MODELING OF DYNAMIC PROCESSES NONHOMOGENEOUS CIRCUITS WITH DISTRIBUTED AND LUMPED PARAMETERS

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REZUMAT. Neomogenitatea circuitelor electrice crează dificultăţi la analiza proceselor staţionare şi dinamice. Urmare a acestui fapt în soluţiile numerice apar componente care fizic nu există şi conduc la diminuarea preciziei soluţiilor obţinute. În lucrarea prezentată s-a popus un procedeu modificat a metodei diferenţelor finite pentru calcularea proceselor dinamice în circuitele neomogene. Soluţiile numerice privind derularea procesele în liniile lungi cu parametri distribuiţi şi concentraţi s-au comparat cu cele obţinute cu metoda FDTD. S-a confirmat robusteţea şi performaţa procedeului propus de calcul numeric a proceselor dinamice în liniile neomogene de transport a energiei electrice.

Cuvinte cheie: circuit neomogen, modelare, metodă numerică, proces dinamic, precizia soluţiei.

ABSTRACT. Circuit's heterogeneity creates difficulties in analyzing stationary and dynamic processes. Following this, the numerical solutions contain components what physical doesn't exist and lead to lower accuracy of obtained solutions. In this paper a modified procedure of finite difference method for calculating the dynamic processes in heterogeneous circuits was proposed. Numerical solutions regarding running processes in long lines with distributed and concentrated parameters were compared with those obtained by the FDTD method. The robustness and performance of the proposed procedure for numerical computation of the dynamic processes in heterogeneous electrical transmission lines was proved.

Keywords: nonhomogeneous circuit, modeling, numerical methods, dynamic process, solution accuracy.

1. INTRODUCTION

Energy transport over long distances poses new problems for the electricity sector, which can be solved in different ways and with different methods. Reality is therefore the most useful and reasonable way to obtain solutions with an accuracy better looking and more lifelike gives us mathematical modelling methods and procedures. In this direction are many efforts made by various research groups [1-11], and electronic computing equipment to implement these opportunities has increased very considerably. Great possibilities gives numerical methods for simulating stationary and dynamic processes in electrical circuits, mainly with parameters and homogeneous structures [2,4,5,12-15].

For a number of issues already raised by theoretical electrical exact analytical solutions are obtained [6.10].

These solutions can serve as reference solutions for the estimation precision of numerical methods, which have an essential share in the use and more extensive possibilities of practical use in solving many problems of calculation and analysis of the regimes in electrical circuits [14]. We also mentioned that real circuits are inhomogeneous by nature, which creates difficulties in using analytical and numerical methods to analyze dynamic processes.

Currently prevailing approach to the calculation procedures used in electrical circuits based processes using the concept of superposition of solutions obtained for inpatient and solutions obtained for transients in circuits. Generally, processes in the electrical circuit runs continuously and has a dynamic change of operating mode parameters. In this context presents an integrated approach to sustainable methodology for calculating the processes in electrical circuits, which takes into account the transition from one regime to another and includes simultaneous determination of the characteristics and parameters stationary and transient processes in their natural sequence of development [14, 16]. Numerical methods offer great possibilities for analyzing processes in their natural sequence of development. However, it is worth mentioning that obtained numerical solutions require a critical review of their correspondence to reality. In this context it may indicate the phenomenon known as Gibbs oscillations in numerical solutions [6]. These oscillations appear to change by jumping values or voltage in the circuit and may form a wrong picture about the dynamic nature of processes running.

In work is proposed and substantiated procedure of exclusion from numerical solution of oscillatory disturbances, which ensures high accuracy numerical solutions of dynamical regimes operating in electrical circuits, including changing regimes by jumping voltage and current values. In order to confirm the benefits of the proposed numerical method was performed comparing numerical solutions with those obtained by FDTD method (Finite-Difference Time-Domain).

2. PROBLEM FORMULATION

The electromagnetic energy transmission along the long line by means of conduction currents can be described by telegraph equations that represent Kirchhoff's laws for closed circuit generated by subcircuit with the length dx:

$$L\frac{\partial i}{\partial t}+\frac{\partial u}{\partial x}+Ri=0;\; C\frac{\partial u}{\partial t}+\frac{\partial i}{\partial x}+Gu=0. \quad (1)$$

To single out the unique solution the system (1) must be completed by boundary and initial conditions. Let suppose that the electrical circuit at the initial time t = 0 is connected to the external voltage or current source:

$$u=U_0(t) \text{ or } i=I_0(t), \quad (2)$$

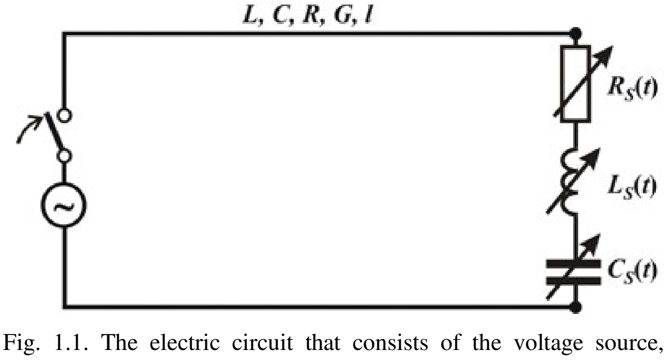

Fig. 1.1. The electric circuit that consists of the voltage source, uniform long line with the lineal parameters *L, C, R, G* and the lumped *RLC* – load at the receiving end

The receiving end is closed to the active-reactive load in the form of series RLC–circuit for which the following integro-differential relation takes place

$$u=R_s i+L_s\frac{di}{dt}+\frac{1}{C_s}\int_0^t i(\tau)d\tau, \text{ when } x=l \quad (3)$$

Obviously, when $R_S = L_S = 0$, $C_S = \infty$ we obtain the short circuited regime: $u = 0$, but the condition $R_S = \infty$ corresponds to idling regime in the line: $i = 0$ (the load is disconnected). The initial conditions usually are assumed to be equal to zero (the electric charge is missing in the circuit before commutation).

Now let consider the situation of power take-off or connection of "bucking out" systems at the intermediate points x = xn of the line. In this case, the currents and voltages (as a functions of spatial variable *x*) can admit the discontinuities of the first kind or other jumps. However, the expression (3) does not change its form if we substitute $i = i_1 - i_2$ and $u = u_1 - u_2$, where the inferior indexes refer to the function values at the left and at the right of the draw-off point. Let remark that the active-reactive lumped loads can consist from the arbitrary set of parallel and series connected *RLC*–circuits.

Starting from the general theory let obtain the energy integral for the hyperbolic system (1). To this end, we multiply the first equation of (1) by i, but the second one – by u and then add the obtained results

$$Li\frac{\partial i}{\partial t}+i\frac{\partial u}{\partial x}+Ri^2+Cu\frac{\partial u}{\partial t}+u\frac{\partial i}{\partial x}+Gu^2=0 \quad \text{or}$$

$$\frac{1}{2}\frac{\partial}{\partial t}(Li^2+Cu^2)+Ri^2+Gu^2+\frac{\partial}{\partial x}(iu)=0.$$

Integrating the last expression over $0\le x\le l$, $0\le\tau\le t$ and taking into consideration the zero initial data, we obtain the following equality:

$$\begin{aligned}&\int_0^t\int_0^l\left(Ri^2+Gu^2\right)dx\,d\tau+\frac{1}{2}\int_0^l\left(Li^2+Cu^2\right)dx=\\&=\int_0^t\left[i(0,\tau)u(0,\tau)-i(l,\tau)u(l,\tau)\right]d\tau\end{aligned}, \text{ when } x=l. \quad (4)$$

The left-hand part of the energy balance equation represents the sum of its active (irreversibly converting to the heat) and reactive (reversible) components, but the right-hand part represents the difference between the energies of the source and the receiver. All components in (4) have the dimension of J (joule).

It is obvious that under the zero initial and boundary conditions the integral equality (4) holds only for trivial solution $i \equiv u \equiv 0$. Then the unicity theorem results from Fredholm alternative in assumption of solution existence.

3. NUMERICAL METHOD

The system of linear equations (1) is of hyperbolic type that implies the finite velocity of electromagnetic wave propagation along the line. The velocity is determined by linear parameters of the line as follows: $a = 1/\sqrt{LC}$.
In order to create the finite-difference scheme we use the grid-characteristic method as follows [14]. At first on the interval $[0, l]$ we generate the uniform grid with integer and half-integer indexes:

$$x_m = mh,\ m = \overline{0, N};\ x_{m-1/2} = x_m - h/2,\ m = \overline{1, N};\ h = l/N$$

The uniform grid is generated over the time coordinate with the step τ also with integer and half-integer indexes: $t_n = n\tau$, $t_{n+1/2} = t_n + \tau/2$, n = 0,1,2,…
Then we integrate the equations (1) by the cell $Q = [x_{m-1}, x_m] \times [t_n, t_{n+1}]$ of the two-dimensional grid. So we obtain the integral relations along the boundary of the cell Q

$$L \int_{x_{m-1}}^{x_m} \left[i(x,t_{n+1}) - i(x,t_n) \right] dx + \int_{t_n}^{t_{n+1}} \left[u(x_m,t) - u(x_{m-1},t) \right] dt +$$
$$+ R \iint_Q i(x,t)\, dt dx = 0;$$

$$C \int_{x_{m-1}}^{x_m} \left[u(x,t_{n+1}) - u(x,t_n) \right] dx + \int_{t_n}^{t_{n+1}} \left[i(x_m,t) - i(x_{m-1},t) \right] dt +$$
$$+ G \iint_Q u(x,t)\, dt dx = 0.$$

Now the one-dimensional integrals we approximate by means of quadrature midpoint rule or rectangle rule, but two-dimensional integrals–by the formula with weights α and β.
As a result we obtain [14]:

$$L \frac{i_{m-1/2}^{n+1} - i_{m-1/2}^{n}}{\tau} + \frac{u_m^{n+1/2} - u_{m-1}^{n+1/2}}{h} + \alpha i_{m-1/2}^{n+1} +$$
$$+ (R - \alpha) i_{m-1/2}^{n} = 0;$$
$$C \frac{u_{m-1/2}^{n+1} - u_{m-1/2}^{n}}{\tau} + \frac{i_m^{n+1/2} - i_{m-1}^{n+1/2}}{h} + \beta u_{m-1/2}^{n+1} + \qquad (5)$$
$$+ (G - \beta) u_{m-1/2}^{n} = 0.$$

The following notations are used here

$$i_{m\pm 1/2}^{n} = i(x_m \pm h/2, t_n),\ i_m^{n+1/2} = i(x_m, t_n + \tau/2)$$

The values $i_m^{n+1/2}$ and $u_m^{n+1/2}$ at the half-integer time layer can be expressed from (5) through the variables $i_{m\pm 1/2}^{n}$ and $u_{m\pm 1/2}^{n}$. We'll use the relations on the characteristics with positive and negative slopes:

$$u_m^{n+1/2} + Z_B i_m^{n+1/2} = u_{m-1/2}^{n} + Z_B i_{m-1/2}^{n};$$

$$u_m^{n+1/2} - Z_B i_m^{n+1/2} = u_{m+1/2}^{n} - Z_B i_{m+1/2}^{n}.$$

Solving this system we obtain the following correlations

$$i_m^{n+1/2} = \frac{u_{m-1/2}^{n} - u_{m+1/2}^{n}}{2Z_B} + \frac{i_{m-1/2}^{n} + i_{m+1/2}^{n}}{2}; \qquad (6)$$

$$u_m^{n+1/2} = \frac{u_{m-1/2}^{n} + u_{m+1/2}^{n}}{2} + Z_B \frac{i_{m-1/2}^{n} - i_{m+1/2}^{n}}{2}.$$

So, the finite difference equations, that approximate (1), have the form of relations (5), (6). The boundary values $i_m^{n+1/2}$ and $u_m^{n+1/2}$ take the form

$$u_0^{n+1/2} = U_0(t_{n+1/2}),$$

$$i_0^{n+1/2} = \left(U_0(t_{n+1/2}) - u_{1/2}^{n} \right) / Z_B + i_{1/2}^{n}; \qquad (7)$$

$$\left(D_h + Z_B \right) i_N^{n+1/2} = u_{N-1/2}^{n} + Z_B i_{N-1/2}^{n},$$

$$u_N^{n+1/2} = D_h i_N^{n+1/2},$$

where D_h is the finite-difference approximation for the integro-differential operator D from (3). The operator D_h can be represented in the following form

$$D_h i_N^{n+1/2} = R_S i_N^{n+1/2} + L_S \frac{i_N^{n+1/2} - i_N^{n-1/2}}{\tau} + \frac{1}{C_S}\sum_{k=1}^{n} \frac{i_N^{n+1/2} + i_N^{n-1/2}}{2}\tau$$

Then the boundary values at the right end $i_N^{n+1/2}$ and $u_N^{n+1/2}$ can be written in the explicit form

$$u_N^{n+1/2} = \frac{1}{B_S + Z_B}\left[Z_B F_S + B_S\left(u_{N-1/2}^n + Z_B i_{N-1/2}^n\right)\right];$$

$$i_N^{n+1/2} = \frac{1}{B_S + Z_B}\left[-F_S + \left(u_{N-1/2}^n + Z_B i_{N-1/2}^n\right)\right];$$

$$B_S = R_S + \frac{L_S}{\tau} + \frac{\tau}{C_S}; \tag{8}$$

$$F_S = \left(-\frac{L_S}{\tau} + \frac{\tau}{2C_S}\right) i_N^{n-1/2} + \frac{1}{C_S}\sum_{k=1}^{n-1} \frac{i_N^{n+1/2} + i_N^{n-1/2}}{2}\tau.$$

The weighting coefficients α, β and the time-step τ in (5), (6) must be chosen in such a way as to ensure the minimal effect of the dispersion and dissipation phenomenon of the finite difference scheme. In order to determine the optimal values of the parameters α, β and τ we apply the first differential approximation method for finite difference relations [14]. Using (6) we eliminate the values $i_m^{n+1/2}$, $u_m^{n+1/2}$ from the (5) and write out the obtained relations in the non-index form:

$$(L + \alpha\tau) i_t + u_{\dot{x}} + Ri - \frac{hLa}{2} i_{\bar{x}x} = 0;$$

$$(C + \beta\tau) u_t + i_{\dot{x}} + Gu - \frac{hCa}{2} u_{\bar{x}x} = 0. \tag{9}$$

So the first differential approximation for this scheme has the following form

$$(L + \alpha\tau)\left(\frac{\partial i}{\partial t} + \frac{\tau}{2}\frac{\partial^2 i}{\partial t^2}\right) + \frac{\partial u}{\partial x} + Ri - \frac{hLa}{2}\frac{\partial^2 i}{\partial x^2} = 0;$$

$$(C + \beta\tau)\left(\frac{\partial u}{\partial t} + \frac{\tau}{2}\frac{\partial^2 u}{\partial t^2}\right) + \frac{\partial i}{\partial x} + Gu - \frac{hCa}{2}\frac{\partial^2 u}{\partial x^2} = 0.$$

These differential equations are approximated by finite-difference equations (9) with second order of accuracy by h and τ.

The original telegraph equations for the line with losses are equivalent to the following:

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} + (LG + RC)\frac{\partial i}{\partial t} + RGi;$$

$$\frac{\partial^2 u}{\partial x^2} = LC\frac{\partial^2 u}{\partial t^2} + (LG + RC)\frac{\partial u}{\partial t} + RGu.$$

Taking into account above representations the first differential approximation can be transformed to the form:

$$L\frac{\partial i}{\partial t} + \frac{\partial u}{\partial x} + Ri + \alpha\tau\frac{\partial i}{\partial t} + \frac{L\tau}{2}\frac{\partial^2 i}{\partial t^2} - \frac{Lah}{2}\frac{\partial^2 i}{\partial x^2} = 0;$$

$$C\frac{\partial u}{\partial t} + \frac{\partial i}{\partial x} + Gu + \beta\tau\frac{\partial u}{\partial t} + \frac{C\tau}{2}\frac{\partial^2 u}{\partial t^2} - \frac{Cah}{2}\frac{\partial^2 u}{\partial x^2} = 0.$$

This implies that the weighting coefficients α, β should be chosen in such a way as to minimize (desirable to zero) differential additions to the original equations:

$$\alpha\tau\frac{\partial i}{\partial t} + \frac{L\tau}{2}\frac{\partial^2 i}{\partial t^2} - \frac{Lah}{2}\left[LC\frac{\partial^2 i}{\partial t^2} + (LG + RC)\frac{\partial i}{\partial t} + RGi\right] \to 0;$$

$$\beta\tau\frac{\partial u}{\partial t} + \frac{C\tau}{2}\frac{\partial^2 u}{\partial t^2} - \frac{Cah}{2}\left[LC\frac{\partial^2 u}{\partial t^2} + (LG + RC)\frac{\partial u}{\partial t} + RGu\right] \to 0.$$

After collecting terms we obtain

$$\frac{\partial^2 i}{\partial t^2}\left(\frac{L\tau}{2}-\frac{Lh}{2a}\right)+$$
$$+\frac{\partial i}{\partial t}\left[\alpha\tau-\frac{Lah}{2}(LG+RC)\right]-\frac{Lah}{2}RGi\to 0;$$

$$\frac{\partial^2 u}{\partial t^2}\left(\frac{C\tau}{2}-\frac{Ch}{2a}\right)+$$
$$+\frac{\partial u}{\partial t}\left[\beta\tau-\frac{Cah}{2}(LG+RC)\right]-\frac{Cah}{2}RGu\to 0.$$

One can readily see that when $\tau = h/a$ and

$$\alpha=\frac{hLa}{2\tau}(LG+RC)=\frac{La^2}{2}(LG+RC)=$$
$$=\frac{1}{2}\left(R+\frac{LG}{C}\right);$$

$$\beta=\frac{hCa}{2\tau}(LG+RC)=\frac{Ca^2}{2}(LG+RC)=$$
$$=\frac{1}{2}\left(G+\frac{CR}{L}\right)$$

the coefficients of time derivatives become zero, but the remaining members do not contain the derivatives and tend to zero with first order when $h\to 0$. The coefficients α and β can be represented as: $\alpha = L\gamma$, $\beta = C\gamma$, $\gamma=(\gamma_G+\gamma_R)/2$, $\gamma_R = R/L$, $\gamma_G = G/C$. Hence, the proposed scheme with the weights minimizes not only the dissipation, but the difference dispersion of the numerical solution as well.

The decomposition on elementary cells by space coordinate x is carried out in such a way as to hold the condition $\tau = h_{n-1/2}/a_{n-1/}2$ = const for any index n.

It is to mention, that the finite-difference relations (5) take into account the linear parameter changes along the longitudinal coordinate x and they can be easily generalized in conformity with multiphase electrical systems with branch points and another complicative factors.

In case of multiwire line when *L, C* are symmetrical square matrixes of self and mutual inductances and capacities, the wave velocities correspond to the eigenvalues of the matrix

$$A=\left\|\begin{matrix}0 & L^{-1}\\ C^{-1} & 0\end{matrix}\right\|,$$

and the matrix of wave resistances is calculated as $Z = L^{1/2}C^{-1/2}$.

If we'll denote by $a_{n-1/2}$ the maximal velocity of the electromagnetic wave propagation and replace scalar values in formulas (5) by corresponding vectors $i = (i_1, i_2,\ldots, i_m)$, $u = (u_1, u_2,\ldots, u_m)$ and matrixes *L, C, R, G, Z*, α, β, then we'll obtain the design equations for distributed system with arbitrary number of conductors.

4.COMPARISON WITH FDTD METHOD

To apply the FDTD method [12,13], at first on the domain $D=\{(x,t): x\in[0,l],\ t\ge 0\}$ we generate two grids with integer $\omega_{h\tau}$ and half-integer $\tilde{\omega}_{h\tau}$ nodes. The grid step h over the space variable is calculated as $h = l/N$, and the step τ over the time variable is chosen according to the scheme stability condition as $\tau = h/a$, where $a = 1/\sqrt{LC}$ is the velocity of the electromagnetic wave propagation. Thus, we have

$$\omega_{h\tau}=\{(x_m,t_n): x_m = mh;\ t_n = n\tau;\ m=\overline{0,N};\ n=0,1,2,\ldots\}$$

$$\tilde{\omega}_{h\tau}=\{(x_{m-1/2},t_{n-1/2}): x_{m-1/2}=x_m-h/2;$$

$$t_{n-1/2}=t_n-\tau/2;\ m=\overline{0,N+1};\ n=1,2,3,\ldots\}$$

The main idea of the FDTD method is the follows: the current function $i(x,t)$ is calculated at the integer nodes of the grid $\omega_{h\tau}$, but the voltage function $u(x,t)$ is calculated at the half-integer nodes of the grid $\tilde{\omega}_{h\tau}$. In this case, the derivatives from the equations (1) can be approximated by finite differences with second order of accuracy with respect to parameters h and τ. In this way, we obtain the following finite difference scheme

$$L\frac{i_m^{n+1}-i_m^n}{\tau}+\frac{u_{m+1/2}^{n+1/2}-u_{m-1/2}^{n+1/2}}{h}+$$
$$+R\frac{i_m^{n+1}+i_m^n}{2}=0;\ m=\overline{0,N},\ n=0,1,2,...$$
$$C\frac{u_{m-1/2}^{n+1/2}-u_{m-1/2}^{n-1/2}}{\tau}+\frac{i_m^n-i_{m-1}^n}{h}+$$
$$+G\frac{u_{m-1/2}^{n+1/2}+u_{m-1/2}^{n-1/2}}{2}=0;\ m=\overline{1,N},\ n=1,2,3,... \qquad (10)$$

The equations (6) are to be completed by the approximations of the initial and boundary conditions. In order to obtain the second order approximation for the initial condition we assume

$$i_m^0=I(x_m);\ u_{m-1/2}^{1/2}=\left(1-\frac{\tau G}{2C}\right)U(x_{m-1/2})-$$
$$-\frac{\tau}{2Ch}\left[I(x_m)-I(x_{m-1})\right] \qquad (11)$$

where $I(x)$ and $U(x)$ are the values of current and voltage at the initial time moment $t=0$.

The boundary condition at the input of the line (2) takes the form

$$u_{-1/2}^{n-1/2}=-u_{1/2}^{n-1/2}+2U_0(t_{n-1/2}) \qquad (12)$$

and the condition (3) at the output (when $L_S=1/C_S=0$) becomes

$$u_{N+1/2}^{n-1/2}=\frac{1}{1+\frac{\tau R}{2L}+\frac{\tau R_S}{hL}}\times$$
$$\times\left[2R_S i_N^{n-1}-\left(1+\frac{\tau R}{2L}-\frac{\tau R_S}{hL}\right)u_{N-1/2}^{n-1/2}\right] \qquad (13)$$

Let compare the accuracy of the solution of the problem (1) – (3) by the scheme (5) and by FDTD method (10) – (13) with the following values of the non-dimensional parameters: $L = C = 1$; $R = G = 0.48$; $R_S = 3$; $l = 0.7$. We consider the voltage and current equal to zero at $t = 0$ ($U(x)\equiv 0$; $I(x)\equiv 0$) and the sinusoidal voltage at the input point of the line: $U_0(t)=\sin(2\pi t)$. The computations were carried out up to the time moment $t = T_{\max} = 4$.

The table 1 contains the results of the comparison of exact analytical solution for current at the output point of the line $I_a(t)$ with the approximate solution $I_F(t)$, obtained by FDTD method and with approximate solution $I_G(t)$, obtained by scheme (5). There are used the following notations: $\|I_F-I_a\|_C$ and $\|I_G-I_a\|_C$ are the maximal values of the differences between the solutions, $\|I_F-I_a\|_{L_2}$ and $\|I_G-I_a\|_{L_2}$ are the corresponding mean square deviations. The first column of the table contains the numbers of grid nodes over the space variable.

Table 1

The absolute error values of the solutions obtained by exact analytical method (I_a) and by approximate methods: FDTD method (I_F) and scheme (5) (I_G)

N	$\|I_F-I_a\|_C$	$\|I_G-I_a\|_C$	$\|I_F-I_a\|_{L_2}$	$\|I_G-I_a\|_{L_2}$
10	0.011108	0.026568	0.013233	0.027565
20	0.002734	0.013365	0.003232	0.013599
40	0.000681	0.006638	0.000800	0.006736
80	0.000170	0.003308	0.000200	0.003357
160	0.000043	0.001651	0.000050	0.001674

The analysis of the given data clearly illustrates the theoretical accuracy of these two methods: the decreasing in two times of the grid step leads to the four times decreasing of the FDTD method error and to the two times decreasing of the scheme (5) error (the second order of accuracy for the FDTD method and the first order – for the scheme (5)). In such a way, for continuous solutions of the problem (1) – (3) the FDTD method is more exact and, correspondingly, more preferable.

Now let consider the same problem, but with condition that during some period of time at the input of the line the short-circuit occurs, i.e. the input voltage becomes zero in some period of time: $U_0(t) = \sin(2\pi t)$ when $0 \le t \le 0.6$ or $2.6 \le t \le 4$ and $U_0(t) = 0$ when $0.6 < t < 2.6$.

The time dependences of the voltages and currents at the input of the line (x = 0) for N = 10; 20; 40 are represented in the fig. 1. The exact analytical solution is marked with thick line and the solution obtained by FDTD method is marked with thin line. The solution obtained by means of scheme (5) practically does not differ from the exact solution.

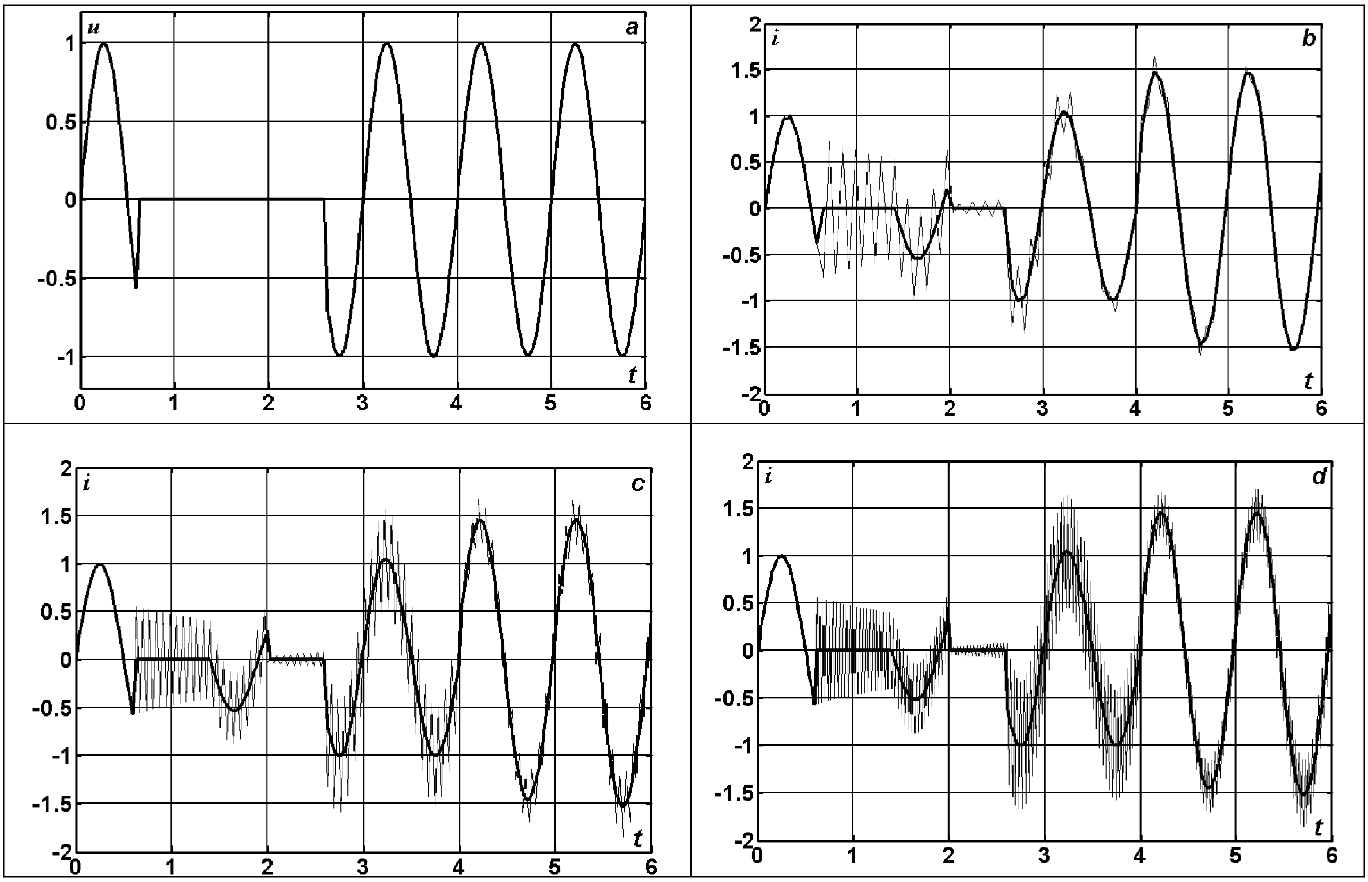

Fig. 2. The time dependence of the voltage at the input of the line (a) and of the current when N = 10 (b); N = 20 (c); N = 40 (d).

The given figures demonstrate that the FDTD method on discontinuous solutions leads to the appearance of large oscillations that do not decrease with decreasing of the grid step. But there is no reason to be surprised since as early as 1959 it was proved by Godunov [15] that among the linear finite difference schemes with second order of accuracy for the equation $\partial u/\partial t + \partial u/\partial x = 0$ there is no one satisfying the condition of monotony, i.e. no one that does not lead to appearance of the oscillations when computing the discontinuous solutions.

Thus, the FDTD method, in spite of the fact that it is of second order of accuracy, can be restrictedly applied to the telegraph equations as it does not permit the modeling of such regimes as short-circuit and idling.

5. CONCLUSIONS

✓ Since any steady-state process is always preceded by transient process, then their computations must be realized in the same consecution. The elaborated numerical scheme is conservative with zero finite-difference dissipation and minimal dispersion. These properties result in the fact that the computational error does not accumulate, that gives the possibility to realize the transparent calculations of nonstationary solutions without loss of accuracy at large time intervals corresponding to 300…500 electromagnetic wave runs along the line length right up to steady-state regime. At the same time the parameters of the line, of the generator and loading units can change instantaneously modeling the load droppings or load pickups, the emergency situations as short-circuits, breaks in circuit, jump changes of the voltage, lightning strokes, etc.

✓ The finite-difference scheme not only repeats with any order of accuracy the solutions obtained by classical methods, but gives the possibility to extend essentially the class of solvable problems from the theory of electrical circuits with variable parameters in comparison with known methods.

✓ It is established and proved by numerical experiments that well-known FDTD method elaborated for Maxwell equations is not applicable or give a poor accuracy in case of discontinuous solutions in transient processes.

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