ij} \right]_{n \times n} \tag{2.2}$$

where $a_{ij}, i \neq j$, represents the transition rate from state j to state i .

For an element of the main diagonal (i.e., $i = j$), the value is calculated as a sum of the other elements in the column taken with the minus.

Based on this matrix of the transition rates, the steady state probabilities $p_1, p_2, \ldots, p_n$ can be obtained by solving a linear equation system, as presented in (2.3), where $P = [p_1, p_2, \ldots, p_n]^T$.

$$\begin{cases} A \times P = 0 \\ \sum_{i=1}^{n} p_i = 1 \end{cases} \tag{2.3}$$

With the steady state probabilities in hand, the efficiency of the machines can be calculated as the sum of the probabilities of all the states of success.

In the following section, a case study in which an weaver serves three different weaving machines is presented.

3. CASE STUDY

Let us consider three weaving machines ($S = 3$) that produce different articles, identified by 1, 2 and 3. The mean breakage rates for these machines are $\lambda_1, \lambda_2, \lambda_3$, and the mean remedying rates of the broken yarns are μ_1, μ_2, μ_3 . All these weaving machines are allocated to a weaver.

The Markov chain describing the evolution of the system composed of the weaving machines and the weaver is presented in Fig. 3.1.

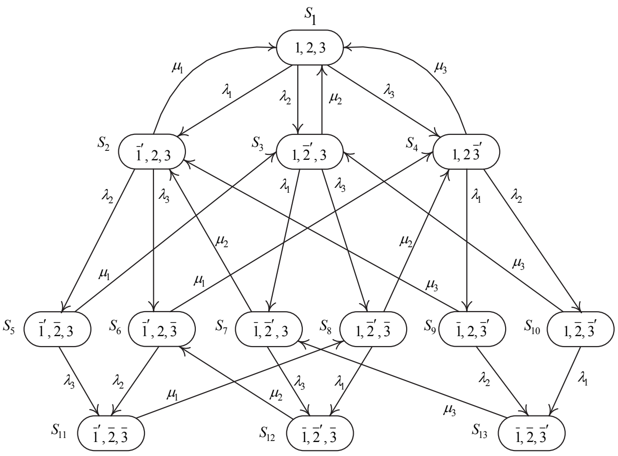

Fig. 3.1 Markov chain: three machines, one service worker.

The state of each weaving machine (up or down) is specified for any node in this graph. A down state is highlighted by using a denied symbol, and the weaving machine under remedying is specified by using the symbol „'". The transition rate matrix of this Markov model is presented in Fig. 3.2.

$$A=\begin{bmatrix}
-(\lambda_1+\lambda_2+\lambda_3) & \mu_1 & \mu_2 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\lambda_1 & -(\lambda_2+\lambda_3+\mu_1) & 0 & 0 & 0 & 0 & \mu_2 & 0 & \mu_3 & 0 & 0 & 0 & 0\\
\lambda_2 & 0 & -(\lambda_1+\lambda_3+\mu_2) & 0 & \mu_1 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0\\
\lambda_3 & 0 & 0 & -(\lambda_1+\lambda_2+\mu_3) & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0\\
0 & \lambda_2 & 0 & 0 & -(\lambda_3+\mu_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \lambda_3 & 0 & 0 & 0 & -(\lambda_2+\mu_1) & 0 & 0 & 0 & 0 & 0 & \mu_2 & 0\\
0 & 0 & \lambda_1 & 0 & 0 & 0 & -(\lambda_3+\mu_2) & 0 & 0 & 0 & 0 & 0 & \mu_3\\
0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & \mu_1 & 0 & 0\\
0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & -(\lambda_2+\mu_3) & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & -(\lambda_1+\mu_3) & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \lambda_3 & \lambda_2 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & \lambda_1 & 0 & 0 & 0 & -\mu_2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & \lambda_1 & 0 & 0 & -\mu_3
\end{bmatrix}$$

Fig. 3.2 The transition rate matrix for the Markov chain presented in Fig. 3.1.

By solving the linear equation system (2.3), the steady state probabilities $p_1, p_2, \ldots, p_{13}$ are obtained.

The efficiency of the three weaving machines can be calculated by applying the following equations:

$$EF_1 = p_1 + p_3 + p_4 + p_8 + p_{10}$$
$$EF_2 = p_1 + p_2 + p_4 + p_6 + p_9 \qquad (3.1)$$
$$EF_3 = p_1 + p_2 + p_3 + p_5 + p_7$$

The weaver loading (O) is given by the equation:

$$O = 1 - p_1 . \qquad (3.2)$$

For a numerical analysis, let us consider the following values for the breaking and the remedying rates:

λ_1= 3, λ_2= 2, λ_3= 1 yarn breaks/ hour,
and
μ_1= 20, μ_2= 17, μ_3= 15 remedies operations/ hour.

With these values, the following numerical results are obtained:

$EF_1 = 84.58\%$,
$EF_2 = 87.79\%$,
$EF_3 = 92.39\%$, and
$O = 29.17\%$.

For these three different weaving processes, the mean efficiency is
$EF_m = 88.25\%$.

This case study shows that this analytical method based on Markov chain is very well formalized and gives complete information regarding the process under study. Nevertheless, this method is more difficult to apply when the number of the machines allocated to the weaver increases. Observe that, for this case study, the weaver loading (29.17 %) is very low. That means that in fact the weaver serves usually much more weaving machines: up to 10 machines, 15 machines or even more.

The main problem for the models based on Markov chains derives from the fact that the number of states increases exponentially with the number of machines allocated to the worker.

For example, when $S = 4$, the Markov chain comprises 33 states, when $S = 5$, 81 states, and when $S = 10$, 5121 states (!). In practice, to overcome this major difficulty, approximate methods are applied.

4. AN APPROXIMATE METHOD

The approximate method usually used to reduce the Markov chain is that in which the machines allocated to the weaver are considered identically. In that case, equivalent rates for yarn breakages (λ) and remedying operations (μ) must be determined. To obtain almost equivalent rates, the following equations can be used:

$$\lambda = \sum_{i=1}^{S} \lambda_i \qquad (4.1)$$

$$\mu = \frac{1}{S}\sum_{i=1}^{S} \mu_i \qquad (4.2)$$

For $S = 3$, a reduced Markov chain for this approximate method and the corresponding transition rate matrix are presented in Fig. 4.1 and Fig 4.2, respectively.

The steady state probabilities p_1, p_2, p_3 and p_4 are obtained by solving the linear equation system (2.3). Based on these state probabilities, the mean efficiency of the weaving machines (EF_m) and the weaver loading (O) can be calculated by using the following equations:

$$EF_m = \left(p_1 + \frac{2}{3}p_2 + \frac{1}{3}p_3\right)\times 100 \ (\%) \qquad (4.3)$$

$$O = (1 - p_1)\times 100 \ (\%) \qquad (4.4)$$

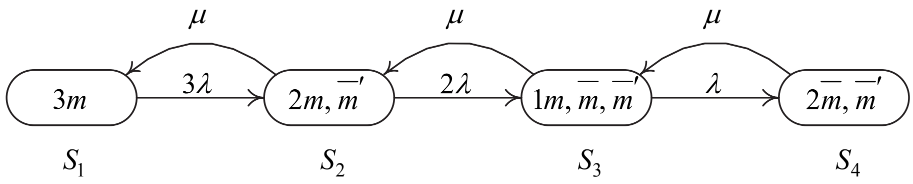

Fig. 4.1 Reduced Markov chain for the approximate method..

$$A = \begin{bmatrix} -3\lambda & \mu & 0 & 0 \\ 3\lambda & -(2\lambda+\mu) & \mu & 0 \\ 0 & 2\lambda & -(\lambda+\mu) & \mu \\ 0 & 0 & \lambda & -\mu \end{bmatrix}$$

Fig. 4.2. The transition rate matrix for the reduced Markov chain..

For example, for the three weaving processes considered in the case study, the mean efficiency of the weaving machines and the weaver loading are:

$$EF_m = 87.61\%\,,\ O = 30.33\%\,.$$

Comparing with the exact values presented in section 3, we can say that the accuracy of estimation is quite good. More exactly, for the mean efficiency of the weaving machines, the relative estimation error is 0.73%, and for the weaver loading, the relative estimation error is something bigger, 3.98%. We have checked by simulation all these analytical results.

5. CONCLUSIONS

The prediction accuracy for the efficiency of the weaving machines explains the applicability of this approximate method based on a reduced Markov chain – a model much easier to compute. Nevertheless, using this approximate method, we can obtain only partial information regarding to the mean efficiency of the machines, whereas, by applying the model presented in section 3 we can obtain complete information regarding the processes under study. But, having in view the complexity of a complete Markov chain, we also recommend the simulation techniques as an alternative approach for this prediction problem.

As simulation model, we recommend the formalism based on the stochastic colored Petri nets. This is a subject for an upcoming paper.

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