ECG Data Communication
Using Chebyshev Polynomial Compression Methods
Transmiterea datelor ECG utilizând metoda Chebyshev de compresie polinomială

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Abstract. Nowadays the communication services market is more grasping for frequency band. The telemedicine is one of the applications that require large band and reliable channels for data transmitting. This paper presents a method based on polynomials with application in telecardiology for ECG data compression in order to transmit the vital signal with less effort. The principle of ECG data compression using Discrete Chebyshev Transform (DChT) is presented and the relevant examples for the capability of the method are provided. The compression and the signal reconstruction processes are described in terms of the ECG teletransmission.

Cuvinte cheie: telemedicină, telecardiologie, compresie semnal, telecomunicații, ECG
Key words: telemedicine, telecardiology, compession signal, telecommunication, ECG

1. Introduction

Telemedicine and other applications focusing on portable devices for online heart monitoring of ECG are increasingly required. Telemedicine, with both synchronous and asynchronous types of services require reliable data transmission. A permanent requirement in data communication frequency band is the economy of bandwidth in association with reducing of channel time allocation for certain transmission.

Converting a great number of samples from the continuous signal into an appropriate number of bits, results a large and vulnerable string of data that has to be transmitted and then stored. As the matter of fact, the reliable techniques to sense, to transmit remotely and to store the ECG signals are highly required [1]. Under these circumstances, it is necessary to look for new methods of analogue signal modelling that target to reduce the volume of the data describing exactly the ECG signals, in order to be transmitted and stored with less effort and resources [2], [3].

Several compression algorithms have then been elaborated for ECG data compression and some approximation schemes including polynomial approximations and polynomial interpolation have been proposed for this matter. Basically, both analog to digital conversion and polynomial modeling of ECG signals are approximate methods. The difference, however, is the amount of data required to describe the signal by the two methods. The advantage of the polynomial approximation is that it requires only polynomial coefficients describing the data signal. So it takes a much smaller amount of data than with analog-digital conversion. However, polynomial methods are valuable to the extent that they manage to approximate the original ECG signal acceptable.

Many approximation schemes including polynomial approximations and polynomial interpolation have then been proposed for this matter. Most of polynomial mappings of ECG are restricted to very low degrees polynomials. The basic idea is the estimation of a sample of signal \( y_n \) by its approximation \( \hat{y}_n \) through Newton interpolation formulae [4]:
\[ \hat{y}_n = y_{n-1} + \Delta y_{n-1} + \Delta^2 y_{n-1} + \ldots + \Delta^m y_{n-1} \]  

(1)

where \( \Delta \) is the finite difference operator and \( m \) is the highest degree of polynomials used. In case of \( m = 0 \), is obtained \( \hat{y}_n = y_n \) and the signal is approximated by horizontal lines called plateaus. In case of \( m = 1 \) is obtained the following:

\[ \hat{y}_n = y_{n-1} + \Delta y_{n-1} = 2y_{n-1} + y'_{n-2} \]  

(2)

The approximated signal is represented by slopes passing within the two previous samples. AZTEC (Amplitude Zone Time Epoch Coding) is very popular ECG compression algorithm via linear approximations: adjacent samples with amplitude difference less than a predefined threshold are mapped by plateaus. Slopes are next generated to interconnect plateaus [5], [6]. An illustration of the AZTEC scheme is shown in Figure 1. Some other linear approximation schemes for ECG are CORTES (Coordinate Reduction Time Encoding System) [7], and SAPA (Scan Along Polygonal Approximation) [8].

![Figure 1. An illustration of the AZTEC compression method](image)

2. **Survey on polynomial methods**

Polynomials of maximum degree 3, including splines functions have been proposed for ECG interpolation in [9] and [10]. The representation of ECG signals using second degree quadratic polynomials is studied by Nygaard et al in [11]. When using cubic splines or quadratic polynomials for ECG compression, the signal should be pre-processed in order to extract some particular points such as extrema, zero crossing and inflexion points, that could be used as the interpolation nodes.

High degrees Legendre polynomials were used for ECG data compression in [12], [13] and [14]. Generalized Jacobi polynomials are tested for ECG compression in [15]. High degree polynomial approximations of a signal is similar to spectral methods since the signal is decomposed into a set of orthogonal polynomials basic functions, the same way do Fourier Transform (with trigonometric functions) and Wavelets Transform (with wavelets). Although Chebychev polynomials are widely used in mathematical interpolation and in spectral methods for solving differential equations systems, propositions for ECG compression through Chebychev polynomials are hardly encountered in the literature.

In this paper, we are presenting a scheme to model ECG signals through Chebyshev polynomials. Portions of ECG signals are decomposed into Chebyshev polynomials base. This transformation gives polynomial coefficients which are sorted. Only those coefficients that are significant compared to a predefined threshold are selected, transmitted and stored.

The rest of this article is organized as follows: in the next section, we give a brief introduction to Chebychev polynomials from where we derive the Discrete Chebychev Transform (DChT). The implementation in order to achieve ECG compression is described in section 5. The results obtained are presented and discussed in section 6. At the last section, conclusions are provided.
3. Chebychev polynomials

Chebychev polynomials are orthogonal set of functions recursively defined on the interval \([-1,1]\). In following we provide the definitions for both first kind and second kind Chebychev polynomials. The Chebychev polynomials of first kind are defined by

\[ T_0(x) = 1, \quad T_1(x) = x, \]

respectively

\[ T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad \text{for} \quad n \geq 1 \]  

(3)

For instance, some few Chebychev polynomials of first kind are following:

\[ T_2(x) = 2x^2 - 1, \quad T_3(x) = 4x^3 - 3x, \quad T_4(x) = 8x^4 - 8x^2 + 1, \]

\[ T_5(x) = 16x^5 - 20x^3 + 5x, \]

(4)

Chebychev polynomials of second kind are defined by

\[ U_0(x) = 1, \quad U_1(x) = 2x, \]

respectively

\[ U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x), \quad \text{for} \quad n \geq 1 \]  

(5)

For instance, the next few Chebychev polynomials of second kind are following:

\[ U_2(x) = 4x^2 - 1, \quad U_3(x) = 8x^3 - 4x, \quad U_4(x) = 16x^4 - 12x^2 + 1 \]

\[ U_5(x) = 32x^5 - 32x^3 + 6x, \]

(6)

The difference between the two sets of polynomials lies only in the initial conditions, i.e. for \( n=1 \). In this paper, we will use the Chebychev polynomials of first kind, whose interesting properties make them very attractive for the design of filters and for optimal polynomials interpolation. They form a complete orthogonal set in the interval \([-1, 1]\) with respect to following the weighting function:

\[ \omega(x) = \frac{1}{\sqrt{1-x^2}} \]

(7)

The orthogonality is expressed as follows:

\[ \langle T_m(x), T_n(x) \rangle = \int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0 & \text{if} \ m \neq n \\ d_n^2 & \text{if} \ m = n \end{cases} \]

(8)

Where

\[ d_n^2 = \begin{cases} \pi & \text{if} \ n = 0 \\ \frac{\pi}{2} & \text{if} \ n \geq 1 \end{cases} \]

The Chebychev polynomials also satisfy a discrete orthogonal relation. If \( x_k \) (\( k = 1, 2, \ldots, m \)) are the \( m \) zeros of \( T_m(x) \), and if \( i, j < m \), then

\[ \sum_{k=1}^{m} T_i(x_k)T_j(x_k) = \begin{cases} 0 & \text{if} \ i \neq j \\ m/2 & \text{if} \ i = j \neq 0 \\ m & \text{if} \ i = j = 0 \end{cases} \]

(9)

The trigonometric form of the Chebychev polynomials of first kind is given by

\[ T_n(x) = \cos(n \arccos(x)) \]

(10)

In figure 2 are plotted curves of some first kind Chebychev polynomials. These polynomials are closely related to cosine trigonometric functions [16].

The zeros of \( T_n(x) \) are derived from (10) as \( T_n(x_j) = \cos(\arccos(x_j)) = 0 \), which implies the following:
\[ x_j = \cos \left( \pi \frac{2j-1}{2n} \right), \quad 1 \leq j \leq n \]  

There are exactly \( n \) distinct zeros of \( T_n(x) \) in \([-1, 1]\).

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\[
\sum_{k=1}^{m} T_i(x_k)T_j(x_k) = \begin{cases} 
0 & \text{if } i \neq j \\
\frac{m}{2} & \text{if } i = j \neq 0 \\
m & \text{if } i = j = 0 
\end{cases}
\]  

The extreme of \( T_n(x) \) are also derived from equation (10) as

\[ T_n(y_j) = \cos(\arccos(y_j)) = \pm1, \quad \text{thus} \]

\[ y_j = \cos(\pi \frac{j}{n}), \quad 0 \leq j \leq n \]

At all of the maxima, \( T_n(x) = 1 \) while at all of the minima, \( T_n(x) = -1 \). This is the property that makes the Chebyshev polynomials extremely useful in polynomial approximation of functions. Many other properties of Chebyshev polynomials can be found in [17].

4. Discrete Chebychev Transform (DChT)

Let us expand a signal \( s(t) \) in terms of Chebychev polynomials series, that is

\[ s(t) = \sum_{k=0}^{n} c_k T_k(t) \]
Since Chebyshev polynomials form a complete system, they make a base of $L^2[-1, 1]$. A coefficient $c_k$ is then the projection of the signal $s(t)$ on the base component $T_k(t)$. The coefficients $c_k$ are calculated as following:

$$c_k = \left\langle s, T_k \right\rangle = \frac{1}{\sqrt{1-t^2}} \int_{-1}^{1} s(t)T_k(t) \, dt = \frac{1}{d_k^2} \int_{-1}^{1} s(t)T_k(t) \, dt$$

(15)

where $d_k^2$ is given in equation (9).

Gauss-Lobatto method is a powerful tool for numerical integration, especially dedicated to orthogonal polynomials [17] and [15]. Gauss quadratures method for numerical integrations easy the evaluation of coefficients $c_k$. It stipulates that for a given family of orthogonal polynomials $\{y_n(x)\}$ in a real interval $[a, b]$, with respect to weight function $\omega(x)$, the following approximation holds:

$$\int_a^b f(x)\omega(x)dx \approx \sum_{j=1}^{M} G_j f(x_j)$$

(16)

where $f(x) \in L^2[a, b]$, $x_j$ are roots of $y_n(x)$ and $G_j$ are called Christoffel numbers.

Equation (16) is Gauss quadratures formulae, it is exact for all polynomials of degree inferior or equals to $2M - 1$.

Applying Gauss-Lobatto integration method on Chebychev polynomials let to

$$\int_{-1}^{1} z(t) \, dt = \frac{\pi}{n} \sum_{j=1}^{n} z(x_j)$$

(17)

Where $x_j$ are roots $T_n(t)$ and given by (11) and all the Christoffel numbers are equal to $\frac{\pi}{2}$.

To compute $c_k$ in (15), we use zeros of $T_{n+1}$. After combining (9), (15) and (17), we obtain

$$c_k = \frac{2}{n+1} \sum_{j=1}^{n+1} s(x_j) \cos\left(\frac{k(j-1)\pi}{2(n+1)}\right) = \frac{2}{n+1} \sum_{j=1}^{n+1} s(x_j) \left(\cos\left(\frac{\pi(2j-1)}{2(n+1)}\right) \cos\left(\frac{k(j-1)\pi}{2(n+1)}\right)\right)$$

for $1 \leq k \leq n$ and

$$c_0 = \frac{1}{n+1} \sum_{j=1}^{n+1} s(x_j) = \frac{1}{n+1} \sum_{j=1}^{n+1} s(x_j) \left(\cos\left(\frac{\pi(2j-1)}{2(n+1)}\right)\right)$$

(18)

We denote equation (18) as Discrete Chebychev Transform (DChT) while equation (14) is the inverse Chebychev Transform. It is obvious that DChT is much closed to Discrete Cosine Transform as we have already mentioned the relationship between Chebychev polynomials and cosine functions. Discrete Cosine Transform is one of the widely used in the area of signal processing, particularly, in the transform coding of image data. For examples, the JPEG standard, the MPEG-I and the MPEG-II use this transform. There exist several rapid algorithms for Discrete Cosine Transform as the Discrete Chebychev Transform could be computed with similar fast algorithms [18], [19].

5. Compression and Transmission of ECGs

Basically, the transmission of ECG data meaning the polynomial coefficients is the subject of polynomial compression of the signal. The processing chain includes some stages and few steps that are depicted in figure 3 and briefly described in following.

The current acquired ECG signal is passing through the preprocessing stage because of the polynomial transformations are not applied to the entire signal, but to portions (windows)
that we call blocks. In addition, a signal that is decomposable within Chebyshev base polynomials must be a function of \( L^2 \([-1, 1]\).

Dividing ECG signals into blocks leads to satisfy the above condition. Each block \( s(t), t \in [0, t_B] \) is of finite energy and should be transposed into \([-1, 1]\) domain by a simple linear transformation as follows:

\[
x = -1 + \frac{2}{t_B} t
\]

where \( t_B \) is the duration of the sampled (into blocks) of signals.

Figure 3. The signal processing chain for a complete transmission session

All methods of polynomial decomposition of the ECG signals proposed so far segment the signal into blocks that coincide exactly with the cardiac cycle [12]-[15]. In most cases, polynomial transformations are applied to R-R intervals of ECGs. In such schemes, a preliminary step, (which is also the subject of the preprocessing stage) that consists in the detection of QRS complexes is necessary to achieve correct segmentation. For DChT instead, it is possible to use blocks signals made of multiple cardiac cycles. There is no requirement on the positions of the ECG’s characteristic waves inside a block. Thus, the segmentation can be carried out blindly; only the duration of blocks must be specified. The use of blocks within multiple cardiac cycles increases the compression ratio and the omission of the step of QRS complexes detection before the segmentation makes DChT faster than other polynomial methods for the compression of the ECGs.

The next stage is the compression mechanism consisting in the certain steps: segmentation, decomposition into the basis of Chebychev polynomials and the selection of significant coefficients.

In the formula of DChT given by equation (18), the coefficients are calculated only with the roots of \( T_{n+1} \). These roots are established analytically by equation (11) and we do not need numerical methods to find them. It is a specificity of the DChT that we do not use all the signal samples in a window to calculate the coefficients of decomposition. If \( n \) (the highest degree of polynomials used for decomposition) is chosen not too great, then the number of computation for DChT will be quite small comparatively to those of other transform schemes for ECGs compression. Sometimes certain roots of \( T_{n+1} \) are not images of signal samples within the interval \([-1, 1]\), in these cases, we calculate \( s(x_j) \) very easily using linear
interpolation from the two adjacent samples around $x_j$. The adjacent samples are obtained by sampling the signal with a frequency $f_s$ according to the basic rule: $f_s \geq 2/t_b$.

The selection of the coefficients that should be used to synthesize the signal during the reconstruction phase uses the principle of thresholding. The absolute values of polynomial coefficients are compared to a predefined positive threshold. Only the coefficients whose absolute values are above this threshold are retained. Because the range of variation of the values of the coefficients fluctuates depending on the signal being processed, we chose to define the threshold as a fraction of the largest absolute value available. By varying some kind of ratio, we modify the parameter of coefficients selection, thus we adjust the compression ratio.

The signal reconstruction stage consists in two steps: the synthesis of the signal and the blocks assembling. We conclude that the core of the signal compression is DChT decomposition, which is made with the equations (18), while the signal reconstruction is based on the signal synthesis with the equation (14), which plays the role of the inverse Chebychev Transform.

6. The performance of DChT: Results and discussions

As the validity of the method is depending on the efficiency of the signal approximation with the polynomial we provide in the following some evaluations for Chebychev polynomials and the Discrete Chebychev Transform. We use two criteria in order to evaluate the performance of the proposed method: the compression ratio (CR) and the quality of the reconstructed signal that is quantified by the well known Percent Root square Difference (PRD).

We conducted our numeric experiments in Matlab environment, using signals from the MIT - BIH arrhythmia database [20], and also records available online [21]. Each record consists of two channels of signals. These signals are sampled at a rate of 360 Hz and use 11 bits/sample resolution. The compression efficiency is measured using the compression ratio (CR), which is expressed as follows:

$$CR = \frac{NB_{or}}{NB_{com}}$$  \hspace{1cm} (20)

where $NB_{or}$ is the total number of bits used to code the original ECG signal and $NB_{com}$ is the total number of bits in the compressed ECG signal. The quality of the reconstructed signal is theoretically quantified by the Percent Root square Difference (PRD) is given by the following expression:

$$PRD = 100 \sqrt{\sum_n (s_n - \hat{s}_n)^2 \over \sum_n s_n^2}$$  \hspace{1cm} (21)

where $s(n)$ and $\hat{s}(n)$ are the original and the reconstructed signals respectively.
For instance, we show on figure 4, the original and reconstructed of 5 seconds (i.e. 1800 samples) signals of record number 100, channel 1 from the MIT – BIH data base. On the top is the original signal, in the middle is the reconstructed signal and the spectrum of polynomial coefficients is plotted at the bottom. Chebychev polynomials up to degree 2000 were used for that matter.

As shown in figure 4, all polynomials whose degree is greater than 400 have their coefficients very close to zero. This highlights the fact that DChT compact most of the signal energy in a small number of coefficients, demonstrating the ability of DChT to achieve signal compression. Almost all of the signal energy is concentrated in the coefficients of polynomials of low degrees. We can therefore neglect the vast majority of coefficients whose values are very low (almost zero), and the reconstructed signal is not altered significantly.

We carry out many compression experiments through DChT. All results confirmed that DChT is an advantageous tool for signal compression. Some examples are commented below. On figure 5 are shown two signals: on the left is record no 200 channel 1, while on the right is signal reference 207, channel 1.

Figure 4. Applying DChT algorithm to record ref. 100, channel 1 using Chebyshev polynomials up to degree 2000.

Figure 5. Examples of DCT compression of ECG signals (Signals ref. 200 and ref. 207)
The original signals are plotted in the first line in both cases, whereas below each original signal, the compressed versions of the same signal, using different compression ratios are presented. Although it is felt that the two original signals have the same morphology, the quality of the reconstructed signal is given by the certain values of the PRD for each case. Despite this, the PRD obtained at different compression ratios are satisfactory as indicated by the values specified in this figure.

Both the original signals shown in figure 5 correspond to medically abnormal ECG. The values of the PRD obtained at various compression ratios are shown in the same figure. These values are very interesting. We can appreciate the strength of DChT as to faithfully reproduce the abnormalities included in the ECG signal even at very high compression ratios. We applied the DChT over 40 signals from the MIT database [20]. The method is proven sufficiently robust in all circumstances.

It is shown in figure 6, the variation curves of the PRD as a function of compression ratio. It should be noted that the signals with high values of PRD are those incorporating very sharp impulses in their QRS complexes. As already highlighted in figure 5, these regions of the QRS complexes contribute much more than other parts of the signal in the formation of the reconstruction errors. So when we realize compression with high compression, for which the values of the PRD are very large, the reconstructed signals remain faithful to the originals, the changes occur only at the amplitudes of QRS complexes.

In figure 6 is depicted the result of an experiment which involves establishing a constant compression ratio (CR = 6.26) for all signals and evaluate the effects of DChT on each. Recomposed signals after compression (in red) are superimposed on the original (blue) in order to show the differences. It appears that the original and reconstructed signals are almost alike, even for cases of relatively large PRD (figure 7).
A zoom of some cases confirms the coincidence of the original and reconstructed signals at the microscopic level. For instance, it is shown in figure 8; a zoom on the cases referenced signals. This is the signal with the PRD=6.18 which is considered a medium value among the signals in figure 7.

It can be seen in figure 8(a) that the coincidence of the reconstructed signal with the original signal is acceptable. Zooming the graphics in the case of signal referenced 112 which has the lowest PRD among the other signals presented in figure 7, It can be seen in Figure 8(b) that the coincidence of the reconstructed signal with the original signal is almost perfect.

7. Conclusions

We were inspired by mathematical methods of polynomial interpolation and polynomial approximations to develop a compression algorithm for ECG signals. DChT as we called it is based on the principle of signal expansion in series of Chebychev polynomials. We used DChT to achieve compression of ECG signals with much success. The impact of the proposed method on the data communication in telemedicine is very optimistic. The results
obtained are higher than those of other similar algorithms in terms of signal reconstruction error at a given compression rate. The DChT also appeared to be quite suitable in terms of computations. This method can be used for compressing other types of signals without any modification of the algorithmic structure of the DChT.

References


