MATHEMATICAL AND PHYSICAL MODELING OF ENERGY SAVING IN THE CONSTRUCTIONS OF PROTECTED GROUND

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Rezumat. *Lucrarea prezintă metoda şi rezultatele obţinute în urma analizei economisirii energiei în cazul culturilor vegetale şi a altor exploataţii agricole.*
Cuvinte cheie: *transfer de căldură nestaţionar, ecranare termică în sere, câmp de temperatură în sol.*
Abstract. *We describe a method and results of the investigations of energy saving in agricultural production areas.*
Keywords: *nonstationary heat transfer, thermal screens in greenhouses, temperature field in the soil.*

The problem of reducing heat losses in the course of cultivation of vegetables and other products in greenhouses is quite important. The installation of energy saving screens can decrease significantly these losses. The use of screens favors a decrease in the heat losses in greenhouses by 40 - 50%.

The data on the use of screens being transformed or established permanently, which can be find in the literature, are often contradictory and not always substantiated. This problem is worthy of studying in order to develop specific recommendations for the installation of energy saving screens in the constructions of protected ground.

In the mathematical formulation of the problem, we assume that the initial temperature field is assigned by an arbitrary function $F_I(x)$.

For the filler structure under consideration, the system of equations will be one-dimensional without internal heat sources. The system of differential equations describing nonstationary heat transfer in an n-layer body has the form

$$\frac{\partial t_i}{\partial \tau} = a_{oi}\left[\frac{\partial^2 t_i}{\partial x^2} + \frac{\Gamma}{x}\frac{\partial t_i}{\partial x}\right] + \varepsilon i \frac{a_{oi}}{2}\left[\frac{\partial^2 t_i^2}{\partial x^2} + \frac{\Gamma}{x}\frac{\partial t_i^2}{\partial x}\right] - \frac{\beta}{2}\frac{\partial t_i^2}{\partial \tau};$$

$$i = 1, 2, ..., n. \quad (1)$$

The initial and boundary conditions can be written in the following way:

$$t_i\,(x, 0) = F_t\,(x) \quad (2)$$

$$-\frac{\partial t_i\,(l_0,\tau)}{\partial x} + H_{00}[t_1(l_0,\tau) - \varphi_1(x)] - \varepsilon_1 H_{01} t_1(l_0,\tau)[t_1(l_0,\tau) - \varphi_1(\tau)] = 0; \quad (3)$$

$$-\frac{\partial t_n(l_n,\tau)}{\partial x} + H_{0n}[t_n(l_n,\tau) - \varphi_n(x)] - \varepsilon_n H_{0n} t_n(l_n,\tau)[t_n(l_n,\tau) - \varphi_n(\tau)] = 0; \quad (4)$$

$$\lambda_{0i}\left[\frac{\partial t_i\,(l_i,\tau)}{\partial x} + \frac{\varepsilon_i}{2}\frac{\partial t_i^2(l_i,\tau)}{\partial x}\right] = \lambda_{0(i+1)}\left[\frac{\partial t_{i+1}(l_i,\tau)}{\partial x} + \frac{\varepsilon_{i+1}}{2}\frac{\partial t_i^2(l_i,\tau)}{\partial x}\right]; \quad (5)$$

$$\lambda_{0i}\left[\frac{\partial t_i\,(l_i,\tau)}{\partial x} + \frac{\varepsilon_i}{2}\frac{\partial t_i^2(l_i,\tau)}{\partial x}\right] = \frac{1}{R_i}\left[t_{(i+1)(l_i,\tau)} - t_i\,(l_i,\tau)\right]; \quad (6)$$

$$i = 1,2, ..., (n-1).$$

We can represent the temperature and heat flux in each layer as a series in terms of powers of the parameters ε and β:

$$t_i(x,\tau) = t_{0i}(x,\tau) + \sum_{K=1}^{n}\varepsilon_K t_{Ki} + \sum_{m=n+1}^{2n}\beta_{m-n} t_m; \quad (7)$$

$$q_i(\tau) = q_{0i}(\tau) + \sum_{K=1}^{n}\varepsilon_K q_{Ki} + \sum_{m=n+1}^{2n}\beta_{m-n} q_{mi}, \quad (8)$$

restricting ourselves only to the zero and first power by virtue of the smallness of subsequent terms of these series.

In these equations, l_0 and l_n are the distances from the coordinate origin to the internal and external surfaces, respectively, and l_i is the distance to the boundary of the ith layer. In addition, $\Gamma = 0, 1, 2$ is the form factor, τ is the time, a_0 is the thermal diffusivity, $H_0 = \frac{a_0}{\lambda_0}$ is the relative heat transfer coefficient, a_0 is the heat transfer coefficient, λ_0 is the thermal conductivity, and R is the thermal resistance.

The linearization of our problem can be carried out as follows: substituting the values of $t_i\,(x, \tau)$ and $q_i(\tau)$ from relations (7) and (8) in Eqs. (1) - (6), we obtain $(2n + 1)$ linear boundary-value heat conduction problems for the functions t_{oi}, $t_{\varepsilon ki}$, and $t_{\beta mi}$ for each layer; $i = 1,2, ... ,n$; $k = 1, 2, ... n$; $m = (n+ 1), (n + 2), ..., 2n$.

To solve the formulated problem, we apply the method of finite integral transformations [11. This method reduces an inhomogeneous boundary-value heat conduction problem for the image space in the case of multilayer walls to an $(n - 1)$-dimensional vector system of $(2n + 1)$ Volterra integral equations of second kind, the solution of which does not present difficulties.

Calculations were carried out for three values of the ambient temperature: t_n = –30°C, –20°C, and –10°C. The temperature inside the greenhouse was taken equal to +18°C.

Using the results of calculations, we have constructed plots of temperature variation in the filler structure of the greenhouse with a screen for different values of the ambient temperature t_n (Fig. 1).

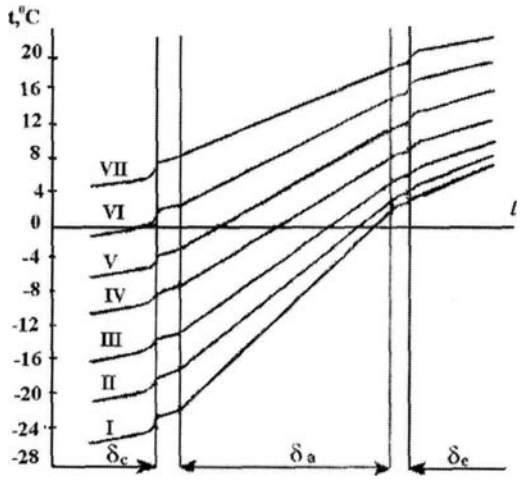

Fig. 1. Temperature variation in the filler structure in the case of installation of an energy saving screen. Curves I-VII correspond to t_n = –26 °C, –21 °C, – 16 °C, – 11 °C, – 6 °C, – 1 °C, and + 4 °C, respectively.

We have shown that the temperature at the internal surface of transparency of the greenhouse differs slightly from its value in the environment. At the same time, the temperature at the internal surface of the screen is positive irrespective of the temperature of outdoor air. The installation of a screen not only leads to a decrease in the heat losses through the filler structures but simultaneously provides for more favorable climatic conditions for the plants growing near the external walls of the greenhouse.

For evaluating the efficiency of the use of thermal screens in greenhouses, we applied the model of an eleent of translucent railing with a movable film screen. Experiments were carried out in a climatic chamber, which enabled one to model the parameters of environent in a wide range, i.e., at different temperatures and wind heads.

The experimental data demonstrate a substantial energy saving role of the screen. For example, in the case of a wind velocity of 6 m/sec and an ambient temperature of –25 °C, the temperature constitutes –23,9 °C at the internal surface of transparency and +1,18 C at the screen surface.

A rational organization of the thermal conditions in the constructions of protected ground is connected with the calculation and evaluation of the factors determining the formation of temperature field in the soil and air adjoining it.

The analysis and calculation of temperature field in the soil is usually based on the analytical theory of heat conduction.

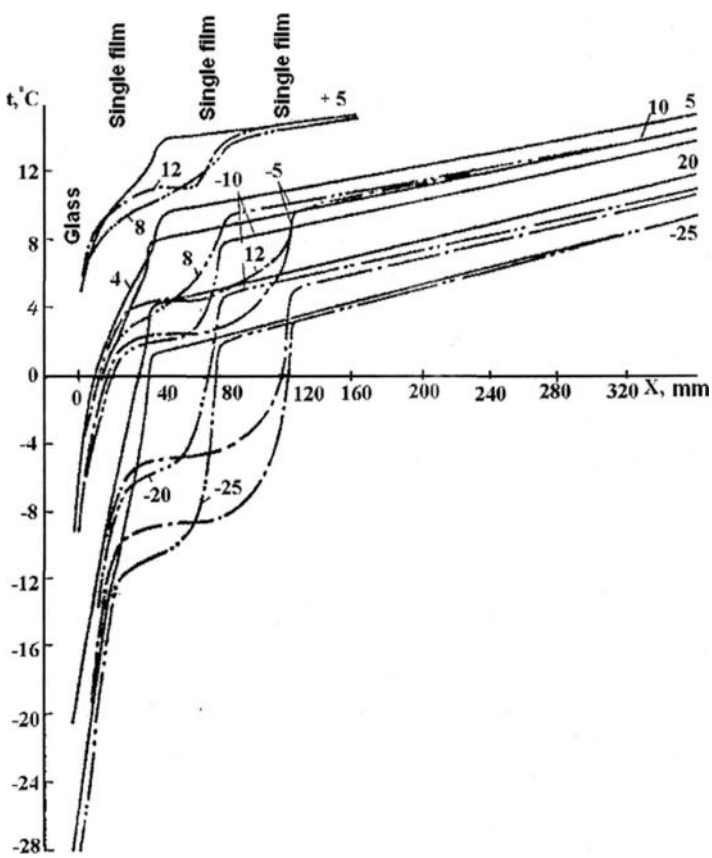

Fig. 2. Plots of temperature variation in the railing with a screen depending on the value of ambient temperature.

In view of the inhomogeneity and multiphase composition of soils, which contain, along with the skeleton, intergranular gas and soil moisture, it is customary to apply here the equations describing the process of effective heat conduction. It is more expedient to consider the soil as a piecewise homogeneous medium with plane interfaces. Such a model to a greater degree takes into account the inhomogeneity of thermophysical characteristics of the soil over depth. In addition, it is desirable here to develop a calculating model that allows for the presence of discrete heat sources in the soil. In what follows, we describe a method for determining the temperature field in the inhomogeneous soil with discrete heat sources.

In this problem, we deal with the temperature field, i.e., with a potential harmonic function E, which will be denoted by $\varphi(x, y, z)$. Let us find it for each layer of a multilayer domain.

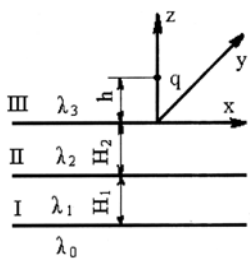

Fig. 3. A scheme of calculation of the temperature field in the multilayer medium with a point source.

We calculate the potential function $\varphi(x, y, z)$ from a point source with a heat intensity q, located in one of the layers of the medium under study (Fig. 3). In the medium where the source is situated, the temperature field is three-dimensional and satisfies the Poisson equation, and, in the other media, it satisfies the Laplace equation. We choose the coordinate system in such a way that the plane XOY should coincide with one of the interfaces between the media, e.g., with the lower. We place the Z axis in such a way that it should pass through the point source. The coordinates of the latter $p(x_m, y_m, z_m)$ are equal to 0, 0, h, respectively.

The medium has three interfaces, and the layers between them are denoted by 0, I, II, III and have thermal conductivities λ_0, λ_1, λ_2, and λ_3, respectively. The thicknesses of the corresponding layers are denoted in Fig. 3 by H_1 and H_2.

The main equations for the field in such a four-layer medium $\delta(x, y, z - h)$ are

$$\Delta\varphi_3 = -q\delta(x, y, z-h);$$
$$\Delta\varphi_2 = 0;\ \Delta\varphi_1 = 0;\quad \Delta\varphi_0 = 0. \tag{9}$$

The first expression in (9) is a Poisson equation. It contains the Dirac delta function $\delta(x, y, z - h)$ since we have a point source. The Dirac function $\delta(x - x_0, y - y_o, z - z_0)$ (it takes zero values everywhere, except the point x_0, y_o, z_0, where it becomes infinite) is applied for the mathematical description of a point source, and, having determined the field for the latter (the Green function), we easily find the field for any source distribution by the method of superposition. The boundary conditions at infinity and at the interfaces between the media are

$$\varphi_3(\infty) = 0;\ \varphi_0(-\infty) = 0;\ \varphi_3(\infty) = 0;$$
$$\varphi_3(\infty) = \varphi_2(0) = 0;\ \varphi_2(-H_2) = \varphi_1(-H_2) = 0;$$
$$\varphi_1(H_1 - H_2) = \varphi_0(H_1 - H_2) = 0;$$
$$\lambda_3 \left.\frac{d\varphi_3}{dz}\right|_{z=0} = \lambda_2 \left.\frac{d\varphi_2}{dz}\right|_{z=0};$$
$$\lambda_2 \left.\frac{d\varphi_2}{dz}\right|_{z=H_2} = \lambda_1 \left.\frac{d\varphi_1}{dz}\right|_{z=H_2}; \tag{10}$$
$$\lambda_1 \left.\frac{d\varphi_1}{dz}\right|_{z=-(H_1-H_2)} = \lambda_{01} \left.\frac{d\varphi_0}{dz}\right|_{z=-(H_1+H_2)}.$$

Using these equations and boundary conditions, we can find the function $\varphi(x, y, z)$ for any point of the space.

Having applied the twofold Fourier integral transformation, we replace problems for partial differential equations by simpler boundary-value problems for ordinary differential equations [2]. The general solutions of these equations, containing constants of integration C_{mn} (mn is the number of the layer), are well known.

Let us write them:

$$\varphi_3\big|_{-\infty\le z_s\le h} = \frac{1}{2\varepsilon}\left[e^{\varepsilon z_s}C_{13} + e^{-\varepsilon z_s}C_{23}\right],$$
$$\varphi_3\big|_{-\infty\le z_s\le h} = \frac{1}{2\varepsilon}\left[e^{\varepsilon z_s}C_{13} + C_0 e^{-\varepsilon h} + e^{-\varepsilon z_s}\left(C_{23} - C_0 e^{\varepsilon h}\right)\right], \tag{11}$$

where z_s is the coordinate of a point S, where we search for the field, and

$$C_0 = q / (\lambda_3 4\pi^2).$$

The solutions for $\varphi_0 < \varphi_1$, and φ_2 are

$$\varphi_2 = C_{12}e^{sZ_s} + C_{22}e^{sz_s};$$
$$\varphi_1 = C_{11}e^{sz_s} + C_{21}e^{sz_s}; \tag{12}$$
$$\varphi_0 = C_{10}e^{sz_s} + C_{20}e^{sz_s}.$$

The constants of integration can be found from the solution of a system of equations for these constants. Substituting the obtained values in (11) and (12), we find the image of the required function for all layers.

Returning to the original of the function, i.e., carrying out the inverse Fourier transformation, we write

$$\varphi_0(x, y) = C_0\mu_{32}\mu_{10}\mu_{21} \Big/ 2\int_{-\infty}^{-\infty}\int e^{-\varepsilon h}e^{\varepsilon z_s}/(\zeta_x+\zeta_y)\times$$
$$\times d\zeta_x d\zeta_y / \Big[\varepsilon(1 + \gamma_{32}\gamma_{10}e^{-2\varepsilon H_1} + \gamma_{10}\gamma_{21}e^{-2\varepsilon H_2} + \tag{13}$$
$$+ \gamma_{32}\gamma_{10}e^{-2\varepsilon(H_1+H_2)})\Big],$$

where j is the imaginary unit, $\varepsilon = \zeta_x^2 + \zeta_y^2$ and ζ_x and ζ_y are the variables of Fourier transformation with respect to the coordinates.

Passing to polar coordinates and using the Bessel function $I_0(\aleph, R)$, we arrive at the following expression for the potential:

$$\varphi_0(x, y) = \frac{\mu_{32}\mu_{10}\mu_{21}q}{4\pi\lambda_3}\int_0^\infty e^{(z_s-h)\aleph} I_0 = (\aleph, R)d\aleph/[1+$$

$$+\gamma_{32}\gamma_{21}e^{-2\aleph H_1}+\gamma_{10}\gamma_{21}e^{-2\aleph H_2}\gamma_{32}\gamma_{10}e^{-2\varepsilon(H_1+H_2)}], \qquad (14)$$

where $\aleph=\zeta_x^2+\zeta_y^2$, $R=x_s^2+y_s^2$ (it follows that $\aleph=\varepsilon$).

The form of the other expressions for the potential in all four layers will be similar.

A product of the Bessel function by an exponential and a certain function

$$f(\aleph)=1/\left[1+ae^{-2H_1\aleph}+be^{-H_2\aleph}+ce^{2(H_1+H_2)\aleph}\right]$$

figures under the integral in (14). As is well known from the theory of Bessel functions, the integral

$$\int_0^\infty e^{-\alpha\aleph}I_0(\aleph,D)\mathrm{d}\aleph=1/\sqrt{\alpha^2+D^2}, \qquad (15)$$

where α and D) are certain parameters.

Hence, expanding the function $f(\aleph)$ into a converging series of exponentials, we can represent each expression for the potential in the layers as a series of integrals (13) and calculate it exactly. The expansion of the function $f(\aleph)$ into a series of exponentials has the form

$$f(\aleph)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}P_{mn}e^{-2[(n-m)H_1+mH_2]\aleph}, \qquad (16)$$

where the coefficient

$$P_{mn}=(-1)^n a^{n-2m}\times$$
$$\times\left\{\left[n-m+1+\Sigma\frac{\prod_{i=0}^{p-1}(m-1-i)\prod_{i=0}^{p}(m-n+1-i)}{p!(p+1)!}\right]\times a^m b^m+\right.$$
$$+\sum_{\varepsilon=1}^{m}\left[n-m+1-\varepsilon+\sum_{p=1}^{m-n}\frac{\prod_{i=0}^{p-1}(m-1-i)\prod_{i=0}^{p}(m-n+1-\varepsilon-i)}{p!(p+1)!}\right]\times$$
$$\left.(-1)\varepsilon\frac{\prod_{i=0}^{\varepsilon-1}(m-i)}{\varepsilon!}ab^{m-\varepsilon}c^{\varepsilon}\right\}, \qquad (17)$$

n, m, p, and i are summation indices, $a=\gamma_{32}\gamma_{21}$, $b=\gamma_{10}\gamma_{21}$, and $c=\gamma_{32}\gamma_{10}$.

Substituting the P_{mn} value in (16) and afterwards the functions from (16) in (14), changing the order of integration and summation of the series, and taking the integral in (14), we obtain

$$\varphi_0(x,y)=\frac{\mu_{32}\mu_{10}q}{4\pi\lambda_3}\times$$
$$\times\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}P_{mn}\int e^{-2[(n-m)H_1+mH_2]\aleph}\times I_0(\aleph,R)\mathrm{d}\aleph=$$
$$=\frac{\mu_{32}\mu_{10}\mu_{21}q}{4\pi\lambda_3}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}P_{mn}(-1)/ \qquad (18)$$
$$/\sqrt{-2[(n-m)H_1+mH_2]+(z_s-h)^2+R^2}.$$

Carrying out similar operations and transformations, we can find expressions for the potentials of all layers of the multilayer medium under consideration. This method is applicable for the solution of various practical problems connected with heat transfer to the soil (or from it) in agricultural production areas.

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