ON THE EFFICIENCY AND POWER FACTOR OF THE CRUCIBLE INDUCTION FURNANCES

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ABSTRACT. The calculation of the crucible induction furnaces uses complex equations. The main purpose of the paper is to transform these well-known designing equations in expressions based on per unit variables (ratio parameters), obtaining a higher generality degree. One emphasizes their dependence on the ratio diameter, when the furnace capacity and destination are known. The material proprieties and slenderness factors influence the resistances, the reactances and their correction functions. The proposed detailed calculation equations allow an easier utilization and getting of some qualitative conclusions on the electric efficiency and power factor of this kind of furnaces.

Keywords: crucible induction furnace, efficiency, power factor.

1. INTRODUCTION

The induction furnaces are wide used in industry because their good performances. Their operating principle is similar to the transformer and determines the heating of a workpiece placed in the alternating magnetic field of a coil called inductor. The heating is the result of the Joule-Lenz looses due to the induced eddy currents. Because depends on the magnitude and frequency of the inducing field and the physical properties of the workpiece, the induced current density and consequent heating effect is always non-uniform.

So, the choice of the operating frequency is very important and done usual depending on the inner medium diameter of the heat-resisting crucible, imposing

\[ f \geq \frac{25 \cdot 10^6 \cdot \rho_2}{\mu_{r2} \cdot d_2^2} \]  

(1)

where \( \rho_2 \), \( \mu_{r2} \) are the resistivity and magnetic permeability of the heated material, considered for the final process temperature.
The crucible induction furnaces are fed at the industrial frequency by power transformers with special construction and different other frequencies by ferromagnetic frequency multipliers or, more and more often, by static frequency converters. In the last case, the furnace operation belongs to the non-sinusoidal permanent regime.

In order to do several assessments on the energetic parameters of the crucible furnace, the paper transforms the well-known calculation equations in a more detailed forms, where the ratio parameters were used, obtained a much higher generality character.

The calculation takes into account the operation regime (sinusoidal or non-sinusoidal) and is usual very complex, one purpose of the paper being the identification of simplified and easier ways.

2. FURNACE ELECTRIC EFFICIENCY DEPENDING ON THE SLENDERNESS FACTOR

The electric efficiency is defined as the ratio between the power transferred to the load and the power absorbed by the inductor:

\[ \eta_e = \frac{P_2}{P_1} = \frac{P_2}{\Delta P_1 + P_2} \]  

where \( \Delta P_1 \) represents the power losses in the inductor coil. In the following, the index 1 refers to the inductor and 2 to the load.

2.1. The case of sinusoidal permanent regime

By feeding the furnace from the power grid or from a rotational medium-frequency converter and its operation belongs to the sinusoidal permanent regime, the previous equation can be written:

\[ \eta_e = \frac{R_2 I_2^2}{R_1 I_1^2 + R_2 I_2^2} = \frac{\rho^2 R_2}{R_1 + \rho^2 R_2} = \frac{A}{1 + A} \]  

where the resistance of the circuit element can be determined with the generic equation

\[ R = \rho \frac{\pi d}{\delta h} K R N^2 = \frac{\pi s \sqrt{\rho f}}{503 \cdot g} K R N^2 \]  

in that the diameters \( d \), the heights \( h \) and the penetration depths \( \delta \) are defined in Fig.1, \( g \) is the filling factor, \( K_R \) is the correction function of the internal resistance, \( N \) – the number of turns, \( I_1 \) and \( I_2 = p I_1 \) are the inductor current and respectively the current through the workpiece, while \( p^2 = \chi N^2 \) is the square of the transformation ratio and \( \chi \) is the adjusting (correction) factor of the transformation ratio.

The variable \( A \) has the expression

\[ A = \frac{\rho^2 R_2}{R_1} = g \cdot \chi \cdot \frac{K_{R2}}{K_{R1}} \cdot \frac{s_2}{s_1} \cdot \sqrt{\frac{\rho_2}{\rho_1}} \]  

and represents a ratio between the resistances of the load and of the inductor, both seen at the inductor’s terminals.

The resistances correction factors are functions of the type \( K_R = f(x) \) in that the argument \( x = \sqrt{f' / \delta} = d / (\sqrt{2} \cdot \delta) \) represents a ratio diameter, which is dependent to the frequency and material proprieties. So, for determining the correction of the inductor’s internal resistance, few cases can be differentiate:

- if \( x_1 = d_1 / (\sqrt{2} \cdot \delta) < 10 \) the inductor is considered an empty cylinder, inward excited,
with the thickness $a_1$ (Fig.1); the correction function $K_{R1} = f(x_1,e_1)$, where $e_1 = a_1/\delta_1$, is given by graphics or tables.

- if $x_1 \geq 10$ and $e_1 < 3$ the inductor is considered a flat plate, one-side excited; the values of the correction function can be found in graphs and tables, or can be determined with

$$K_{R1} = \frac{sh2e_1 + \sin 2e_1}{ch2e_1 - \cos 2e_1} \quad (6)$$

In practice, in order to reduce the inductor looses, one recommends $e_1 \geq \pi/2$ and the minimal value of the correction function $K_{R1,\min} \equiv 0.92$ corresponds to $e_1 = \pi/2 \equiv 1.57$.

- if $x_1 \geq 10, e_1 \geq 3$ the inductor behaves like an infinite semi-spatial conductor and $K_{R1} = 1$.

The determination of the correction functions for the charge internal resistance takes into account two situations:

- if $x_2 = d_1/h_2 < 4$ - the melt is considered a full cylindrical conductor, longitudinal excited, the correction function being

$$K_{R2} = \frac{berx_2 \cdot ber'x_2 + berx_2 \cdot bei'x_2}{ber'x_2 + bei'x_2} \quad (7)$$

where the zero-order modified Bessel function can be decomposed in real and imaginary part like $I_0(x_2 \sqrt{j}) = ber x_2 + j \cdot bei x_2$, $ber x_2$ and $bei x_2$ are the first order derivates. The values of $K_{R2} = f(x_2)$ are usual given in literature by tables and graphs.

- if $x_2 \geq 4$, the correction function can be approximated, with an error less than 1%, through:

$$K_{R2} = 1 - \frac{1}{\sqrt{2}x_2} \quad (8)$$

The practical experience shows that, in the condition of $x_2 \geq 4$, the energetic performances become acceptable. That is why only this case is discussed in the followings. In this hypothesis, the adjusting factor of the transforming ratio becomes

$$\chi = \frac{(\alpha_M)^2}{\alpha_2^2 + \left(\frac{2}{\sqrt{2}x_2 - 1}\right)^2} \quad (9)$$

and corresponds to the case when the system inductor-charge is constituted by two coaxial pipes with thin walls having the thicknesses $\delta_1$ and $\delta_2$ (Fig.2).

Taking into account the equivalent scheme, the system operating equations are:

$$\begin{align*}
U_1 &= (R_1 + j\omega L_1)I_1 + j\omega M L_2 \\
0 &= (R_2 + j\omega L_2)I_2 + j\omega M L_1
\end{align*} \quad (10)$$

Because the electromagnetic field is distorted at the ends of the pipes with finite lengths $h_1$ and $h_2$, it becomes necessary to introduce auxiliary correction functions noted $\alpha_1, \alpha_2$ and $\alpha_M$ for the external inductances $L_1$ (inductor), $L_2$ (charge) and mutual $M$, given by:

$$\begin{align*}
\alpha_1 &= E_1 \left(\frac{d_1}{h_1}\right) = E_1 \left[s_1 \left(1 + \frac{1}{\sqrt{2}x_1}\right)\right] \quad (11a) \\
\alpha_2 &= E_2 \left(\frac{d_2}{h_2}\right) = E_2 \left[s_2 \left(1 - \frac{1}{\sqrt{2}x_2}\right)\right] = E_2 (s_2 K_{R2}) \quad (11b) \\
\alpha_M &= E_M \left(\frac{d_1}{h_1} \cdot \frac{d_1}{h_1}\right) = E_M \left[\chi \cdot s_1 \left(1 + \frac{1}{\sqrt{2}x_1}\right)\right] \quad (11c)
\end{align*}$$

where $d'_1$ and $d'_2$ are the average calculation diameters for inductor and charge, given by:

$$\begin{align*}
d'_1 &= d_1 + \delta_1 = d_1 \left(1 + \frac{1}{\sqrt{2}x_1}\right) \quad (12) \\
d'_2 &= d_2 - \delta_2 = d_2 \left(1 - \frac{1}{\sqrt{2}x_2}\right) = d_2 \cdot K_{R2}
\end{align*}$$

The ratio diameters $x_1$ and $x_2$ are in interdependence as results from the relation:

$$x_1 = \overline{h} \cdot s_1 \cdot \sqrt{\overline{p}_1 \cdot \frac{1}{x_2}} \cdot \frac{1}{\sqrt{2}x_2} \quad \overline{h} = h_1 / h_2 \quad (13)$$

It can be noticed that the penetration depths $\delta_1$ and $\delta_2$ are explicit present in the expressions of $x_1$, $x_2$ and $e_1$ and also implicit in the $K_{R1}$, $K_{R2}$, $\alpha_1, \alpha_2$, $\alpha_M$, $L_1$, $L_2$ variables, so that the numerical values are influenced both by the operating frequency and material proprieties of the inductor and charge.

Finally we can conclude that the electric efficiency is complex function as follow:
2.2. The case of non-sinusoidal permanent regime

When the furnace is supplied by a static frequency converter, usual a non-sinusoidal permanent regime is installed. To express easier the terms from the definition (3) one way is to:
- do the harmonic analyze of the current wave
- apply the effects superposition principle for the active and reactive powers
- express the variables of the high order harmonics, having the frequency \( f_k = k f \), using their values from the sinusoidal permanent regime.

So, when the order of harmonics takes the values \( k = 3, 5 \ldots (2n + 1) \) the specified terms yields:

\[
K_{R1,k} = G(\varepsilon_1, k) \quad \text{with} \quad K_{R1,k} = 1 \quad \text{for} \quad \varepsilon_1 \geq \pi/2
\]

\[
K_{R2,k} = 1 - \frac{1}{k \cdot \sqrt{2 \cdot x_2}}
\]

\[
R_{1,k} = \frac{K_{R1,k}}{K_{R1}} R_1 \sqrt{k}
\]

\[
R_{2,k} = \frac{K_{R2,k}}{K_{R2}} \frac{R_2 \sqrt{k}}{R_1}
\]

\[
\eta_e = \frac{\alpha_2^2 + \frac{2}{\sqrt{k} \cdot \sqrt{x_2}}}{\alpha_2^2 + \frac{2}{\sqrt{x_2}}}
\]

Examining the graphical representation of \( \alpha_2 = E_2 (d_2 / h_2) \) and \( \alpha_M = E_M (d_1 / h_1)_{\text{tot}} \) given in Fig. 3 it can be seen that the increasing of the harmonic order from \( k \) to \( k+1 \) leads to \( \alpha_{2,k} > \alpha_{2,k+1} \) and \( \alpha_{M,k} > \alpha_{M,k+1} \) and so to an increasing of the transformation ratio because \( \chi_{k+1} > \chi_k \).

Based on the superposition principle, the total powers for the inductor and the charge can be written as the sum between the power corresponding to the fundamental and the powers generated by the high order harmonics

\[
\begin{align*}
\Delta P_{1\text{tot}} &= P_1 + \sum \Delta P_{1k} \\
\Delta P_{2\text{tot}} &= P_2 + \sum \Delta P_{2k}
\end{align*}
\]

\[
\begin{align*}
P_1 &= R_1 I_1^2 \\
P_2 &= R_2 N^2 I_1^2 \\
\Delta P_1 &= R_1 I_1^2 \\
\Delta P_2 &= R_2 \chi \eta_1 N^2 I_1^2 \\
\end{align*}
\]

The ratio \( \psi_k = I_{1k} / I_1 \) being the weight of the \( k \)-order current harmonic related to the current fundamental. So, the electric efficiency for the case of the non-sinusoidal permanent regime is

\[
\eta_{ed} = \frac{P_{2\text{tot}}}{\Delta P_{1\text{tot}} + P_{2\text{tot}}}
\]
The ratio diameter related to the permanent regimes:

\[
\eta_{en} = \frac{\chi N^2 R_1}{R_1 + \sum K_{R1k} \frac{X_k}{\chi} \psi_1 \sqrt{k} + \chi N^2 R_1 \left[ 1 + \sum R_{2k} \frac{X_k}{\chi} \psi_1 \sqrt{k} \right]}
\]

(18)

Taking into account the notation (5), it can be expressed a biunique correspondence between the electric efficiency in the both sinusoidal and non-sinusoidal efficiencies.

\[
\eta_{en} = \frac{A \cdot B}{1 + A \cdot B} = \frac{\eta_{es} \cdot B}{1 - \eta_{es} + \eta_{es} \cdot B}
\]

(19)

in that

\[
B = \frac{1 + \sum K_{R2k} \frac{X_k}{\chi} \psi_1 \sqrt{k}}{1 + \sum K_{R1k} \frac{X_k}{\chi} \psi_1 \sqrt{k}}
\]

(20)

can be called link coefficient between the two electric efficiencies.

Analyzing the development of the new resulted forms of the electric efficiency equations, one can conclude the followings:

- The resistances and inductances (interns, externs, mutual) of the circuit elements can be synthetically expressed using the slenderness factor \( s = \frac{d}{h} \), the ratio diameter related to the penetration depth \( \chi = \frac{d}{\sqrt{2 \delta}} \) and the medium relative height \( \tilde{h} = h_1 / h_2 \);

- when \( x_2 \geq 4 \) the resistance correction factor can be calculated easier with (8), expression that leads to errors less than 1\% compared to the calculation by means of modified Bessel function;

- The adjusting factor for the transforming ratio, (9) and respectively (15e), is easy to be used because has a simplified form and is independent on the geometrical real dimensions of the inductor-charge system.

- The electric efficiency is higher for the furnaces having the inductor built with rectangular cross-section compared to circular cross-section, the thickness of the charge-oriented wall being \( a_1 = 1.57 \delta_1 \);

- The electric efficiencies calculated for both sinusoidal and non-sinusoidal permanent regime have a biunique correspondence by means of the link coefficient \( B \), its value can be determined by means the histogram of the inductor current in the case of the deforming regime.

3. THE FURNACE POWER FACTOR

The induction crucible furnace is single-phase equipment that has the natural power factor defined by the ratio

\[
k_p = \frac{P}{S}
\]

(21)

where \( P \) and \( S \) are the active and apparent powers. Because the electric equipments are designed for the maximum allowable effective values for the voltage and current (that means, for the maximum apparent power) results that a high power factor leads to a better energetic efficiency of the analyzed equipment.

If the sinusoidal permanent regime is considered, the power factor can be written

\[
k_p = \frac{R}{\sqrt{R^2 + X^2}}
\]

(22)

where the notations are: \( R = R_1 + p^2 R_2 \) - the total resistance; \( X = X_1 + p^2 X_2 + X_1 - p^2 X_2 \) - the total inductive reactance; \( X_1 = R_1 \frac{K_{X1}}{K_{R1}} \) - the inner reactance of the inductor; \( X_2 = R_2 \frac{K_{X2}}{K_{R2}} \) - the inner reactance of the charge; \( X_1 = \omega L_1 \) and \( X_2 = \omega L_2 \) - the leakage reactances of the inductor and the charge.

The correction functions of the reactances \( K_{X1} \) and \( K_{X2} \) are established, as well the ones for the resistances, in correlation with the ratio diameter. Thus:

- if \( x_1 > 10, \ v_1 < 2 \) then \( K_{X1} = \frac{sh2\epsilon_1 - \sin 2\epsilon_1}{ch2\epsilon_1 - \cos 2\epsilon_1} \);

- if \( x_1 > 10, \ v_1 \geq 2 \) then \( K_{X1} = 1 \) for \( x_2 \geq 4 \).

Because the induction crucible furnaces are high inductive equipments for that the reactances are much lower than the reactances, \( R << X \), in practice the power factor value is calculated with the simplified relation

\[
k_p = \frac{R}{X}
\]

(23)

for that the error is lower than 1\%. By replacement of the terms, the authors found the detailed expression (24) for the power factor, which is different to the one presented in [1].
Similar to the electric efficiency, the power factor can be graphically represented depending on the ratio diameter $x_2$.

For this new expression of the furnace power factor, one can notice the following:

- For the power factor in the permanent sinusoidal regime was established an expression that explicitly includes the electric efficiency for this operating regime. One notices an inverse proportional dependence between the power factor and electric efficiency, so the charges consisted in low resistivity materials (like copper, for example) will have a better power factor than the one with high resistivity materials (cast iron, steel).

- The power factor for the case of the permanent non-sinusoidal regime cannot be expressed only depending on the current harmonics weighting factor $\psi_k$, in this situation a deep harmonic analysis is necessary in order to assess the deforming power.

\[
\lambda = \frac{1}{(1 - \eta_e) \cdot \frac{K_{e1}}{K_{r1}} + \eta_e \cdot \frac{K_{e2}}{K_{r2}} + \eta_e \cdot \frac{\sqrt{2}x_2 - 1}{2} \cdot \alpha_2 + \left[ \frac{1}{\frac{1}{\sqrt{2}x_1}} \cdot \frac{\alpha_1}{\alpha_2} \cdot \frac{\alpha_2 + \left( \frac{2}{\sqrt{2}x_1} \right)^2}{\alpha_M} \right]}
\]

\( (24) \)

**11. CONCLUSION**

The presented study on the energetic indicators for an induction crucible furnace gave new detailed expressions for this indicators and leads to some conclusions with both theoretical and practical meaning. Because using the ration parameters, the new equation forms have a higher degree of generality and allow a graphical representation of the energetic parameters as a function of ratio diameters. The correction functions got analysis and has been identified, for the particular case $x_2 \geq 4$, a simplified calculation formula for that the error don’t excess 1% compared to the classical calculation. All this are quicker and easier to be used and represent a help for the designing or exploiting engineers.

The authors intend to extend the presented study looking for an analytic approximation of the experimental correction functions $\alpha_1$, $\alpha_2$ and $\alpha_M$.

**REFERENCES**