THEORETICAL CHARACTERIZATION OF AN ELECTROMAGNETIC GENERATOR FOR VIBRATION ENERGY HARVESTING

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REZUMAT. In această lucrare prezentăm un studiu teoretic al generatorilor electromagnetici pentru recuperarea energiei din vibrații ambientale. Generatorul este modelat ca un oscilator de ordinul doi subamortizat. Optimizarea generatorului pentru obținerea valorilor maxime ale transmisibilității şi puterii electrice s-a făcut prin considerarea unei pulsații de rezonanță diferită de pulsația naturală care este folosită curent în literatură. Răspunsul la intrarea treaptă a sistemului este util în analiza generatoarelor supuse la variații bruşte ale amplitudinii vibrațiilor aleatoare.

Cuvinte cheie: recuperarea energiei, generatori electromagnetici, fenomen de rezonanță, oscilator subamortizat.

ABSTRACT. In this paper we present a theoretical study of the electromagnetic generators for harvesting energy from ambient vibrations. The generator is modeled as an oscillator of second order under-damped. Generator optimization for achieving the maximum values of transmissibility and electrical power has performed by considering a resonance frequency different to natural frequency that is usually used in literature. Step response of the system is useful to analysis of the generators subjected to suddenly variations of the random vibration amplitude.

Keywords: energy harvesting, electromagnetic generators, resonance phenomenon, under-damped oscillator.

1. INTRODUCTION

Energy harvesting generators from vibration, as alternative energy source, have become increasingly widespread because vibrations are common. Vibration-driven generators based on electrostatic (capacitive), piezoelectric or electromagnetic (inductive) technologies have been demonstrated [1]. Energy harvesting generators can be oscillatory or non oscillatory. The oscillatory generators have an elastic suspended inertial mass that damped oscillates due to the external applied vibrations. These can be classified as resonant or non resonant. The resonant oscillatory generators have a resonance frequency where the amplitude of the mass displacement is largest. In literature the resonant generators have been classified as Velocity Damped Resonant Generator (VDRG) and Coulomb Damped Resonant Generator (CDRG) [2]. The first one deals when the electromechanical force is proportional to velocity, whereas the last is applicable when electromechanical force is a constant force. The maximum power generation at the resonance frequency for both generators is same. The two categories of generators (VDRG and CDRG) can work and as non resonant generators when the damped devices oscillates with a different frequency than the excitation frequency of the vibration.

The non oscillatory generators are called as Coulomb Force Parametric Generator (CFPG). Such generator CFPG is non-linear; it does not permit adjustment on a resonance frequency, having not a suspension with resort, the generator converting mechanical energy into electricity when it is at maximum acceleration.

Class of generators VDRG can be implemented using electromagnetic phenomena (the movement of a permanent magnet in a coil or vice versa) while the other two (CDRG and CFPG) using electrostatic phenomena (the movement of the plates of a capacitor in a direction parallel to them, thus maintaining a constant distance between the fixed and mobile plates, developing a constant force to movement).

While the class VDRG includes both mini and micro energy generating devices, the classes CFPG and CDRG includes especially the micropower generators fabricated using MEMS technique.

In literature the VDRG generators are modeled by a reduced second order model where the output power is maximum at resonance, when the excitation vibration frequency is matched to the natural frequency [2],[3],[4],[5]. This is true only when the damping system is very low ($\zeta \cong 0$). If the damping is increased, the maximum power and frequency at which it is obtained may differ significantly from the case mentioned.

In this paper we achieve a theoretically study based on analytical modeling of devices VDRG considered as underdamped ($0 \le \zeta < 1$) systems. The effect of the damping in obtaining the largest values for the mass inertiale amplitude, output powers and voltage is rated, which is of great importance in the analysis and design of such generators for harvesting energy from vibration.

2. THEORY OF VIBRATION-POWERED ELECTROMAGNETIC GENERATORS

Electromagnetic induction generator. In the following we will make reference to a generator that has a levitated magnet (Fig. 1).

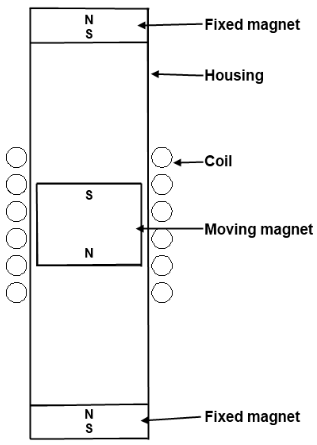

Fig. 1. Principle scheme of electromagnetic generator with levitated magnet.

Frame excitation of the generator results the vibration of the magnet relative to stationary coil causing an induced current. One can formulate a general model for the conversion of the kinetic energy of an oscillating magnetic mass to electricity based on the linear system theory. This model based on the schematic in Figure 2 is the reduced second order system model that is described by equation (1).

$$m\frac{d^2z}{dt^2} + c\frac{dz}{dt} + kz = -m\frac{d^2y}{dt^2} \quad (1)$$

where, m is magnet mass; z - mass relative displacement; y - the displacement of the frame; k - the magnetoelastic constant of the magnetic suspension; $c = c_e + c_m$ - the total damping coefficient, where c_e and c_m are electrical damping coefficient and mechanical damping coefficient, respectively.

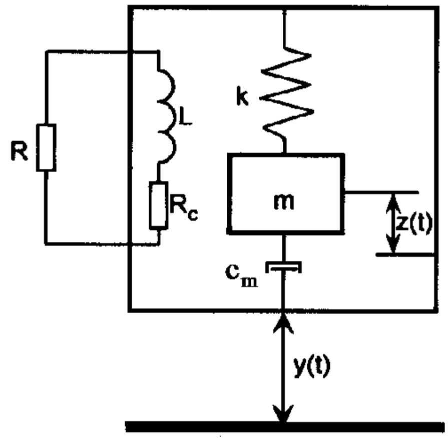

Fig. 2. Electromagnetic induction generator model.

The electric damping c_e expresses the mechanism of conversion of the kinetic energy into electricity, while the mechanical damping c_m is due to air damping.

Diferrential equation (1) can be also writen:

$$\frac{d^2z}{dt^2} + 2\zeta\omega_n\frac{dz}{dt} + \omega_n^2 z = -\frac{d^2y}{dt^2}, \quad (2)$$

where $\zeta = \frac{c}{c_0} = \frac{c}{2\sqrt{mk}}$ is the total damping ratio; $c_0 = 2\sqrt{mk}$ - critically damping coefficient; $\omega_n = 2\pi f_n = \sqrt{\frac{k}{m}}$ - natural angular frequency (in rad/s), where f_n is the natural frequency (in Hz).

If $c < c_0$ or $\zeta < 1$, the system from Figure 2 can oscilate due of an externally aplied displacement that has a certain variation in time, *y(t)*, being such an oscilattory system, without to be a indispensable resonant system, for which, as we will see, there must be another condition.

Because the energy is extracted from the system by relative movement from mass and system we need the solution of the equation (1).

Performing a Laplace transformation on the eq. (1) we get to the transfer function of the system, G(s),

$$G(s) = \frac{Z(s)}{Y(s)} = \frac{-ms^2}{ms^2 + cs + k} \quad (3)$$

For the mathematical model of the system expressed by the equation (2) the transfer function of the system is:

$$G(s)=\frac{-s^2}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{4}$$

Response of the system to sinusoidal vibration. If it assumed that the input is a sinusoidal vibration like $y=Y\sin(\omega t)$, where Y is the vibration amplitude and ω is the vibration angular frequency, substituting this into the governing equation (2) result:

$$\frac{d^2z}{dt^2}+2\zeta\omega_n\frac{dz}{dt}+\omega_n^2 z=\omega^2 Y\sin(\omega t) \tag{5}$$

It can find the frequency response of the system described by transfer function $G(s)$, by substituting s by $j\omega$ in the equation (4):

$$G(j\omega)=\frac{\omega^2}{\omega_n^2-\omega^2+2\zeta\omega_n\omega j} \tag{6}$$

The modulus and the phase angle of the function $G(j\omega)$ represent the amplitude-response and the phase-response, respectively, of the system:

$$G(\omega)=\left|G(j\omega)\right|=\frac{Z(\omega)}{Y}=\frac{\omega^2}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega_n\omega\right)^2}} \tag{7}$$

$$\varphi(\omega)=\mathrm{arctg}\frac{2\zeta\omega_n\omega}{\omega^2-\omega_n^2} \tag{8}$$

Then, the particular solution (steady-state solution) is found as:

$$z_p(t)=Z(\omega)\sin(\omega t-\varphi)=$$

$$=\frac{\omega^2}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega_n\omega\right)^2}}Y\sin(\omega t-\varphi) \tag{9}$$

This is the response of the system to sinusoidal vibrations.

For domains which studies how propagation or damp vibration, often use the term of transmissibility, $T(\omega)$, to designate the ratio of the amplitude of the vibration transmitted to the excitation vibration amplitude:

$$T(\omega)=\frac{Z(\omega)}{Y}=\frac{\omega^2}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\zeta\omega_n\omega\right)^2}} \tag{10}$$

For $\omega=\omega_n$, the amplitude of the inertial mass is:

$$Z(\omega_n)=\frac{Y}{2\zeta}=\frac{\sqrt{mk}}{c}Y \tag{11}$$

In literature is considered that the resonance occurs when the external vibration frequency is matched to the natural angular frequency, $\omega=\omega_n$, that is true only when the damping is very small, as we will still show.

To find the resonance frequency at which the magnitude of the relative movement of the inertial mass is the maximum, express the eq. (10) using the ratio between the excitation frequency and the natural frequency, $r=\omega/\omega_n$:

$$T(r)=\frac{r^2}{\sqrt{\left(1-r^2\right)^2+4\zeta^2r^2}} \tag{12}$$

The function (12) has a maximum for

$$r_r=1\Big/\sqrt{1-2\zeta^2}\,, \tag{13}$$

when is obtained the maximum value of the transmissibility:

$$T_{max}=\frac{1}{2\zeta\sqrt{1-\zeta^2}} \tag{14}$$

Therefore, the resonance angular frequency is,

$$\omega_r=\frac{\omega_n}{\sqrt{1-2\zeta^2}} \tag{15}$$

and the amplitude of the mass displacement or the resonance amplitude is given as:

$$Z_r=\frac{Y}{2\zeta\sqrt{1-\zeta^2}} \tag{16}$$

From eqs. (13) and (15) results that the resonance phenomenon only occurs when $\zeta<1/\sqrt{2}$ (practically, $\zeta<0.5$).

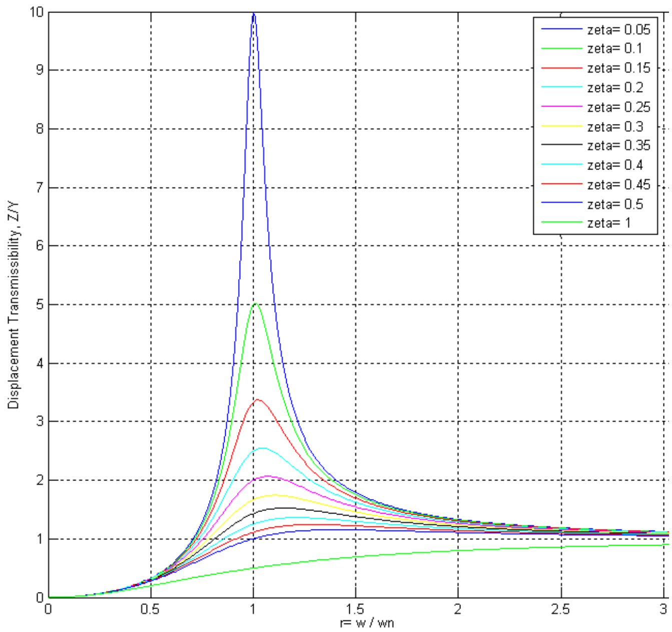

Fig. 3. Displacement transmissibility versus r for different ζ values.

The displacement transmissibility is optimized when the excitation frequency is equal to the resonance frequency ω_r, eq. (15). This can be well observed from Figure 3. The maximum transmissibility is obtained when $r_r = 1/\sqrt{1-2\zeta^2}$. In other words, the frame vibration is totally coupled to magnet mass. In this way, the relative motion of the moving magnet relative to coil is maximized, eq. (16). This is an important result. As will be seen, the generator gives the maximum power at the mechanical resonance frequency.

Response of the system to step input. Using a step input displacement, $y(t)= y_0$, for $t \geq 0$, where y_0 is the step amplitude (Fig. 4), the inertia force on the levitated magnetic mass is theoretically infinite at the initial moment, according to the second term of the eq. (5), as well as the relative movement $z_i(t)$ that can be expressed by:

$$z_i(t) = \begin{cases} -\infty, t = 0 \\ 0, t > 0 \end{cases} \qquad (14)$$

Both the force and the relative displacement have contrary senses to y_0. Due to this impulse $z_i(t)$, if ζ<1, the system oscillates with the amplitude gradually decreasing to zero.

The analitical expressions of the step response for the three cases, underdamped system (*ζ*<1), critically damped system (*ζ*=1) and overdamped system (*ζ*>1) are [6]:

$$z_T(t)=\begin{cases}\left[1-\dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}sin\left(\omega_n\sqrt{1-\zeta^2}t+arctg\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]z_i(t); \quad 0\leq\zeta\leq1 \\ \left[1-(1+\omega_n t)e^{-\omega_n t}\right]z_i(t); \quad \zeta=1 \\ \left[1-\dfrac{e^{-\zeta\omega_n t}}{\sqrt{\zeta^2-1^2}}sh\left(\omega_n\sqrt{1-\zeta^2}t+arcth\dfrac{\sqrt{\zeta^2-1}}{\zeta}\right)\right]z_i(t); \quad \zeta>1\end{cases} \qquad (15)$$

where $z_i(t)$, given by eq. (14), is the forced component or the component of steady state of the system.

In practical cases, we consider that the force and displacement impulses have limited values at *t=0*, z_0 and F_0, respectively, and relationship between these is given of eq. (2), $z_0 = F_0/\omega_n^2$. Step response for the practical cases is plotted in Figure 5.

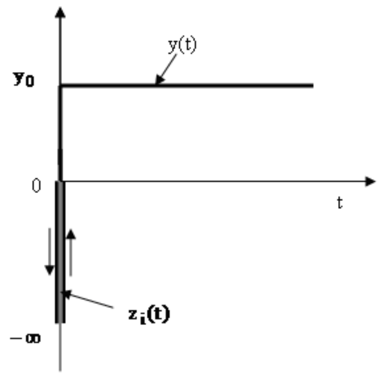

Fig. 4. Forced component of the step response of the system.

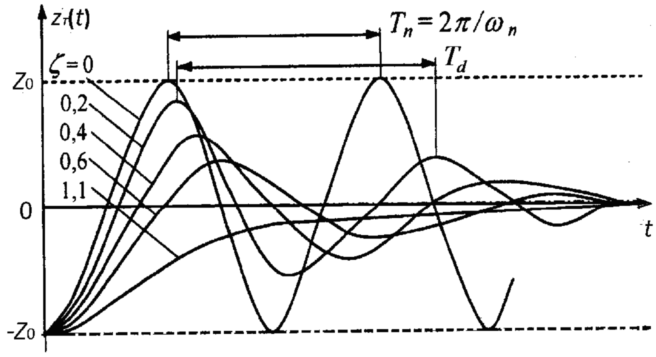

Fig. 5. Step response curves for different ζ values.

For 0<ζ<1, when the system is underdamped, also to oscilatting generators, the frequency of oscillation is called the damped frequency or ringing frequency ω_d,

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \qquad (16)$$

and the damped period T_d will be:

$$T_d = 2\pi/\omega_d \qquad (17)$$

Because the damping coefficient *c* of the system is generally difficult to be theoretically determined (there are some experimental methods for this), the value of the damping ratio ζ is not accurately known. Consider that the magnetic mass of electromagnetic generator presents a damped oscillating response when 0 < ζ < 1, or $c<c_c$. Free oscilations of the magnetic mass for a step input displacement y_0 are shown in Figure 6.

The logarithmic decrement *δ* for underdamped oscilations is defined as the natural log of the ratio of any two successive amplitudes,

$$\delta = ln\frac{z_1}{z_2} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}, 0<\zeta<1 \qquad (18)$$

which shows the damping dependence only of the damping ratio *ζ*.

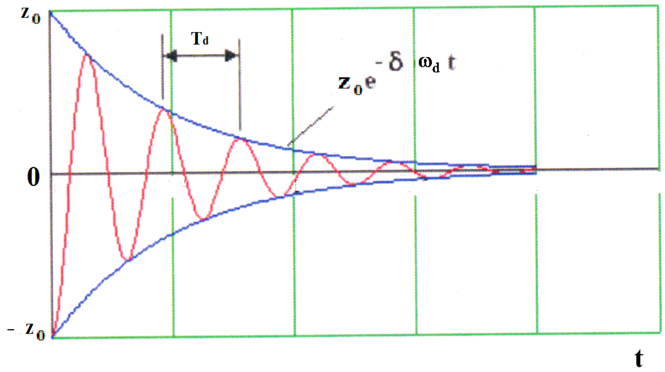

Fig. 6. Ilustrative with damped oscillations for calculus of the logarithmic decrement (18), damping ratio (19) and natural frequency (20).

Eq. (18) can be used to experimentally determine the ratio damping ζ, by measuring the ratio of two succesive amplitudes z_1 and z_2:

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}, \ \delta > 0 \tag{19}$$

Also, the natural frequency can be determined by measuring the damped period T_d (Fig. 6), using the relationship:

$$\omega_n = \frac{\sqrt{4\pi^2 + \delta^2}}{T_d} \tag{20}$$

Power analysis. The electrical average power of the vibration-induced generator can be derivated as:

$$P_e = \frac{m\zeta_e \left(\frac{\omega}{\omega_n}\right)^3 \omega^3 Y^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2} \tag{21}$$

where ζ_e is the electrical damping ratio.

This is the general form of the extracted power from a resonant electromagnetic generator. In this form, or other equivalent forms, the relationship is well known in the literature.

Power losses are expressed in a similar relationship in which appears the mechanical damping ratio ζ_m,

$$P_m = \frac{m\zeta_m \left(\frac{\omega}{\omega_n}\right)^3 \omega^3 Y^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}, \tag{22}$$

so the total power developed in a electromagnetic damper will be:

$$P = \frac{m\zeta \left(\frac{\omega}{\omega_n}\right)^3 \omega^3 Y^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2} \tag{23}$$

where $\zeta = c/2m\omega_n = \zeta_m + \zeta_e$ is the total damping ratio.

When $\omega = \omega_n$ power output P is at largest if $\zeta << 1$ and eq. (23) has the simplified forms:

$$P_{\omega_n} = \frac{m\omega_n^3 Y^2}{4\zeta} \tag{24}$$

$$P_{\omega_n} = \frac{mA^2}{4\omega_n \zeta} \tag{25}$$

Relationship (25) uses the acceleration amplitude of the external vibration $A = \omega_n^2 Y$.

Similarly the electrical power at the natural frequency can be expressed as:

$$P_{e\omega_n} = \frac{m\zeta_e \omega_n^3 Y^2}{4\zeta^2} \tag{26}$$

and

$$P_{e\omega_n} = \frac{m\zeta_e A^2}{4\omega_n \zeta^2} \tag{27}$$

Electrical power is maximised wherever possible when $\zeta_e = \zeta_m$, that is the power from the electrical damping is equal to the mechanical losses. In this case relationship (27) gives:

$$P_{e\omega_n} = \frac{m\omega_n^3 Y^2}{16\zeta_m} \tag{28}$$

Relationships (24)...(27) show that the powers in the oscillating system are theoretically infinite when $\omega = \omega_n$ and the total damping is equal to zero.

However, in some cases, when the damping is notable and the resonant system is nearing the limit of existence of rezonance (ζ = 0.5-0.7), the maximum electrical power can be found in relation to the resonance phenomenon described at the beginning of the chapter. In order to find this power, the ratio of the frequences $r = \omega/\omega_n$ is substituted into eq. (21) and rearranged as:

$$P_e = \frac{m\zeta_e r^6 \omega_n^3 Y^2}{\left(1-r^2\right)^2 + 4\zeta^2 r^2} \qquad (29)$$

Using the resonance ratio $r_r = 1\big/\sqrt{1-2\zeta^2}$, electrical power in resonance is given by:

$$P_{e\omega_r} = \frac{m\zeta_e \omega_n^3 Y^2}{4\zeta^2\left(1-\zeta^2\right)\left(1-2\zeta^2\right)},\ 0\le \zeta < 0,7 \qquad (30)$$

By using eq. (26), one can write:

$$P_{e\omega_r} = \frac{P_{e\omega_n}}{\left(1-\zeta^2\right)\left(1-2\zeta^2\right)} \ge 1 \qquad (31)$$

Eq. (31) shows that the electrical power to resonance frequency $\omega_r = \omega_n\big/\sqrt{1-2\zeta^2}$ is bigger than the power at natural frequency ω_n, if ζ>0. Only when ζ=0 the two powers are equal.

For example, with values ζ of 0.2, 0.3 and 0.4, the power ratio $P_{e\omega_r}\big/P_{e\omega_n}$ is 1.13, 1.34 and 1.75, respectively.

Own equation (30) for electrical power to resonance frequency ω_r comprise the power expression to natural frequency ω_n, eq. (31), the last being obtained from (30) for the particularly case when ζ^2 and $2\zeta^2$ have negligible values relative to the unit.

The electrical damping ratio ζ_e can be written as [3],

$$\zeta_e = \frac{B^2 l^2}{2m\omega_n\left(R+R_c\right)} \qquad (32)$$

where B is the average flux density; l – the length of the coil; R – the load resistance; R_c - the coil resistance.

The voltage in resonance V_{max} can be now expressed as:

$$V_{max} = \sqrt{P_{e\omega_r}\left(R+R_c\right)} = \frac{BlY\omega_n}{2\zeta\sqrt{2\left(1-\zeta^2\right)\left(1-2\zeta^2\right)}} \qquad (33)$$

Eq. (33) shows that the output voltage of the generator is maximum in resonance, and proportional to ω_n, while the corespoding electrical power $P_{e\omega_r}$ is proportional to the third power of ω_n.

3. CONCLUSIONS

✓ The basic theory of the electromagnetic generators for harvesting energy from vibration has been developed. This theory refers to: analytic modeling of the electromagnetic generators, system response to sinusoidal vibration and to step input displacement, and power analysis.

✓ In the frequency response of the generator system analised for the working resonant domain where the damping ratio has the values 0<ζ<0.7, we established the expressions for the resonance frequency and the resonance amplitude of the inertial mass, eqs. (15), (16), which depend on ζ. In the literature dedicated to energy harvesting generators the resonance phenomenon is described only for $\omega = \omega_n$.

✓ The step response of the generator system for a step input displacement of vibration, help in the analysis of the resonant (0<ζ<0.7) and non-resonant (0.7<ζ<1) oscillatory generators when the external vibrations applied to the harvester system presents suddenly variations in amplitude at variable time intervals. We did not find in literature the analysis of the step response of the electromagnetic generators.

✓ We found the expression for the largest electrical power of the generators using the resonance frequency $\omega_r = \omega_n\big/\sqrt{1-2\zeta^2}$, which is bigger than the power to natural frequency ω_n, if ζ>0, given in literature as the maximum electrical power extracted from the system. For example, if ζ=0.4, the power calculated with own expression (eq. (30)) is with 75% greater than that calculated with the relationship known in literature, for the maximum power (21)). Own general equation (30) for electrical power to resonance frequency ω_r includes the power expression to natural frequency ω_n, as shows eq. (31).

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