MAXIMUM PROFIT PERFORMANCE FOR A GENERALIZED IRREVERSIBLE CARNOT HEAT PUMP

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1. INTRODUCTION

Since 1970s, the research into identifying the performance limits of thermodynamic processes and optimization of thermodynamic cycles has made a tremendous progress by scientists and engineers with finite time thermodynamics [1-8]. The objective functions in finite time thermodynamics are often pure thermodynamic parameters including power, efficiency, entropy production, effectiveness, cooling load, heating load, COP, loss of exergy, etc. Salamon and Nitzan [9] viewed the operation of the endoreversible heat engine as a production process with work as its output. They carried out the economic optimization of the heat engine with the maximum profit as the objective function [10].

A relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the design and performance of energy systems is exergoeconomic (or thermoeconomic) analysis [11, 12]. Salamon and Nitzan’s work [9] combined the endoreversible model with exergoeconomic analysis. It was termed as finite time exergoeconomic analysis [13-20] to distinguish it from the endoreversible thermodynamic analysis with pure thermodynamic objectives and the exergoeconomic analysis with long-run economic optimization. Similarly, the performance bound at maximum profit was termed as finite time exergoeconomic performance bound at maximum profit, which is termed as the finite time exergoeconomic performance bound. Moreover, the effects of cycle parameters on the performance of the cycle are analyzed by using numerical example.

This paper presents the finite time exergoeconomic performance of a Newtonian law system generalized irreversible Carnot heat pump cycle taking into account several irreversibilities, such as heat resistance, heat leakage and other undesirable irreversible factors. The operation of the generalized irreversible Carnot heat pump cycle is viewed as a production process with exergy as its output. The finite time exergoeconomic performance optimization of the generalized irreversible Carnot heat pump cycle is performed by taking profit as the objective. The relations between the profit and the temperature ratio of refrigerant (working fluid), between the COP (coefficient of performance) and the temperature ratio of refrigerant, as well as the optimal relation between profit and the COP of the cycle are derived. The focus of this paper is to obtain the compromised optimization between economics (profit) and the energy utilization factor (COP) for the generalized irreversible Carnot heat pump cycle, by searching the optimum COP at maximum profit, which is termed as the finite time exergoeconomic performance bound. Moreover, the effects of cycle parameters on the performance of the cycle are analyzed by using numerical example.

Keywords: Finite-time thermodynamics, generalized irreversible Carnot heat pump, exergoeconomic performance, generalized thermodynamic optimization
optimal relation between profit and the COP of the cycle are derived.

2. CYCLE MODEL AND PERFORMANCE ANALYSIS

To simulate the performance of a real heat pump more realistically, some authors [21-23] established a generalized irreversible steady flow Carnot heat pump cycle model as shown in Fig. 1, considering heat resistance, heat leakage, and internal irreversibilities. The assumptions made for the irreversible heat pump with its constant-temperature heat-reservoirs are:

1. The refrigerant flows through the system in a steady-state fashion.
2. The cycle consists of two isothermal and two adiabatic processes. All four processes are irreversible.
3. External irreversibilities are caused by heat-transfers in the high- and low-temperature side heat-exchangers between the heat pump and its surrounding heat reservoirs. Because of the heat-transfers, the refrigerant temperature \( T_{HC} \) and \( T_{LC} \) are different from the heat-reservoir temperature \( T_H \) and \( T_L \). These temperatures are related to one another in the following order:

\[
T_{HC} > T_H > T_L > T_{LC}
\] (1)

![Fig. 1. The generalized irreversible Carnot heat pump cycle model.](image)

4. There is a constant rate of heat leakage \( q \) from the heat sink at the temperature \( T_H \) to heat source at \( T_L \) such that

\[
Q_H = Q_{HC} - q
\] (2)

\[
Q_L = Q_{LC} - q
\] (3)

where \( Q_{HC} \) is the rate of heat flow from the warm refrigerant to the heat sink in the high-temperature heat-exchanger (condenser) across a temperature difference of \( (T_{HC} - T_H) \), \( Q_{LC} \) is the rate of heat flow from the heat source to cold refrigerant in the low-temperature heat-exchanger (evaporator) across a temperature difference of \( (T_L - T_{LC}) \). \( Q_H \) and \( Q_L \) are the rates of total heat-transfer released to the heat sink and absorbed from the heat source, and the heating load of the heat pump is \( Q_H \).

5. Other than irreversibilities due to heat resistance between the refrigerant and the heat reservoirs, as well as the heat leakage between the heat reservoirs, there are more irreversibilities such as friction, turbulence, and non-equilibrium activities inside the refrigerant. Thus when compared with an endoreversible Carnot heat pump of the same heating load, a larger power input is needed. In other words, the rate of heat flow \( Q_{LC} \) from the heat source to the cold refrigerant for the real heat pump is less than that \( Q'_{LC} \) of the endoreversible Carnot heat pump with the same heating load. A constant coefficient \( \varphi \) is introduced in the following expression to characterize the additional miscellaneous irreversible effects:

\[
\varphi = Q'_{LC}/Q_{LC} \geq 1
\] (4)

From the second law of thermodynamics, one has

\[
Q_{HC}/T_{HC} - Q'_{LC}/T_{LC} = 0
\] (5)

Combining Eqs. (4) with (5) gives

\[
Q_{LC} = x Q_{HC}/\varphi
\] (6)

where \( x = T_{LC}/T_{HC} \) (0 < \( x \) ≤ \( T_L/T_H \)) is the temperature ratio of the refrigerant.

Assuming the rates of the heat flow in the heat-exchangers follow the Newton’s law, one has

\[
Q_{HC} = k_1 F_1 (T_{HC} - T_H)
\] (7)

\[
Q_{LC} = k_2 F_2 (T_L - T_{LC})
\] (8)

where \( k_1 \) and \( k_2 \) are the overall heat-transfer coefficient of high- and low-temperature side heat-exchangers, \( F_1 \) and \( F_2 \) are the heat transfer surface area of high- and low-temperature side heat-exchangers. The total heat transfer surface area of the heat exchangers is taken as a constant, that is

\[
F_1 + F_2 = F_T
\] (9)

And a ratio (\( f \)) of heat exchanger area is defined as

\[
f = F_1/F_2
\] (10)

Combining Eqs. (6)- (10) yields

\[
Q_L = Q_{LC} - q = B' x (T_L x^{-1} - T_H) - q
\] (11)

\[
\beta = \frac{Q_L}{P} = \frac{Q_{LC} - q}{Q_{HC} - Q_{LC}} = \frac{B' x (T_L x^{-1} - T_H) - q}{B' (x - \varphi x (T_L x^{-1} - T_H))}
\] (12)

where \( B' = k_1 k_2 F_T [[(1 + f)/(\beta_k + \varphi k_1 f)]] \).

Assuming the environmental temperature is \( T_{en} \), the rate of exergy input of the heat pump is:

\[
A_{en} = Q_H (1 - T_L/T_{H}) - Q_L (1 - T_H/T_L) = Q_H \eta_i - Q_L \eta_2
\] (13)

where \( \eta_i \) is the Carnot coefficient of the reservoir \( i \).
Assuming that the prices of exergy output rate and the power input be $\psi$ and $\psi_2$, the profit of the irreversible Carnot heat pump is:

$$\pi = \psi_1 A_{rev} - \psi_2 P$$  \hspace{1cm} (14)

Combining Eqs. (11)-(14) gives

$$\pi = B(T_L x^1 - T_H) \left[ \psi(\psi_1 \eta_1 - \psi_2) - x(\psi_1 \eta_2 - \psi_2) \right] - \psi q(\eta_1 - \eta_2)$$  \hspace{1cm} (15)

Maximizing $Q_H$, $\beta$ and $\pi$ with respect to $f$ by setting $dQ_H/df = 0$, $d\beta/df = 0$ and $d\pi/df = 0$ in Eqs. (11), (12) and (15) yields the same optimal ratio of heat-exchanger area ($f_o$)

$$f = f_o = (\phi k_2/k_1)^{0.5}$$  \hspace{1cm} (16)

Substituting Eq. (16) into Eqs. (11), (12) and (15), respectively, yields the optimal heating-load, COP, and profit in the following forms:

$$Q_H = B(\psi T_L x^1 - T_H) - q$$  \hspace{1cm} (17)

$$\beta = \frac{Q_L - q}{P} = \frac{Q_{LC} - q}{Q_{HC} - Q_{LC}}$$  \hspace{1cm} (18)

$$\pi = B(T_L x^1 - T_H) \left[ \phi(\psi_1 \eta_1 - \psi_2) - x(\psi_1 \eta_2 - \psi_2) \right] - \psi q(\eta_1 - \eta_2)$$  \hspace{1cm} (19)

where $B = k_1 k_2 F_T / \left[ k_1^{0.5} + (j k_2)^{0.5} \right]^2$.

Maximizing $\pi$ with respect to $x$ by setting $d\pi/dx = 0$ in Eq. (19) yields the optimal temperature ratio and the maximum profit of the heat pump:

$$x_{opt} = \left[ \phi T_L (\psi_2 - \psi_1 \eta_1) / (T_H (\psi_2 - \psi_1 \eta_2)) \right]^{0.5}$$  \hspace{1cm} (20)

$$\pi_{max} = B(T_L \phi (\psi_2 - \psi_1 \eta_1) + T_H (\psi_2 - \psi_1 \eta_2))$$  \hspace{1cm} (21)

Substituting Eq. (20) into Eq. (18) gives $\beta_x$, which is the finite-time exergoeconomic bound of the generalized irreversible Carnot heat pump:

$$\beta_x = \left[ B x_{opt} (T_L x_{opt} - T_H) - q \right] / [B (\phi - x_{opt}) (T_L x_{opt} - T_H)]$$  \hspace{1cm} (22)

## 3. DISCUSSION

If $\phi = 1$ and $q \neq 0$, equations (18), (19), (21) and (22) become:

$$\beta = x/(1-x) - q / [B_1 (1-x)(T_L x^1 - T_H)]$$  \hspace{1cm} (23)

$$\pi = B(T_L x^1 - T_H) \left[ (\psi_1 \eta_1 - \psi_2) - x(\psi_1 \eta_2 - \psi_2) \right]$$  \hspace{1cm} (24)

$$\pi_{max} = B(T_H (\psi_2 - \psi_1 \eta_1) + T_L (\psi_2 - \psi_1 \eta_2))$$  \hspace{1cm} (25)

$$\beta_x = 1/(1-x_{opt}) - q / [B_1 (1-x_{opt})(T_L x_{opt} - T_H)]$$  \hspace{1cm} (26)

where $B_1 = k_1 k_2 F_T / \left[ k_1^{0.5} + (j k_2)^{0.5} \right]^2$.

And $x_{opt} = \left[ T_L (\psi_2 - \psi_1 \eta_1) / (T_H (\psi_2 - \psi_1 \eta_2)) \right]^{0.5}$.

Equation (26) is the finite-time exergoeconomic performance bound of the irreversible Carnot heat pump with heat resistances and heat leakage losses.

If $\phi > 1$ and $q = 0$, equations (19), (21) and (22) become:

$$\pi = B(T_L \beta / (\beta - 1) - T_H \phi)$$  \hspace{1cm} (27)

$$\beta_x = \phi \left[ \left[ \phi [T_L \phi (\psi_2 - \psi_1 \eta_1) / (T_H \psi_2 - T_H \psi_1 \eta_2)]^{0.5} \right] \right]$$  \hspace{1cm} (28)

$$\pi_{max} = B(T_H \phi (\psi_2 - \psi_1 \eta_1) + T_L (\psi_2 - \psi_1 \eta_2))$$  \hspace{1cm} (29)

Equation (29) is the finite-time exergoeconomic performance bound of the irreversible Carnot heat pump with heat resistance and internal irreversibility losses.

If $\phi = 1$ and $q = 0$, equations (19), (21) and (22) become:

$$\pi = B(T_L \beta / (\beta - 1) - T_H \phi)$$  \hspace{1cm} (30)

$$\beta_x = \phi \left[ \left[ \phi [T_L \phi (\psi_2 - \psi_1 \eta_1) / (T_H \psi_2 - T_H \psi_1 \eta_2)]^{0.5} \right] \right]$$  \hspace{1cm} (31)

$$\pi_{max} = B(T_L \phi (\psi_2 - \psi_1 \eta_1) + T_L (\psi_2 - \psi_1 \eta_2))$$  \hspace{1cm} (32)

Equations (32) is the finite-time exergoeconomic performance bound of the endoreversible Carnot heat pump.

The finite-time exergoeconomic performance bound at the maximum profit is different from the classical reversible bound and the finite-time thermodynamic bound at the maximum heating load. It is dependent on $T_H$, $T_L$, $\alpha_{HC D}$ and $\psi_2$. Note that for the process to be potential profitable, the following relationship must exist: $0 < \psi_2 / \psi_1 < 1$, because one unit of power input must give rise to at least one unit of exergy output rate. As the price of exergy output rate becomes very large compared with that of the power input, i.e., $\psi_2 / \psi_1 \rightarrow 0$, equation (19) becomes

$$\pi = B \psi_1 \phi (x_{opt} - T_H) + T_L \eta_1 x - T_H \phi \eta_1 - T_H \eta_2)$$  \hspace{1cm} (33)

One can see that the heating load and the profit are both monotonic decreasing function of temperature ratio of refrigerant. That is the profit maximization approaches the heating load maximization. When $T_H \rightarrow T_0$, equation (33) becomes

$$\pi = \psi_1 \eta_1 Q_{HT}$$  \hspace{1cm} (34)

On the other hand, as the price of exergy output rate approaches the price of the power input, i.e., $\psi_2 / \psi_1 \rightarrow 1$, equation (19) becomes

$$\pi = -\psi_1 T_0 \left[ (Q_{HC D} - q) / T_H - (Q_{HC D} - q) / T_L \right] = -\psi_1 T_0 \sigma$$  \hspace{1cm} (35)

where $\sigma$ is the rate of entropy production of the heat pump cycle. That is the profit maximization approaches the rate of entropy production minimization, in other
word, the minimum waste of exergy. Eq. (35) indicates that the heat pump is not profitable regardless of the COP is at which the heat pump is operating. Only the heat pump is operating reversibly \((\beta = \beta_c)\) will the revenue equal to the cost, and then the maximum profit will be equal zero. The corresponding rate of entropy production is also zero.

Therefore, for any intermediate values of \(\psi_2/\psi_1\), the finite-time exergoeconomic performance bound \((\pi_{\beta})\) lies between the finite-time thermodynamic performance bound and the reversible performance bound. \(\beta_c\) is related to the latter two through the price ratio, and the associated COP bounds are the upper and lower limits of \(\beta_c\).

4. NUMERICAL EXAMPLE

To illustrate the preceding analysis, a numerical example is provided. In the calculations, it is set that:

\[
T_H = 300\,\text{K}, \quad T_L = 260\,\text{K}, \quad T_0 = T_L, \quad k_1 = k_2, \quad F_p = 4\,\text{kW/K}, \quad k = 1.4, \quad \psi_1 = 1000\,\text{yuan/kW}, \quad \psi = C_j (T_H - T_L) \quad \text{and let} \quad C_j = 0.0 - 0.04\,\text{kW/K} ;
\]

\(C_j\) is the thermal conductance inside the heat pump.

Fig. 2 shows the effects of the heat leakage and internal irreversibility losses on the profit versus temperature ratio of the refrigerant and COP versus temperature ratio of the refrigerant. Fig. 3 shows the optimal relation between the profit and COP with different loss items. It illustrates that when \(0 < \psi_2/\psi_1 < 1\) the curves of \(\pi - \beta\) are parabolic-like ones in the case of \(q = 0\), while the curves are a loop-shaped ones in the case of \(q \neq 0\), which is to say there exists the maximum profit. Fig. 4 shows the optimal relation between the profit and the ratio of the refrigerant and between heating load and the ratio of the refrigerant with \(\psi_2/\psi_1 \to 0\). It illustrates that both the heating load and the profit are monotonic decreasing functions of temperature ratio of refrigerant. It means that the profit maximization approaches the heating load maximization when \(\psi_2/\psi_1 \to 0\). Fig. 5 shows the optimal relation between the profit versus temperature ratio of the refrigerant and COP versus temperature ratio of the refrigerant with different loss items and \(\psi_2/\psi_1 \to 1\). It illustrates that the profit maximization approaches minimization of the rate of entropy production when \(\psi_2/\psi_1 \to 1\), which indicates that the heat pump is not profitable regardless of the COP is at which the heat pump is operating. Only the heat pump is operating reversibly \((\beta = \beta_c)\) will the revenue equal to the cost, and then the maximum profit will equal zero. Furthermore, the relations between the finite-time exergoeconomic performance bound \((\beta_c)\) and \(\psi_2/\psi_1\), and between the maximum profit \((\pi_{\max})\) and \(\psi_2/\psi_1\) with different loss items are shown in Figs. 6 and 7. The corresponding optimal temperature ratio of the refrigerant is also shown in Figs. 6 and 7.
5. CONCLUSION

This paper analyzes the exergoeconomic performance of the generalized irreversible Carnot heat pump cycle. One seeks the economic optimization objective function instead of pure thermodynamic parameters by viewing the heat pump as a production process. It is shown that the economic and thermodynamic optimization converged in the limits $\psi_2/\psi_1 \rightarrow 0$ and $\psi_2/\psi_1 \rightarrow 1$.

When the power for exergy conversion is small, the maximum profit operation is near the minimum loss of exergy operation, while when the power input is very cheap compared to the price of exergy output, the maximum profit operation is near the maximum heating load operation. The heat leakage and the internal irreversibility affect the finite time exergoeconomic performance of the heat pump obviously. It is necessary to investigate the optimal performance of a generalized irreversible Carnot heat pump.

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REFERENCES