ANALYSIS OF NONLINEAR AND LINEAR HEAT CONDUCTION PROBLEMS FOR A MULTILAYER FILLER STRUCTURE

Acad. Gleb DRAGAN1, Acad. Boris DRAGANOV2

1ROMANIAN ACADEMY
2INSTITUTE OF ENGINEERING THERMOPHYSICS, UKRAINIAN ACADEMY OF SCIENCES

Abstract. We describe a method and results of the solution of nonlinear and linear nonstationary heat conduction problem for a multilayer external filler structure. We also give a comparative analysis of the results of calculations.

Keywords: nonstationary heat conduction.

Multilayer external walls are widespread in building. The problem of heat transfer in such structures is also interesting for other branches of engineering. In calculating the thermal conditions of external filler structures, it is customary to consider a linear nonstationary heat conduction problem with boundary conditions of third kind. At the same time, experiments in a thermal pressure chamber and preliminary calculations show that this way can lead to appreciable errors. Therefore, it seems interesting to compare the results of calculations based on linear and nonlinear mathematical models.

Experience shows that, for numerous building materials, the temperature dependence of their thermophysical properties is fairly substantial in a wide range of changes in the temperature [1, 2]. Without significant errors, the thermal conductivity and specific heat of different materials can be approximated as

\[
\lambda = \lambda_0 (1 + \varepsilon t), \quad (1)
\]

\[
c = c_0 (1 + \beta t), \quad (2)
\]

where \(\lambda_0\) is the thermal conductivity of the material at 0°C, \(c_0\) is its specific heat at 0°C, and \(\varepsilon\) and \(\beta\) are the coefficients of the linear temperature dependence of thermal conductivity and specific heat.

A scheme of the calculation of heat transfer through external three-layer walls and data for the physical and mathematical statement of the problem are presented in Fig. 1.

Fig. 1. A scheme of the calculation of heat transfer through a three-layer external wall under boundary conditions of third kind.

The system of equations for a multilayer wall \((i = 1, 2 \ldots n)\) looks like

\[
\frac{\partial t_i}{\partial x} = a_i \left[ \frac{\partial^2 t_i}{\partial x^2} + \frac{F}{x} \frac{\partial t_i}{\partial x} \right] + \frac{\varepsilon_i}{2} \left[ \frac{\partial^2 t_i}{\partial x^2} + \frac{F}{x} \frac{\partial t_i}{\partial x} \right] \frac{\partial t_i}{\partial t}. \quad (3)
\]

Correspondingly, the initial and boundary conditions can be written as

\[ t_i(x, 0) = F_i(x) \quad (4) \]
In these relations, \( t \) is the temperature, \( \tau \) is the time, \( a \) is the thermal diffusivity, \( \Gamma = 0, 1, 2 \) is the form factor (for a plane body, \( x = x, \)), \( H_0 = a_0 \lambda_c \) and \( H' = a_0 \lambda_c \) are the relative heat transfer coefficients, \( \alpha \) is the heat transfer coefficient, \( l_0 \) and \( l_a \) are the distances from the coordinate origin to the internal and external surfaces, respectively, and \( l_i \) is the distance to the boundary of the layer \( i \).

The temperatures \( t_i(\tau) \) and \( t_a(\tau) \) are arbitrary functions of time \( \phi(\tau) \) and \( \phi_a(\tau) \).

The problem consists of finding the temperature distribution over the thickness of the railing for any moment of time.

Consider the linearized system of equations. Here, the \( n \)-layer formulation of the problem can be divided into \( n \) boundary-value heat conduction problems for single-layer media. All internal problems will have boundary conditions of fourth kind, but the problems for the first and \( n \)th layers include additional boundary conditions at the free surfaces. Thus, the problem can be written in the following way:

\[
\begin{align*}
\frac{\partial t_{01}}{\partial \tau} &= a_{01} \left[ \frac{\partial^2 t_{01}}{\partial x^2} + \frac{\Gamma}{x} \frac{\partial t_{01}}{\partial x} \right]; \\
\tau &> 0; \quad l_0 \leq x \leq l_1; \\
t_{01}(x,0) &= F_1(x); \\
-\frac{\partial t_{01}}{\partial x}(l_0, \tau) + H_{01} t_{01}(l_0, \tau) &= H_{01} \phi_1(\tau); \\
\frac{\partial t_{01}(l_1, \tau)}{\partial x} &= \frac{q_{01}(\tau)}{\lambda_{01}}. \\
\end{align*}
\]

(9)

\[
\begin{align*}
\frac{\partial t_{0i}}{\partial \tau} &= a_{0i} \left[ \frac{\partial^2 t_{0i}}{\partial x^2} + \frac{\Gamma}{x} \frac{\partial t_{0i}}{\partial x} \right]; \\
\tau &> 0; \quad l_{i-1} \leq x \leq l_i; \quad i = 2, 3, \ldots, (n-1); \\
t_{0i}(x,0) &= F_i(x); \\
-\frac{\partial t_{0i}}{\partial x}(l_{i-1}, \tau) + H_{0i} t_{0i}(l_{i-1}, \tau) &= H_{0i} \phi_i(\tau); \\
\frac{\partial t_{0i}(l_i, \tau)}{\partial x} &= \frac{q_{0i}(\tau)}{\lambda_{0i}}. \\
\end{align*}
\]

(10)

We solve all single-layer problems by the method of finite integral transformations [3, 4].

The solution of problem (9) has the form

\[
t_{0i}(x, \tau) = V_{0i}(x, \tau) + \sum_{\gamma=1}^{\infty} \left[ \frac{P_i(x)}{C_i} \int_{0}^{x} e^{-\mu_0 \gamma} \frac{\partial t_{0i}}{\partial x}(l_i, \tau) \right]_0^x a_{0i} \lambda_{0i} \int_{0}^{\tau} e^{-\mu_1 \gamma} \frac{q_{0i(\tau)}}{\lambda_{0i}} d\tau.
\]

(12)

The solution of internal problems (10) can be presented as

\[
t_{0i}(x, \tau) = V_{0i}(x, \tau) + \sum_{\gamma=1}^{\infty} \left[ \frac{P_i(x)}{C_i} \int_{0}^{x} e^{-\mu_0 \gamma} \frac{\partial t_{0i}}{\partial x}(l_i, \tau) \right]_0^x a_{0i} \lambda_{0i} \int_{0}^{\tau} e^{-\mu_1 \gamma} \frac{q_{0i(\tau)}}{\lambda_{0i}} d\tau.
\]

(13)

For the \( n \)th layer (problem (11)), we have

\[
t_{0n}(x, \tau) = V_{0n}(x, \tau) + \sum_{\gamma=1}^{\infty} \left[ \frac{P_{0n}(x)}{C_{0n}} \int_{0}^{x} e^{-\mu_{0n} \gamma} \frac{\partial t_{0n}}{\partial x}(l_n, \tau) \right]_0^x a_{0n} \lambda_{0n} \int_{0}^{\tau} e^{-\mu_{1n} \gamma} \frac{q_{0n(\tau)}}{\lambda_{0n}} d\tau.
\]

(14)

Acting in a similar way for problems of the I approximation in the parameters \( \epsilon_{m\alpha} \), we obtain (\( m = 1, 2, \ldots, n \))

\[
\begin{align*}
t_{m1}(x, \tau) &= V_{m1}(x, \tau) + \sum_{\gamma=1}^{\infty} \left[ \frac{a_{m1} P_m(x)}{C_{m1}} \int_{0}^{x} e^{-\mu_{0m} \gamma} \frac{\partial t_{m1}}{\partial x}(l_1, \tau) \right]_0^x a_{0m} \lambda_{0m} \int_{0}^{\tau} e^{-\mu_{1m} \gamma} \frac{q_{m1(\tau)}}{\lambda_{0m}} d\tau - \\
&\quad - \frac{P_m(x)}{C_m} \int_{0}^{1} e^{-\mu_1 \gamma} \frac{t_{m1}(l_1, \tau)}{\lambda_{0m}} d\tau - \frac{1}{2} \int_{0}^{1} e^{-\mu_1 \gamma} \frac{\partial t_{m1}(l_1, \tau)}{\lambda_{0m}} d\theta + \\
&\quad + H_{m1} t_{m1}(l_1, \tau) \int_{0}^{1} e^{-\mu_1 \gamma} \frac{t_{m1}(l_1, \tau)}{\lambda_{0m}} d\theta - \\
&\quad - H_{m1} t_{m1}(l_1, \tau) \int_{0}^{1} e^{-\mu_1 \gamma} \frac{t_{m1}(l_1, \tau)}{\lambda_{0m}} d\theta - \\
&\quad - \frac{1}{2} \int_{0}^{1} e^{-\mu_1 \gamma} \frac{\partial t_{m1}(l_1, \tau)}{\lambda_{0m}} d\theta + \\
&\quad + \frac{1}{2} \int_{0}^{1} e^{-\mu_1 \gamma} \frac{\partial t_{m1}(l_1, \tau)}{\lambda_{0m}} d\theta + \frac{1}{2} \int_{0}^{1} e^{-\mu_1 \gamma} \frac{\partial t_{m1}(l_1, \tau)}{\lambda_{0m}} d\theta.
\end{align*}
\]

(15)
\[ t_{ewn}(x, \tau) = V_{ewn}(x, \tau) + \sum_{j=1}^{\infty} a_{0j} P_{j}(l_{j}) \frac{P_{j}(l_{j}) \rho_{0j} \frac{\partial}{\partial \omega} q_{n}(\omega) d\omega}{C_{j}} \]

\[ -P_{j}(l_{j-1}) p(l_{j-1}) \int_{0}^{\frac{\tau}{\omega}} \exp \left[ -\frac{\mu_{j}^{2} a_{0j}}{\delta_{j}^{2}} (\tau - \omega) q(\omega)d\omega \right] - W_{ewn}(x, \tau) \]
Fig. 4. Time dependences of the temperatures (a) and specific heat fluxes (b) penetrating through a three-layer panel (damp state):

-----------linear case; ____________ nonlinear case.

Similar calculations were also carried out for a multilayer filler structure. For this purpose, we chose a three-layer railing with aluminum casings of thickness 0.01 m and a warmth-keeping jacket ($\rho = 60 \text{ kg/m}^3$ in the damp state, $\varphi = 30\%$) (Fig. 4).

Analysis of the time dependences of temperatures (Fig. 4, a) shows that the effect of nonlinearities manifests itself most of all in the thickness of the structure: corrections at the center (point $t_3$) are 19% for dry material and 26.3% for damp. At the internal surface (point $t_6$) the effect of nonlinearities is insignificant.

In should be noted that, for high-porous materials at moistures corresponding to the air-dry state, the heat fluxes calculated with regard for nonlinearities are always lower than the "linear" results, which enables one to decrease the power of heating systems in buildings with railings of such materials. Thus, the proposed method makes it possible to calculate the specific heat flux through a structure. Furthermore, owing to a higher accuracy of calculations, one can save power consumption for the heating of buildings, especially under conditions where the most part of the structure is in the range of negative temperatures, the thermal conductivity of heat insulation becomes lower, and, hence, the heat transfer intensity and heat losses decrease.

REFERENCES