

CONSIDERATIONS ABOUT THE FORCES APPLIED TO ELECTRIC CONDUCTORS LOCATED CLOSE TO FERROMAGNETIC WALLS

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REZUMAT: Conductoarele electrice sunt deseori plasate în vecinătatea pereților feromagnetici sau în interiorul nișelor electromagnetice. Este necesar să se cunoască exact forțele care apar în aceste situații, pentru a asigura o proiectare optimă a mașinilor sau aparatelor electrice. Această lucrare prezintă o metodă generală de calcul a acestor forțe bazată pe tensorii lui Maxwell, care ia în considerație plasarea asimetrică a conductorului între pereți

Cuvinte cheie: Forțe, Conductori electrice, Pereți și nișe feromagnetice

ABSTRACT: Electric conductors are often placed nearby ferromagnetic walls or inside ferromagnetic slots. Knowing exactly the forces which appear in those situations is mandatory in order to insure an adequate design of all machines, apparatus and power equipment. This paper presents a more generalized method to compute these forces, based on Maxwell's tensors, and taking in consideration the asymmetry of the conductor placement between the walls.

Keywords: Forces, Electric Conductors, Ferromagnetic Walls and Slots

1. INTRODUCTION

As we know from [1], electric conductors crossed by different electric currents, are often located or placed nearby ferromagnetic walls.

They are even placed inside ferromagnetic slots or compartments. In all these locations, they are subjected to attraction forces.

The computational methods found in literature [1],[2],[3],[4],[5],[6], are treating those situations in some particular cases like:

- the electric conductors are placed exactly in the symmetry axis or plan belonging to that geometrical structure;
- the length of the ferromagnetic wall is considered as infinite;
- the ferromagnetic slots are rectangular or triangular ones only;

This paper presents a more generalist case, where the electric conductor is placed asymmetrical between two finite length ferromagnetic walls or in such a ferromagnetic slot.

2. ELECTRIC CONDUCTOR PLACED IN THE SYMMETRY AXIS OF A FERROMAGNETIC SLOT

We consider an electrical conductor crossed by a certain current, "i".

First, it is placed in a rectangular ferromagnetic slot. The geometrical dimensions are those represented in Figure 1 a,b

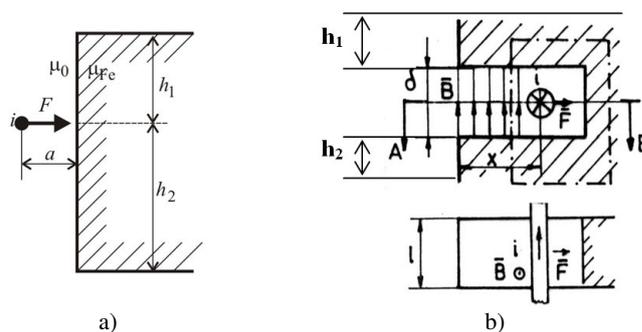


Fig.1. Electric conductor placed close to a ferromagnetic wall a) or inside a ferromagnetic slot b)

By applying the generalized forces theorems for a specific case when the conductor is placed symmetrical inside the slot and the exterior walls are extended up to infinite ($h_1=h_2=\infty$), we will obtain the relation for the

\overline{F} force which tries to push the conductor to the ferromagnetic wall [1], as shown in Fig.1b.

$$F = \left(\frac{\partial W_m}{\partial x} \right)_{i=ct.} = \frac{\mu_0}{2} \cdot \frac{\ell}{\delta} \cdot i^2 \quad (1)$$

where W_m is the magnetic energy.

Next, we consider that the length of the upper and lower ends of that slot are finite, considered as h_1 and h_2 .

When we use the Maxwell Tensors Method in order to compute the same force, taking in consideration the magnetic induction B , we will obtain this relation:

$$F = F_1 + F_2 = \frac{\mu_0 \cdot \ell}{2 \cdot \delta} \cdot i^2 \cdot \left(\frac{h_1}{2 \cdot h_1 + \delta} + \frac{h_2}{2 \cdot h_2 + \delta} \right) \quad (2)$$

In Fig. 1a. the conductor is placed close to a ferromagnetic wall. By using the electrical image method, we obtained a relation for the specific attraction force [1]:

$$f_\infty = \frac{F}{l} = \frac{\mu_0}{4 \cdot \pi \cdot a} \cdot i^2 \quad (3)$$

where l is the length of the conductor and the heights h_1 and h_2 are extended to infinite.

When h_1 and h_2 are finite, (3) becomes:

$$f = f_\infty \cdot \varphi_{h_1} \cdot \varphi_{h_2} \quad (4)$$

where φ_{h_1} and φ_{h_2} are shape coefficients which took in consideration the heights h_1 and h_2 . They are given by:

$$\varphi_{h_1} = \frac{\pi \cdot h_1 - 2 \cdot a}{\pi \cdot h_1 + 2 \cdot a} \quad (5)$$

$$\varphi_{h_2} = \left(1 + \frac{2 \cdot \pi \cdot a}{\pi \cdot h_1 - 2 \cdot a} \cdot \frac{h_2 - h_1}{\frac{3 \cdot \pi}{2} \cdot h_2 + 2 \cdot a - \frac{\pi}{2} \cdot h_1} \right) \quad (6)$$

All relations starting from (1) up to (6) are particular cases, where the conductor is placed close to a ferromagnetic wall.

3. ELECTRIC CONDUCTOR ASYMMETRICALLY PLACED BETWEEN TWO FERROMAGNETIC WALLS

We also consider an electrical conductor crossed by a certain current, " i ". It is asymmetrically placed between two ferromagnetic walls A and B, as shown in Fig. 2.

We appreciate that, with the right approximation that the magnetic lines are closing according to Fig. 2. The two walls are extended up to infinite.

The length of an ordinary field line, starting from A, continuing in air, which does not reach wall B (shape Γ on Fig. 2) is given by:

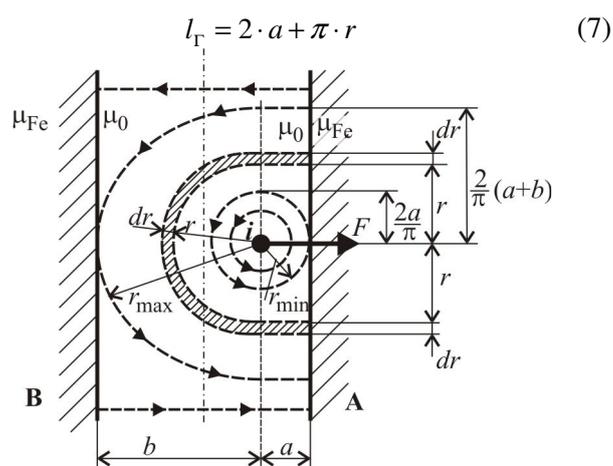


Fig.2. Electric conductor placed between two ferromagnetic walls

By applying the magnetic circuit law for the closed loop Γ , we will obtain the magnetic field's intensity H_m as well as the magnetic field's induction B_M , normal in point M, on the A surface.

$$B_M = \mu_0 \cdot H_M = \frac{\mu_0}{\pi \cdot r + 2 \cdot a} \cdot i \quad (8)$$

By consequent, the Maxwell tensor around point M, normal on A is [1]:

$$f_M = \frac{\mu_0}{2} \cdot \frac{i^2}{(\pi \cdot r + 2 \cdot a)} \quad (9)$$

The force applied by the ferromagnetic wall on the electric conductor is the sum of all Maxwell tensors generated by that field lines starting on wall A and not reaching wall B.

We consider that the minimum radius r_{min} of a field line reaching wall A is:

$$\pi \cdot r_{min} = 2 \cdot a \quad (10)$$

And the maximum radius of a field line which does not reach wall B, r_{max} , is satisfying the next relation:

$$\pi \cdot r_{max} = 2 \cdot (a + b) \quad (11)$$

We consider an elementary surface dS_A , on wall A, given by:

$$dS_A = l \cdot dr \quad (12)$$

The integration limits are from r_{min} up to r_{max} . Taking in consideration the conditions established by (10) and (11), the F force could be written as:

$$F = 2 \cdot \int_{r_{min}}^{r_{max}} f_m dS_A = \int_{\frac{2a}{\pi}}^{\frac{2(a+b)}{\pi}} \frac{\mu_0 \cdot l}{2} \cdot i^2 \cdot \frac{dr}{(\pi \cdot r + 2 \cdot a)^2} \quad (13)$$

After computing the integral, we will obtain:

$$F = \frac{\mu_0 \cdot i^2 \cdot l}{4 \cdot \pi \cdot a} \cdot \frac{b - a}{b + a} \quad (14)$$

And the specific attraction force:

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$$f = \frac{F}{l} = \frac{\mu_0 \cdot i^2}{4 \cdot \pi \cdot a} \cdot \frac{b-a}{b+a} = f_\infty \cdot \varphi_{ab} \quad (15)$$

where:

$$\varphi_{ab} = \frac{b-a}{b+a} \quad (16)$$

and f_∞ is given by the same relation (3).

Relations (14) and (15) allow us to make some significant observations:

- We observe that the presence of a second infinite ferromagnetic wall, called B in this case, diminishes the attraction force applied on the conductor by the ferromagnetic wall A, with a diminution coefficient φ_{ab} given by relation (16).
- If the conductor is placed symmetrical ($a = b$), then the attraction force becomes zero, which is logical, because the attraction forces applied by both walls are canceling each other.
- If dimension "a" becomes greater than "b", the attraction force changes its sign, and the conductor

will be attracted by the B wall, instead of A, which is also a logic thing.

- If wall B is taken away ($b \gg a$), then $\varphi_{ab} \rightarrow 1$ and relation (15) becomes relation (3), which is the one deduced for a single wall.
- If we look at the integration limits, relations (14) and (15) are not necessary applied for infinite walls. They are used for walls having a minimum height equal to $2(a+b)/\pi$.

4. ELECTRIC CONDUCTOR ASYMMETRICALLY PLACED INSIDE A FERROMAGNETIC SLOT

We consider an electrical conductor asymmetrically placed in a ferromagnetic rectangular slot, as shown in Fig.3 a and b.

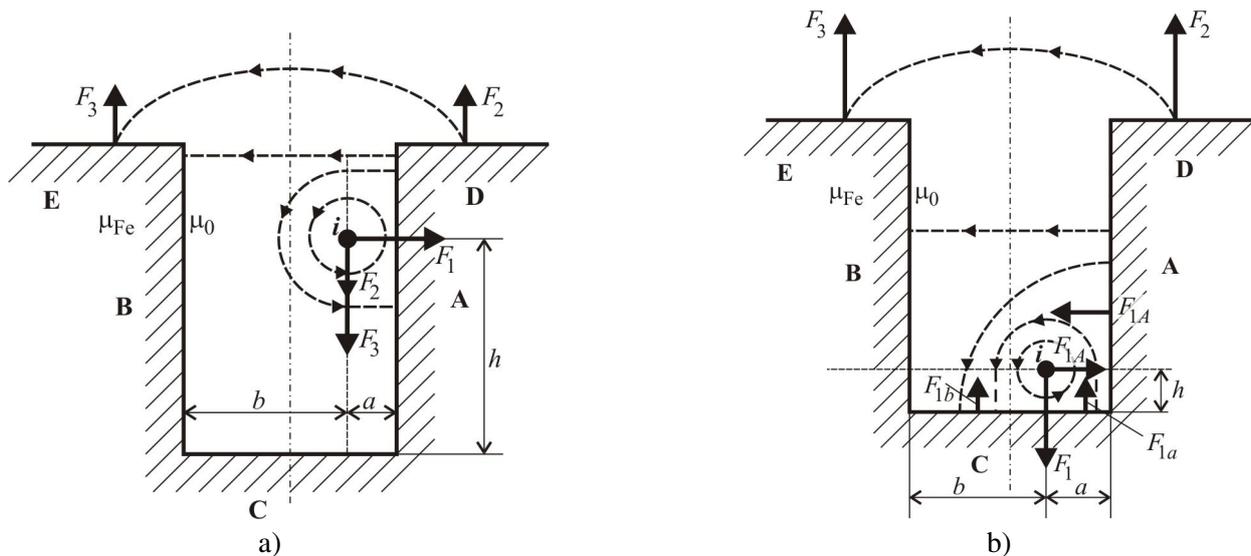


Fig.3. Electric conductor placed inside a ferromagnetic slot. a. Placed far from the bottom; b. Placed close to the bottom

Situation described in Fig.3a could be treated starting from the case previously presented, when the conductor was placed between two ferromagnetic walls. The main condition is that the "h" distance between the conductor and the bottom of the slot, named as C, must fulfill the condition given by (11), which means to be at least equal to $2(a+b)/\pi$.

In this case, the attraction force F_1 , exerted by the A wall on the conductor, is given by a relation similar to (14), which is:

$$F = \frac{\mu_0 \cdot i^2 \cdot l}{4 \cdot \pi \cdot a} \cdot \frac{b-a}{b+a} \quad (17)$$

Forces F_2 and F_3 , generated by Maxwell's tensors are also applied on the conductor. They are produced on the exterior D and E surfaces of the slot, and they are pushing the conductor to the bottom C of the slot. If we consider surfaces D and E extended to infinite, according to (1), the forces are:

$$F_2 = F_3 = \frac{\mu_0 \cdot i^2 \cdot l}{4 \cdot (a+b)} \quad (18)$$

By consequent, the total force F , which is pushing the conductor up to the bottom C of that slot, is given by:

$$F = F_2 + F_3 = \frac{\mu_0 \cdot i^2 \cdot l}{2 \cdot (a+b)} \quad (19)$$

We noticed that if $a = b$, then F_I becomes zero, so only the F force is applied to the conductor. We are finding relation (1).

Case b in Fig.3 does not satisfy the relation from which the distance between the conductor and the bottom must be higher than $2(a+b)/\pi$.

In this situation, the conductor will be attracted to the bottom of the slot with a F_I force and it will be supplementary pushed by a force $F = F_2 + F_3$. It will be also attracted by the A wall with an additional F_{IA} force.

Because forces F_2 and F_3 are the same like in case a), they are given by relation (18).

The F_I force is given by the sum of F_{Ia} and F_{Ib} , forces that appear on the surfaces having an "a" and a "b" length, belonging to the C wall.

By using the Maxwell's tensors on the field lines having the specified shape like in Fig.3b, we will obtain:

a. For the F_{Ia} force, the average field line is considered as:

$$l_{Ia} = 2 \cdot h + \pi \cdot r \quad (20)$$

By repeating all computational procedures like previously described in chapter 3, it results:

$$F_{Ia} = 2 \cdot \int_{r_{\min}}^{r_{\max}} f_m dS_c = \int_{\frac{2h}{\pi}}^{a-h} \frac{\mu_0 \cdot l}{2} \cdot i^2 \cdot \frac{dr}{(\pi \cdot r + 2 \cdot h)^2} = \frac{\mu_0 \cdot l \cdot i^2}{4 \cdot \pi \cdot h} \cdot \frac{\pi(a-h) + 2h}{\pi(a-h) + 2h} \quad (21)$$

b. For the F_{Ib} force, the average field line is considered as:

$$l_{Ib} = h + \frac{\pi}{2} r + a \quad (22)$$

and the integration limits of the Maxwell's tensors are:

$$\begin{aligned} r_{\min} &= a-h \\ r_{\max} &= b-h \end{aligned} \quad (23)$$

It results:

$$F_{Ib} = \int_{a-h}^{b-h} \frac{\mu_0 \cdot l \cdot i^2}{2} \cdot \frac{dr}{\left(\frac{\pi}{2} r + h + a\right)^2} = \frac{\mu_0 \cdot l \cdot i^2}{2} \cdot \frac{b-a}{\frac{\pi}{2}(a-h) + h + a} \cdot \frac{b-a}{\frac{\pi}{2}(b-h) + h + a} \quad (24)$$

c. The F_{IA} force is equal to F_{Ib} , but oriented like in Fig. 3b. On Fig.4, we can distinctly represent the composition of all forces applied on the electric conductor placed inside the ferromagnetic slot, like in Fig. 3b.

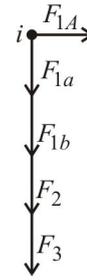


Fig.4. The forces applied on the electric conductor asymmetrically placed near the bottom of the ferromagnetic slot.

According to (24), we still notice that if $a = b$, than $F_{Ib} = 0$ and $F_{IA} = 0$, which is perfectly logical.

3. CONCLUSIONS

The situation of the electrical conductors placed nearby ferromagnetic walls (or inside ferromagnetic slots) is a case frequently seen inside all the electrical machines and apparatus, as well as transformers and power equipment.

Knowing exactly the forces acting in these cases is a priority for all designers of such equipment.

The computational method based on Maxwell's tensors offers a more precise point of view by comparing the classical methods based on the magnetic energy and using many simple models.

All the computational processes are carried out by considering a more general case, when the electric conductor is placed asymmetrical between two ferromagnetic walls.

The new formulas deduced by using those considerations are perfectly compatible with the older cases, all simplified older formulas are particular cases of the newest ones.

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