IRREVERSIBILITY GENERATION ANALYSIS OF REVERSED CYCLE CARNOT MACHINE BY USING THE FINITE SPEED THERMODYNAMICS

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1. INTRODUCTION

The entropy generation estimation becomes an important issue when internal and external irreversibilities of a thermodynamic cycle are considered and an analytical approach of them is sought. The optimization of Carnot cycle and the calculation of the entropy generation is a topic previously developed by the authors for Carnot cycle engines. The model was based on the Direct Method and the First Law of Thermodynamics for processes with Finite Speed [1-6]. In that model a closed system has been studied, no internal irreversibilities generated by throttling process were involved. The study was further developed [7] for an irreversible Carnot cycle with perfect gas as working fluid, achieved in four separate machine components (an isothermal expansion component at \( T_{H} \), an adiabatic expansion component, an isothermal compression component at \( T_{I} \) and an adiabatic compression component) that are connected through tubes and valves, keeping the expansion ratio during the isothermal expansion at the high temperature constant.

The results were obtained for a particular set of engine parameters, namely the optimum piston speed for maximum power and optimum speed for maximum first law efficiency has been found and it has been
shown that minimum entropy generation per cycle occurs at maximum efficiency.

This time, a similar model is developed for Carnot cycle refrigeration machine. The objective of this work is to find an analytical expression of the internal entropy generation per cycle and entropy generation rate, and to compare it with proposed equations in the literature. Also, an attempt of validation is made by using experimental data available for a real operating refrigeration machine.

2. THERMODYNAMICS OF PROCESSES WITH FINITE SPEED APPLIED TO AN IRREVERSIBLE CARNOT CYCLE OF A REFRIGERATION MACHINE ACHIEVED WITH 4 MACHINE COMPONENTS, USING THE DIRECT METHOD

2.1. Basic equations

The entropy generation per cycle and entropy generation rate for the Carnot refrigeration cycle with irreversible processes is computed using the First Law of Thermodynamics for processes with Finite Speed [1, 8]:

\[
dU = \partial Q_{irr} - p \left( \frac{1}{\sqrt{3RT}} \frac{aw_p}{c} + \frac{f \Delta p_f}{p} + \frac{\Delta p_{th}}{p} \right) dV \tag{1}
\]

where the work in irreversible processes is

\[
\partial W_{irr} = -p \left( \frac{1}{\sqrt{3RT}} \frac{aw_p}{c} + \frac{f \Delta p_f}{p} + \frac{\Delta p_{th}}{p} \right) dV \tag{2}
\]

In equation (2) each term in parenthesis takes into account one type of irreversibility:

- \( \frac{aw_p}{c} = \) contribution of finite speed, with \( c = \sqrt{3RT} \);
- \( \frac{\Delta p_f}{p} = \) contribution of mechanical friction between mechanical parts;
- \( \frac{\Delta p_{th}}{p} = \) contribution of throttling through the valves.

The mechanical friction and throttling losses are expressed in a similar manner to the case of internal combustion engines, from [9]:

\[
\Delta p_f = A + B' w^2_p, \quad \Delta p_{th} = C w^2_p \tag{3-4}
\]

In equations (1) and (2), the sign (+) is for compression and the sign (−) is for expansion.

The First Law expression is combined to the internal entropy generation definition given by:

\[
\frac{Q_H}{T_H} + \frac{Q_L}{T_L} + S_i = 0 \tag{5}
\]

respectively, the total entropy generation one, per cycle, given by:

\[
\frac{Q_H}{T_{H,S}} + \frac{Q_L}{T_{L,S}} + S_T = 0 \tag{6}
\]

The internal and total entropy generation can be computed, \( S_i, S_T \), from equations (5) – (6), and also the internal and total entropy generation rate, \( \dot{S}_i, \dot{S}_T \).

Note that the expression of the First Law of Thermodynamics for processes with Finite Speed has been derived taking into account the irreversibilities introduced by the Second Law of Thermodynamics, caused by the piston finite speed and mechanical friction [2].

Equation (1) can be integrated for various processes with finite speed, to obtain the process equations and also the resulting expressions for the irreversible work and heat.

![Fig. 1. Irreversible Carnot cycle on P-V coordinates.](image1)

Each process in the irreversible Carnot cycles shown in Fig 1 and Fig 2 occurs in a separate machine component. The resulting four components are assumed to be connected by tubes and valves as needed. To derive an easy to apply expression of the internal entropy generation, only irreversibility on the adiabatic processes have been considered here, as shown in Fig. 2.

![Fig. 2. Irreversible Carnot cycle on T-S coordinates.](image2)
2.2. Internal entropy generation calculations

For the internal entropy generation per cycle, we get from (5):

\[ S_{i,\text{cycle}} = - \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \]

which leads to:

\[ S_{i,\text{cycle}} = - (S_1 - S_2) - (S_4 - S_3) = DS_{ad,\text{irr},3-4} + DS_{ad,\text{irr},1-2} \]

\[ = - \frac{\Delta S_{ad,\text{irr}}}{T_L} \]

(8)

The internal entropy generation rate is:

\[ \dot{S}_i = S_{i,\text{cycle}} \times \frac{n_p}{60} \]

(9)

The rotation speed is expressed as a function of piston speed and stroke:

\[ n_p = \frac{1}{t_{\text{cycle}}} = \frac{w_p}{2L} \]

(10)

while the stroke is further computed as a function of displacement volumes and piston head area:

\[ L = \left( V_1 - V_3 \right) / A_p \]

(11)

Upon substitution of eq. (10) and (11) in (9):

\[ \dot{S}_i = S_{i,\text{cycle}} \times \frac{w_p \times A_p}{2(V_1 - V_3)} \]

(12)

Simple mathematical and thermodynamical computations allow us to express the volume difference as:

\[ V_1 - V_3 = V_1 \frac{\dot{e}_t}{\dot{e}_t} = V_1 \frac{\dot{e}_t}{\dot{e}_t} = V_1 \frac{1}{\dot{e}_t} \]

\[ = \frac{V_1}{\dot{e}_t} \]

(13)

We note that eq. (15) is valid for perfect gas machine. The temperature ratio is:

\[ q = \frac{T_H}{T_L} \]

(16)

Upon substituting eqs. (13)–(16) into equation (12):

\[ \dot{S}_i = S_{i,\text{cycle}} \times \frac{w_p \times A_p}{2 \frac{V_1}{\dot{e}_t}} \]

(17)

In equation (17), we have to introduce the internal entropy generation per cycle, computed with equation (8), so we need to compute firstly the entropy generation during the two adiabatic irreversible processes.

2.3. Entropy generation during an adiabatic irreversible process

In order to determine the entropy generation during an adiabatic irreversible process, \( \Delta S_{ad,\text{irr}} \), we need to find the irreversible adiabatic process equation, with finite speed, friction and throttling, and thus the equations for the two irreversible adiabatic processes 1 – 2irr and 3 – 4irr. The First Law for processes with Finite Speed, eq. (1), is used and integrated by applying the Direct Method [1-3, 5, 8]; the adiabatic process condition is imposed, \( \delta Q_{irr} = 0 \):

\[ mc_d\Delta T = - \frac{w_p \dot{e}_t}{\sqrt{3RT}} \frac{f \Delta P_f}{\Delta P_{th}} \frac{1}{p_i} \frac{\dot{d}V}{\dot{V}} \]

(18)

where the factor \( f \) shows the part of friction heat that remains inside the system, \( 0 \leq f \leq 1 \). The case \( f = 0 \) corresponds to the case when all the friction heat is „lost” towards the surrounding (at the cold source); the case \( f = 1 \) is the other extreme, when all friction heat remains inside the system. The calculations are done assuming a value of 0.85 for \( f \).

Equation (18) could be integrated in different assumptions in order to avoid cumbersome calculations. The simplest method of integration is described below: we denote the parenthesis that collects the irreversibility causes with \( B = \text{constant} = f(T_{med, 1-2}, p_{med, 1-2}) \):

\[ B = 1 + \frac{w_p \dot{e}_t}{\sqrt{3RT_{med, 1-2}}} \frac{f \Delta P_f}{\Delta P_{th}} \frac{1}{p_i} \frac{\dot{d}V}{\dot{V}} \]

(19)

where the mean temperature and pressure are expressed as:

\[ T_{med, 1-2} = \frac{T_L + T_H}{2} \]

(20)

\[ p_{med, 1-2} = \frac{p_1 + p_3}{2} = \frac{p_1 \dot{e}_t + p_3 \dot{e}_t}{2} \]

(21)

With equation (16) in (20), this becomes:

\[ T_{med, 1-2} = \frac{1}{2} \left( \frac{T_L \dot{e}_t + T_H \dot{e}_t}{T_L \dot{e}_t + T_L \dot{e}_t} \right) \]

(22)

The same for equation (21) yields:

\[ \frac{p_2}{p_1} = \frac{T_L \dot{e}_t + T_H \dot{e}_t}{T_L \dot{e}_t + T_L \dot{e}_t} = \frac{g}{g^{\dot{e}_t-1}} \]

(23)

Upon substitution of (20)-(23) in (19), it results:

\[ B = 1 + \frac{w_p \dot{e}_t}{\sqrt{3RT}} \frac{2f \left( 4 + B \dot{e}_t \right)}{2(1 + q)} \frac{\dot{d}V}{\dot{V}} \]

(24)

Once the coefficient \( B \) expressed as a function of the piston speed and the other gas parameters, we proceed a variable separation in equation (18):

\[ \frac{mc_d\Delta T}{B \dot{m}RT} = - \frac{\dot{d}V}{\dot{V}} \]

(25)

where the pressure is expressed from the state equation:

\[ p = \frac{mRT}{V} \]

(26)
By taking into account that:
\[ c_v = \frac{R}{g - 1}, \quad c_p = \frac{1}{g - 1} \] (27)-(28)
equation (25) becomes:
\[ \frac{1}{B(g - 1)} \frac{dT}{T} = - \frac{dV}{V} \] (29)

By integrating eq. (29) for the adiabatic process 1 – 2 irr, we get:
\[ \ln \frac{T_2}{T_1} = - B(g - 1) \ln \frac{V_2}{V_1} = \ln \frac{V_1}{V_{2_{irr}}} B \theta(g - 1) \] (30)
which leads to the following equation for the irreversible adiabatic process with finite speed and friction:
\[ q = T_H = \frac{T_{2_{irr}}}{T_1} \frac{V_1}{V_{2_{irr}}} B \theta(g - 1) \] (31)

Assuming 0, the volume expression results to be:
\[ V_{2_{irr}} = V_1 q^{-1} B \theta(g - 1) \] (32)

Since the process 1-2 irr is a compression process, in the analytical expression of B, the (+) signs appears.

In a similar manner we get the equation for the irreversible adiabatic expansion process 3 – 4 irr:
\[ T_3 V_3 \theta(g - 1) = T_{4_{irr}} V_{4_{irr}} \theta(g - 1) \] (33)

Thus, it results:
\[ V_{3_{irr}} = V_3 \frac{V_1}{V_4} B \theta(g - 1) \] (34)
with (-) sign in B, for expansion.

We notice the perfect analogy to reversible adiabatic process equation, for the case when \( w_p \to 0, B \to 1 \) when we find the reversible adiabatic equation from classical thermodynamics:
\[ T_1 V_1 \theta(g - 1) = T_4 V_4 \theta(g - 1) \] (35)
The above determined expressions – eq. (32) and (34) – for the irreversible volumes are used to determine the entropy generation during irreversible adiabatic compression and expansion processes with finite speed, friction, and throttling.

Thus, for the compression process we get:
\[ D_{S_{1_{irr},ad irr,exp}} = m c_v \ln \frac{T_2}{T_1} + m R \ln \frac{V_2}{V_1} = m c_v \ln \frac{T_H}{T_1} + m R \ln \frac{V_2}{V_1} = \] (36)
\[ = m R \frac{1}{g - 1} \ln q + \frac{m R}{B_{exp} (1 - g)} \ln q \]
It results:
\[ D_{S_{1_{irr},ad irr,exp}} = m R \frac{1}{g - 1} \ln \frac{\xi}{\beta} + \frac{1}{B_{exp} (1 - g)} \ln q \] (37)
where \( B_{exp} \) is computed with the (+) sign:
\[ B_{exp} = 1 + \frac{2 f' (A + B w_p)^2 - 2 C \theta}{2 \sqrt{\frac{3 R_1 T_1}{T_2} (1 + q)}} - \frac{p_c (1 + q^{g - 1})}{p_1} \] (38)

In a similar way, for the expansion process we get:
\[ D_{S_{3_{irr},ad irr,exp}} = m R \frac{1}{g - 1} \ln q + \frac{m R}{B_{exp} (1 - g)} \ln q \] (39)

It results:
\[ D_{S_{3_{irr},ad irr,exp}} = m R \frac{1}{g - 1} \ln \frac{\xi}{\beta} + \frac{1}{B_{exp} (1 - g)} \ln q \] (40)
where \( B_{exp} \) is computed with the (−) sign:
\[ B_{exp} = 1 + \frac{2 f' (A + B w_p)^2 - 2 C \theta}{2 \sqrt{\frac{3 R_1 T_1}{T_2} (1 + q)}} - \frac{p_c (1 + q^{g - 1})}{p_1} \] (41)

Upon substituting eq. (36) and (39) in (7), the internal entropy generation per cycle results:
\[ S_{i, cycle} = m R \frac{1}{g - 1} \ln q + \frac{1}{B_{exp}} \frac{1}{\beta} \frac{1}{B_{exp}} \frac{1}{\beta} \] (42)

Finally, from eq. (17), the expression of the internal entropy generation rate as a function of the piston speed becomes:
\[ \dot{S}_{i, cycle} = \frac{m R}{g - 1} \ln q + \frac{1}{B_{exp}} \frac{1}{\beta} \frac{1}{B_{exp}} \frac{1}{\beta} \] (43)

Similar expressions can be given as a function of the mass flow rate of the working fluid and the rotation speed respectively:
\[ \dot{S}_{i, cycle} = \frac{m R}{g - 1} \ln q + \frac{1}{B_{exp}} \frac{1}{\beta} \frac{1}{\beta} \] (44)

\[ \dot{S}_{i, cycle} = \frac{m R}{g - 1} \ln q + \frac{1}{B_{exp}} \frac{1}{\beta} \frac{1}{\beta} \] (45)

3. PARTICULAR FORMS

We take as reference equation (43), (38) and (41), for which some particular cases are emphasized in what follows.

By taking into account internal irreversibility generated by finite speed \( w_p \) and pressure losses due to friction \( A_p \) it results the following expression for the internal entropy generation rate when only adiabatic irreversible processes have been considered:
\[ \dot{S}_{i, cycle} = m C_{ad irr} \frac{T_H}{T_1} = c_{ad irr} \ln q \] (46)

Thus, we notice a logarithmic dependence of the internal entropy generation rate on gas temperature ratio.
With respect to the literature [12-14], this time, the “constant” $C_{ad, irr}$ is analytically found to be a function of eleven parameters ($w_p, A_p, e_p, 0, T_1, P_1, P_o, f, A, B'$):

$$C_{ad, irr} = \frac{nR}{g - 1} \times \frac{\ln \left( \frac{T_0}{T_1} \right)}{1 - e_p \frac{1}{T_1} \ln \left( \frac{T_0}{T_1} \right)} \times \frac{1}{B_{exp}} \times \frac{1}{B_{exp}}$$

Some particular cases:

1. The case for which we consider a piston speed $w_p \rightarrow 0$. Even in these conditions, we still have losses introduced by friction since:

$$B_{exp} \big|_{w_p = 0} = 1 + \frac{2 \sqrt{f \times A}}{p_1 + q g^{-1}}$$

and

$$B_{exp} \big|_{w_p = 0} = 1 - \frac{2 \sqrt{f \times A}}{p_1 + q g^{-1}}$$

2. Further more, although if we consider friction between piston and cylinder, but if the fraction $f \rightarrow 0$ (no friction heat remains inside the gas), the state $2_{irr}$ is not perturbed. The process is still adiabatic irreversible, but with external irreversibilities generated by pressure losses due to friction $\Delta P$, that consumes a higher mechanical work to cover the friction. This mechanical work is found in the total entropy generation $S_T$ and respectively, $\tilde{S}_T$.

3. The extreme case when the piston speed $w_p \rightarrow \infty$, which corresponds to the passage of state $2_{irr}$ in state $2_{irr}$ during the compression process. This means that the adiabatic process with infinite speed is equivalent to an isometric process (see Fig. 1 and 2).

4. At the reversible limit, $w_p \rightarrow 0$, $B_{exp} \rightarrow 1$, and $A = 0.94; B' = 0.045; C = 0.035$, and $\gamma = 1.178$.

4. RESULTS AND DISCUSSIONS

A Finite Speed analysis of the entropy generation in a Carnot cycle refrigeration machine achieved in four components was performed. Irreversibility due to the finite speed of the piston, friction, and throttling during the adiabatic compression and expansion has been taken into account. An analytical expression of the entropy generation rate has been derived. Besides its expected logarithmic dependence on the temperature ratio, it yielded as a function of several parameters of the model through the term $C_{ad, irr}$. The comparison of our expression (eq. (43)) with proposed dependence in the literature [12-14]:

$$S_{i, cycle} = c t \left(T_H - T_L\right)$$

obviously shows that these forms are too simple. Even the last one, which is nearest to our result, is still far from an accurate evaluation of the real operating conditions. It can be seen that the term $C_{ad, irr}$ is not a constant, since it depends on eleven parameters. Hence, the present model extends the approximations from literature and clearly indicates their parametric dependence.

![Fig. 3](image-url)

Fig. 3. Entropy generation rate determined analytically (using the DM) and from experiments (exp), for $Q_L = 4$ kW.

![Fig. 4](image-url)

Fig. 4. Entropy generation rate determined analytically (using the DM) and from experiments (exp), for $Q_L = 5$ kW.

Another comparison of our result can be made with experimental data available for a real operating refrigeration machine [10, 11]. In this respect an equivalent reversed Carnot cycle is considered instead of the real cycle. It results from experimental data using appropriate approximations for evaporator and condenser processes. The calculations are done using eq. (45) and assuming the following value [9] of the coefficients $B'$'s given by the eq. (38) and (41): $A = 0.94; B' = 0.045; C = 0.035$, and $\gamma = 1.178$.

Two values of the cooling load for which experimental results have been selected were considered, in order to show that the accurate prediction of our results is not accidentally. As illustrated in Fig. 3 and 4, our results are close to the experimental data.

5. CONCLUSIONS

Important calculations of the entropy generation of a four cylinder irreversible Carnot cycle refrigeration machine, based on a Finite Speed analysis have been made and are presented.

A thermodynamic approach based on the Direct Method and First Law of Thermodynamic for processes...
with Finite Speed is shown to be especially effective for analytical estimation of the entropy generation rate since the comparison with available predictions have shown a more realistic evaluation. Also, the results are shown to be accurate when compared to experimental data.

Nomenclature

\begin{itemize}
  \item A – Area \([\text{m}^2]\)
  \item a, B’ – Coefficients
  \item C – Coefficient
  \item c – Average molecular speed \([\text{m s}^{-1}]\)
  \item L – Length \([\text{m}]\)
  \item m – Mass \([\text{kg}]\)
  \item nr – Rotation speed \([\text{rot/min}]\)
  \item P – Pressure \([\text{Pa}]\)
  \item Q – Heat \([\text{J}]\)
  \item R – Gas constant \([\text{J kg}^{-1} \text{K}^{-1}]\)
  \item S – Entropy \([\text{J K}^{-1}]\)
  \item T – Temperature \([\text{K}]\)
  \item U – Internal energy \([\text{J}]\)
  \item V – Volume \([\text{m}^3]\)
  \item W – Work \([\text{J}]\)
  \item w – Piston speed \([\text{m s}^{-1}]\)
\end{itemize}

Greek symbols

\begin{itemize}
  \item ε – Volumetric compression ratio
  \item γ – Ratio of the specific heats
  \item θ – Temperature ratio
  \item τ – Duration time of a process
\end{itemize}

Subscripts

\begin{itemize}
  \item ad – Adiabatic
  \item cpr – Compression
  \item DM – Direct Method (Figs. 3 and 4)
  \item exp – Expansion or Experimental
  \item f – Friction
  \item H – Related to the gas at the source
  \item HS – Source, at the hot side of the engine
  \item i – Internal
  \item irr – Irreversible
  \item L – Related to the gas at the sink
  \item LS – Sink, at the cold side of the engine
  \item med – Medium
  \item p – Piston
\end{itemize}

REFERENCES