MATHEMATICAL SOLUTIONS TO APPROXIMATE THE PV PANELS CHARACTERISTICS AND PARAMETERS

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REZUMAT. Ultima perioadă a dus la cresterea semnificativa a cercetărilor si aplicatiilor industriale in domeniul energiei soalre deoarece aceasta este considerata ca fiind energia viitorului, este curata si regenerabila. Evolutia sistemelor fotovoltaice (PV) a condus la proiectarea si instalarea a numeroase panouri PV, cu puteri din ce in ce mai mari. Atunci cand panoul PV este mai mare, structura sa este determinata de conexiuni adecvate serie si paralel ale celulelor PV, care se leaga la acelasi convertor DC/AC. Cunoasterea parametrilor si caracteristicilor celulelor PV este deosebit de importanta si necesra pentru sistemele PV si proiectarea aplicatiiilor acestora. Lucrarea de fata descrie doua solutii matematice, metoda interpolarii si metoda analitica, pentru aproximarea parametrilor si caracteristicilor celulelor/panourilor PV la diferite nivele de radiatie solara. Rezultatele au fost comparate cu cele obtinute din datele achizitionate si masurate pentru cazurile reale. Proceduri de calcul specifici au fost implementati in scopul de a proba metodele teoretice propuse.

Cuvinte cheie: celula PV, conexiuni, caracteristici, parametri, interpolare, metoda analitica.

ABSTRACT. The last period brought a significant development of research and industrial application in the field of solar energy because this is considered to be the energy source of the future, it is clean and renewable. The evolution of photovoltaic (PV) array is leading to the design and installation of more and more largely sized PV plants. Then, when the plant size increases, the structure of the PV plant is determined by suitable parallel and series connections of the PV cells, forming the PV array that lies under the same DC/AC converter. The knowledge of the PV cells parameters and characteristics is crucial and needed for PV systems and applications design. This paper describes two mathematical solutions, curve fitting and analytical methods, in order to approximate the PV cells/panels characteristics and parameters under different solar irradiance levels. The results are compared with those obtaining from acquisition and measured data for real cases. Specific computational procedures are implemented in order to prove the theoretical proposed methods.

Key words: PV cell, connections, characteristics, parameters, curve fitting method, analytical method.

1. INTRODUCTION

A significant number of studies and scenarios have investigated the contribution of renewable energy to satisfy global needs in energy, indicating that the first half of XXI century its contribution will increase from 20 to 50%. So, the technical developments in the electricity systems, the political aspects and the actual problems related to air pollution and to climate changes, led to increased use of photovoltaic (PV) energy sources.

The basic element of the PV systems is the PV cell. A typical PV panel is composed of series-parallel connected cells, until the desired current and voltage levels are achieved. In this context, researches in solar energy systems are a significant evolution in order to improve the performances of the PV systems and establish effective techniques for accurate modeling and analysis of their parameters and characteristics. In order to inform users about their products, the PV manufactures present the electrical properties and performances of PV cells [1]: the output characteristics current-voltage (*I*-*V*) and power-voltage (*P*-*V*) curves; some typical parameters as: short circuit current I_{sc}, open circuit voltage V_{oc} and maximum power point (MPP) P_{MPP}, V_{MPP}, I_{MPP}. These values are usually given at the Standard Test Conditions (STC) also known as Standard Reporting Conditions (SRC) which correspond to a cell temperature equal to 25 °C (T_0 = 298 K) and the irradiance level G_0 =1000 W/m^2. Thus, can be defined the *model parameters* (V_{oc}, I_{sc}, V_{MPP}, I_{MPP}, P_{MPP}) of PV cells/panels, resulting from *I*-*V* and *P*-*V* characteristics of PV cells/panels.

All these characteristics and parameters of PV cells depend on the environmental conditions, especially solar irradiance level (*G*). The knowledge of the PV cell parameters and characteristics and their dependence on *G*, is very important and needed for PV systems analysis and design. For example, this knowledge is of a special interest when the maximum power is required from the PV cell/panel. This is known as the MPP, and the PV cell/panel it is necessary to operate at its MPP. On the *P*-*V* curve of one PV cell there is a unique point on the curve which the solar cell will generate maximum power. In order to achieve MPP for series-parallel connections of PV cells under different environmental conditions, several tracking (MPPT) algorithms that iteratively find the MPP are used. This problem has been solved in order to achieve a variety of aims including: accurate tracking, fast response speed and less oscillation due to the change of the solar irradiance [2]-[4].

This paper describes two mathematical solutions to asses the performances of PV cells/panels under different irradiance levels. Based on LabVIEW data acquisition and measured system, the characteristics and *model parameters* (V_{oc}, I_{sc}, V_{MPP}, I_{MPP}, P_{MPP}) of the PV cells/panels are modeled and analyzed by using curve fitting and analytical methods. First, according to the series and parallel interconnection conditions from the curve fitting method are obtained the polynomial functions approximation $I(V)$, $P(V)$, $V(I)$, $P(I)$ and respectively the *model parameters* of one PV cells and of PV panels, under different connections and solar irradiance levels. Second, an analytical method is used to add point by point the measured characteristics of PV cells corresponding to their series and parallel connections. Then, from these equivalent characteristics under different values of G the PV panels' *model parameters* are obtained. Specific computational procedures are implemented in order to compare the theoretical results with those derived from measured data in real cases. Numerical example taken for real PV cells/panels cases proves the correctness and accuracy of one of the two proposed methods.

2. CHARACTERISTICS AND CONNECTIONS OF PV CELLS

A. Equations, characteristics and parameters of PV cells. PV cell consist of a silicon p-n junction that when exposed to light release electrons around a closed electrical circuit. These electrons are then free to move across the junction due to the built-in potential (V_d) and create a current. This is modeled by the light generated current source (photo-current) I_{ph}. The intrinsic silicon p-n junction characteristic is simulated as a diode in the circuit equivalent. During darkness, the solar cell is not an active device; it works as a diode, i.e. a p-n junction. It produces neither a current nor a voltage. The light generated current source is generated by the solar radiation, when the solar cell exposed to light, then flows to the output this current varies linearly with the solar irradiance as

$$I_{ph}(G) = [I_{sc0} + k_I (T - T_0)] \frac{G}{1000} \quad (1)$$

where I_{sc0} is the PV cell short-circuit current at T_0 =298 K, K_I = 0.0017 A/K is the PV cell short-circuit current temperature coefficient, T is the PV cell absolute temperature and G is the solar irradiance in W/m^2. Many equivalent circuits have been proposed in the literature in order to assess the behavior of the PV cell [5]-[6]. In this paper, the one-exponential model with a single diode is used for the equivalent circuit of PV cell, shown in Fig. 1. At the output 2-terminal of the PV cell,

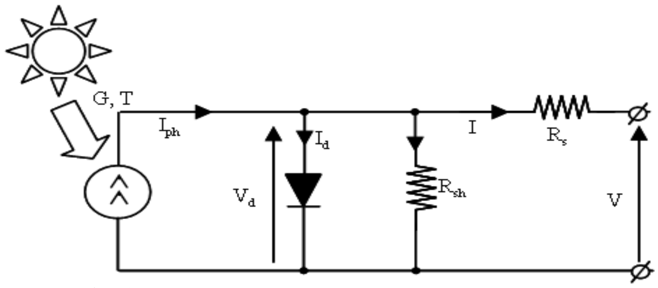


Fig. 1. One-exponential equivalent circuit of PV cell.

the relationship between the outputs current I and voltage V, is known as the *I-V* characteristic of the PV cell. For the directions of currents and voltages indicated by arrows in Fig. 1, the *I-V* characteristic is defined by

$$I = I_{ph} - I_d - \frac{R_s I + V}{R_{sh}} \quad (2)$$

where R_{sh} and R_s are the shunt (representing the effect of the surface current dispersion), and respectively, series resistance (representing the effect of the voltage drop of the semiconductor and contacts) of one PV cell. The diode current is given by

$$I_d = I_{sd}\left[\exp\left(\alpha(R_s I + V)\right) - 1\right] \quad (3)$$

where I_{sd} is diode's saturation current, $\alpha = q / nkT . k$ = 1.3807 x 10^{-23} JK^{-1} is Boltzmann's constant, q = 1.6022 x 10^{-19} C is the electronic charge, and n is the quality factor of the p-n junction of the diode [7].

A solar cell can be operated at any point along its *I-V* characteristic. Two important points on this curve are the open circuit voltage and short-circuit current [8]-[10]. The solar cell output current has its maximum value when it is applying a short-circuit to the cell's output 2-terminal, and the output voltage V = 0, so the short-circuit current I_{sc} is expressed by

$$I_{sc}(G) = \frac{I_{ph}}{1 + \frac{R_s}{R_{sh}}} \quad (4)$$

If the circuit is opened, the output current I = 0 and the maximum value of output voltage is the open-circuit voltage

$$V_{oc}(G) = V_d = \frac{1}{\alpha} \ln\left(\frac{I_{ph}}{I_{sd}} + 1\right) \quad (5)$$

The model parameters V_{oc} and I_{sc} can be evaluated from the *I–V* characteristic of PV cell for different levels of G. Thus V_{oc} and I_{sc} are the intercept points of the *I-V* characteristic with the horizontal respectively vertical axis.

The output power P of the PV cell is defined as

$$P = VI \quad (6)$$

thus, by used (2), can be written explicitly as power-voltage (*P-V*) characteristic of the PV cell

$$P(G) = VI_{ph} - I_{sd}\left[\exp\left(\alpha(R_s I + V)\right) - 1\right]V - \frac{(R_s I + V)V}{R_{sh}} \quad (7)$$

In order to analyze the PV cell efficiency, it is necessary to know the MPP point value of the PV cell, i.e. $P_{MPP} = P\,(V_{MPP}, I_{MPP})$. The *P-V* characteristic curve of one PV cell shows a unique point on the curve which the solar cell will generate maximum power. The MPP value occurs when

$$\left.\frac{dP}{dV}\right|_{V_{MPP},I_{MPP}} = 0 \qquad (8)$$

The P_{MPP} and V_{MPP} values result from *P-V* characteristic as the maximum point, while I_{MPP} value can be determined from *I-V* characteristic as $I(V_{MPP})$. All the model parameters (V_{oc}, I_{sc}, V_{MPP}, I_{MPP}, P_{MPP}) of PV cell are in fact directly related to irradiance level through I_{ph}. So, it is important to consider the effect of irradiance in order to analyze the behavior of PV cell.

B. Parallel and series connections of PV cells. Basically a solar panel comprises n_s PV cells in series and/or n_p PV cells in parallel, interconnected at the output terminals [11]-[14].

A *parallel connection* of n_p PV cells, shown in Fig. 2 for different levels of irradiance, is defined at the output 2-terminals by the Brune's interconnection conditions

$$V_e = V_1 = ... = V_{n_p} = V \; ; \; I_e = \sum_{k=1}^{n_p} I_k \; ; \; P_e = V_e I_e = V_e \sum_{k=1}^{n_p} I_k \qquad (9)$$

In particular case, when all the PV cells are under the same environmental conditions, results

$$I_e = n_p I \;\; ; \;\; P_e = n_p P \qquad (10)$$

where I, I_{ph} and P are the output current, the photo-current, and respectively the output power of one PV cell.

A *series connection* of n_s PV cells, for different level of irradiance shown in Fig. 3, is defined at the output 2-terminals by the Brune's interconnection conditions

$$V_e = \sum_{k=1}^{n_s} V_k \; ; \; I_e = I_1 = ... = I_{n_s} = I \; ; \; P_e = V_e I_e = I_e \sum_{k=1}^{n_p} V_k \qquad (11)$$

For the same environmental conditions, results

$$V_e = n_s V \;\; ; \;\; P_e = n_s P \qquad (12)$$

where V represents the output voltage of a PV cell.

3. APPROXIMATION METHODS OF THE PV PANELS CHARACTERISTICS

Two mathematical methods are used in order to asses the performance of PV cells/panels under different irradiance levels. In order to modeled and approximate the characteristics and parameters of PV cells/panels the curve fitting and analytical methods are used. The both methods extract and process the necessary information obtained by a specific data acquisition LabVIEW application [15].

A. Curve fitting method. The curve fitting defines an appropriate curve to fit the measured values and uses a curve function to analyze the relationship between the variables [16]. The purpose of curve fitting is to find a function $f(x)$ for the input measured data (x_i, y_i) where $i = 1, 2,…, n$ means the number of measurements. The function $f(x)$ minimizes the distance, named residual, between the measured data and $f(x)$. Different fitting methods can evaluate the input data to find the curve fitting model parameters. Each method has its own criteria for evaluating the fitting residual in finding the fitted curve. The general polynomial fits the data set to a polynomial function of the general form

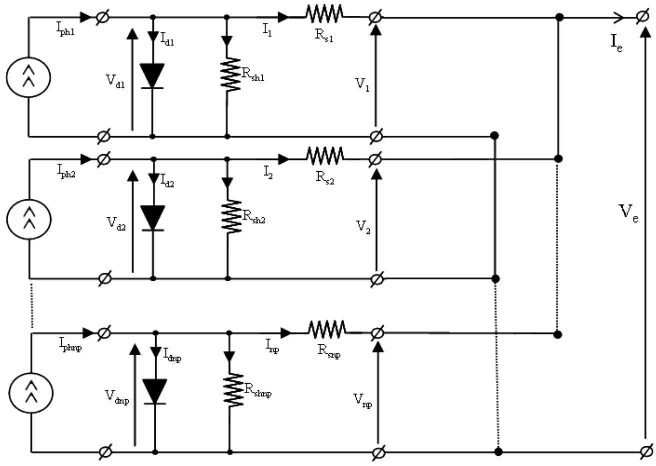


Fig. 2. Parallel connection of PV cells.

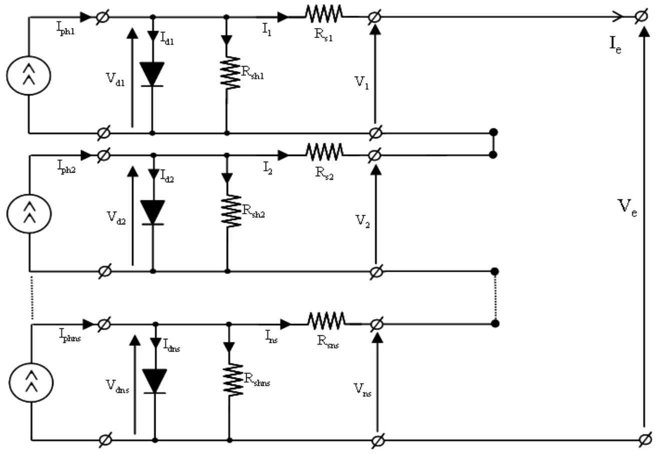


Fig. 3. Series connection of PV cells.

$$f(x) = a_0 + a_1 x + a_2 x^2 + ... \qquad (13)$$

This method calculates the mean square error (MSE) using the equation

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left[f(x_i) - y_i\right]^2 \qquad (14)$$

where $f(x_i)$ is the ith element of the array of y-values of the fitted model and y_i is the ith element of the data set (x_i , y_i). A high polynomial function order does not guarantee a better fitting result and can cause oscillation.

The information resulting from a specific LabVIEW data acquisition and measurement process for one PV cell characteristic I^m-V^m, P^m-V^m, V^m-I^m and P^m-I^m, under different irradiance levels, are used as input measured data in a curve fitting method. In other words, theoretical polynomial functions $I_e^t(V), P_e^t(V)$ for parallel connection, respectively $V_e^t(I), P_e^t(I)$ for series connection of PV cells under different irradiance levels, are obtained from measured data in order to approximate the non-linear equations (2) and (7). If the connection conditions (9) or (11) of PV cells are imposed for these functions, then result, for different values of irradiance, the theoretical polynomial functions which approximate the characteristics of PV panels composed of series or parallel connections of PV cells. Note that the superscripts ^{m} and ^{t} means measured respectively theoretical.

A *computational procedure (C1)* organized into a number of interlinked computational modules is developed in order to obtain the theoretical characteristics of PV panel based on the PV cells' measured data and using curve fitting method. A *description of C1* , shown in Fig. 4, a, implemented for two levels of irradiance G_1 and G_2, is provided below:

C1.1. Data input constructs the measured characteristics $(I^m$-V^m, P^m-V^m, V^m-I^m,P^m-$I^m)_{G1}$ and $(I^m$-V^m, P^m-V^m, V^m-I^m,P^m-$I^m)_{G2}$ for one PV cell under two irradiance levels G_1 and G_2 using the LabVIEW application.

C1.2. Determination of the theoretical polynomial functions of one PV cell which approximates the measured characteristics. Based on the "*Raw Data and Fitted Curve*" procedure of LabVIEW, the theoretical polynomial functions $I^t_{G1}(V)$, $P^t_{G1}(V)$, $V^t_{G1}(I)$, $P^t_{G1}(I)$ and $I^t_{G2}(V)$, $P^t_{G2}(V)$, $V^t_{G2}(I)$, $P^t_{G2}(I)$ for two levels of irradiance are obtained.

C1.3. Determination of the theoretical polynomial functions for PV panel composed of parallel and series connections of PV cells under two levels of irradiance. Consider that the PV panel is composed of n_p PV cells under two irradiance levels in *parallel connection*, then their theoretical polynomial functions $I_e^t(V), P_e^t(V)$ result as

$$I_e^t(V)=\sum_{k=1}^{n_p}[I_{G_1}^t(V)+I_{G_2}^t(V)]_k \; ; P_e^t(V)=V\sum_{k=1}^{n_p}[I_{G_1}^t(V)+I_{G_2}^t(V)]_k \quad (15)$$

where $V_e = V_1 = ... = V_{n_p} = V$. For *series connection* of n_s PV cells under two irradiance levels, the theoretical polynomial functions $V_e^t(I), P_e^t(I)$ of the equivalent PV panel result as

$$V_e^t(I)=\sum_{k=1}^{n_s}[V_{G_1}^t(I)+V_{G_2}^t(I)]_k \; ; P_e^t(I)=I\sum_{k=1}^{n_s}[V_{G_1}^t(I)+V_{G_2}^t(I)]_k \quad (16)$$

where $I_e = I_1 = ... = I_{n_p} = I$.

C1.4. Determination of the theoretical model parameters $(V^t_{oc,e}, I^t_{sc,e}, V^t_{MPP,e}, I^t_{MPP,e}, P^t_{MPP,e})$ for PV panel and calculation of relative errors are performed from the theoretical polynomial functions obtained in module C1.3 and based on the properties defined in section 2, A. Consider, for example, the parallel connection of PV cells, from relations (15) result: for V = 0, results $I^t_{e,sc}$.; the real root of equation $I^t_e(V) = 0$ means $V^t_{e,oc}$; for dP^t_e / $dV = 0$, result $P^t_{e,MPP}$, $V^t_{e,MPP}$, and then $I^t_e(V^t_{e,MPP})= I^t_{e,MPP}$.

For each model parameters determinates above, is defined and calculated the relative error related to the measured value. For example, the relative error of the open-circuit voltage is calculated as

$$\varepsilon_{V_{oc},e} = \frac{\left|V_{oc}^m - V_{oc,e}^t\right|}{V_{oc}^m}\cdot 100(\%) \quad (17)$$

B. Analytical method. The analytical method is used in order to obtain the PV panel characteristics by add point by point the measured characteristics of PV cells under different irradiance levels when the interconnection conditions are imposed. Based on LabVIEW application, for one PV cell results four vectors $\mathbf{I}_{G_1}^m = [I_{1,G_1}^m,..., I_{n,G_1}^m]$, $\mathbf{V}_{G_1}^m = [V_{1,G_1}^m,...,V_{n,G_1}^m]$, $\mathbf{P}_{G_1}^m(\mathbf{V}_{G_1}^m) = [P_{1,V_{G_1}}^m,...,P_{n,V_{G_1}}^m]$, $\mathbf{P}_{G_1}^m(\mathbf{I}_{G_1}^m) = [P_{1,I_{G_1}}^m,...,P_{n,I_{G_1}}^m]$, which contain n measured data for the same level of irradiance G_1. Analogue, if consider the same PV cell under other level of irradiance G_2 the vectors contain p measured data $\mathbf{I}_{G_2}^m = [I_{1,G_2}^m,...,I_{p,G_2}^m]$, $\mathbf{V}_{G_2}^m = [V_{1,G_2}^m,...,V_{p,G_2}^m]$, $\mathbf{P}_{G_2}^m(\mathbf{V}_{G_2}^m) = [P_{1,V_{G_2}}^m,...,P_{n,V_{G_2}}^m]$, $\mathbf{P}_{G_2}^m(\mathbf{I}_{G_2}^m) = [P_{1,V_{G_2}}^m,...,P_{n,V_{G_2}}^m]$. A *computational procedure (C2)* organized into a number of interlinked computational modules is developed in order to obtain the theoretical characteristics of PV panel, by analytical adding, point by point, of the PV cells' measured data. A *description of C2* implemented for two levels of irradiance and for order of the elements of measured vectors given by LabVIEW application, is provided below:

C2.1. Data input constructs the vectors $\mathbf{V}_{G_1}^m$, $\mathbf{V}_{G_2}^m$, $\mathbf{I}_{G_1}^m$, $\mathbf{I}_{G_2}^m$, $\mathbf{P}_{G_1}^m(\mathbf{V}_{G_1}^m)$, $\mathbf{P}_{G_2}^m(\mathbf{V}_{G_2}^m)$, using the measured data from LabVIEW application.

C2.2. Determination of the reduced vectors $\mathbf{V}_{G_1}^{m,r}$, $\mathbf{V}_{G_2}^{m,r}$, $\mathbf{I}_{G_1}^{m,r}$, $\mathbf{I}_{G_2}^{m,r}$ for one PV cell under two irradiance levels. A synthetic description of the modules of this procedure, shown in Fig. 4, b, is presented here:

- *Error Calculation (EC)*: the *i*th element $V_{i,G_1}^m \notin (V_{oc,G_1}^m, V_{oc,G_2}^m)$ of vector $\mathbf{V}_{G_1}^m$, is compared to all the elements

$V^m_{j,G_2} \notin (V^m_{oc,G_1}, V^m_{oc,G_2})$ of vector $\mathbf{V}^m_{G_2}$ and is calculated the absolute error

$$\varepsilon_{Vi,j} = \left| V^m_{i,G_1} - V^m_{j,G_2} \right| \leq 10^{-3}, j = 1,.., p \quad (18)$$

Analogue, the *i*th element $I^m_{i,G_1} \notin (I^m_{sc,G_1}, I^m_{sc,G_2})$ of vector $\mathbf{I}^m_{G_1}$, is compared to all the elements $I^m_{j,G_2} \notin (I^m_{sc,G_1}, I^m_{sc,G_2})$ of vector $\mathbf{I}^m_{G_2}$ and is calculated the absolute error

$$\varepsilon_{Ii,j} = \left| I^m_{i,G_1} - I^m_{j,G_2} \right| \leq 10^{-3}, j = 1,,, p \quad (19)$$

- *Determination of Minimum Error (DME)*: for all the error values that verify the relation (17) respectively (18), the lowest is choose, i.e.

$$\varepsilon_{\min,V_{i,j}} = \min_j \left| V^m_{i,G_1} - V^m_{j,G_2} \right|, j = 1,.., p \quad (20)$$

respectively

$$\varepsilon_{\min,I_{i,j}} = \min_j \left| I^m_{i,G_1} - I^m_{j,G_2} \right|, j = 1,.., p \quad (21)$$

- *Vectors Reduction (VR)*: all the values V^m_{i,G_1}, V^m_{j,G_2} respectively I^m_{i,G_1}, I^m_{j,G_2} that not verify the conditions (17) or (19), respectively (18) or (20) are removed from the initial vectors $\mathbf{V}^m_{G_1}$, $\mathbf{V}^m_{G_2}$ respectively $\mathbf{I}^m_{G_1}, \mathbf{I}^m_{G_2}$. Thus are obtained the reduced vectors $\mathbf{V}^{m,r}_{G_1}, \mathbf{V}^{m,r}_{G_2}$ respectively $\mathbf{I}^{m,r}_{G_1}, \mathbf{I}^{m,r}_{G_2}$ which contain $e \leq \min(n, p)$ elements of the closest measured values of voltage and current. On the other hand, the initial measured values of voltage located in the set $(V^m_{oc,G_1}, V^m_{oc,G_2})$ and the initial measured values of the current located in the set $(I^m_{sc,G_1}, I^m_{sc,G_2})$ are kept in the reduced vectors $\mathbf{V}^{m,r}_{G_1}, \mathbf{V}^{m,r}_{G_2}$ respectively $\mathbf{I}^{m,r}_{G_1}, \mathbf{I}^{m,r}_{G_2}$.

The procedures *EC*, *DME* and *VR* are performed for all the elements, *i*=1,..,*n* of vectors $\mathbf{V}^m_{G_1}$ and $\mathbf{I}^m_{G_1}$ located outside the set $(V^m_{oc,G_1}, V^m_{oc,G_2})$, and outside the set $(I^m_{sc,G_1}, I^m_{sc,G_2})$.

C2.3. Determination of the reduced vectors $\mathbf{P}^{m,r}_{G_1}(V^{m,r}_{G_1})$, $\mathbf{P}^{m,r}_{G_2}(V^{m,r}_{G_2})$, $\mathbf{P}^{m,r}_{G_1}(I^{m,r}_{G_1})$, $\mathbf{P}^{m,r}_{G_2}(I^{m,r}_{G_2})$ of one PV cell under two irradiance levels. This procedures constructs the reduces vectors of the values of power by using the corresponding values of voltage and current from reduces vectors obtained in module *C2.2*. In other words, are removed all the elements of vectors $\mathbf{P}^m_{G_1}(V^m_{G_1}) = [P^m_{1,V_{G_1}},..,P^m_{n,V_{G_1}}]$, $\mathbf{P}^m_{G_1}(I^m_{G_1}) = [P^m_{1,I_{G_1}},..,P^m_{n,I_{G_1}}]$, $\mathbf{P}^m_{G_2}(V^m_{G_2}) = [P^m_{1,V_{G_2}},..,P^m_{n,V_{G_2}}]$, $\mathbf{P}^m_{G_2}(I^m_{G_2}) = [P^m_{1,V_{G_2}},..,P^m_{n,V_{G_2}}]$ which correspond to the values V^m_{i,G_1}, I^m_{i,G_1}, V^m_{j,G_2}, I^m_{j,G_2} eliminated by the procedure *VR*.

C2.4. Determination of the theoretical characteristics for PV panel composed of parallel and series connections of PV cells under two levels of irradiance. For parallel connection of n_p PV cells the theoretical characteristic $I^t_a(V)$ is obtained by add the elements of reduced vectors $\mathbf{I}^{m,r}_{G_1}$ with the elements of reduced vector $\mathbf{I}^{m,r}_{G_2}$ which correspond to the elements of reduced vectors $\mathbf{V}^{m,r}_{G_1}$ and $\mathbf{V}^{m,r}_{G_2}$. Similarly, the characteristic $P^t_a(V)$ is obtained by add the corresponding elements of reduced vector $\mathbf{P}^{m,r}_{G_1}(V^{m,r}_{G_1})$ with $\mathbf{P}^{m,r}_{G_2}(V^{m,r}_{G_2})$. For example, consider that n_{G1} cells are under the irradiance G_1, then n_p - n_{G1} cells are under irradiance G_2. In this case, is added each element of vector $\mathbf{I}^{m,r}_{G_1}$ multipled by n_{G1} with each corresponding element of vector $\mathbf{I}^{m,r}_{G_2}$ multiplied by (n_p-n_{G1}). Analogue for the elements of vectors $\mathbf{P}^{m,r}_{G_1}(V^{m,r}_{G_1})$ and $\mathbf{P}^{m,r}_{G_2}(V^{m,r}_{G_2})$.

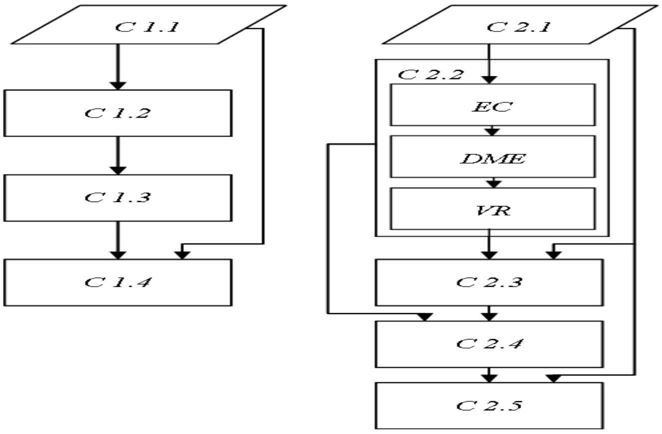
For series connection of PV cells the theoretical characteristics $V^t_a(I)$ and $P^t_a(I)$ are obtained by add the elements of reduced vectors $\mathbf{V}^{m,r}_{G_1}$ with $\mathbf{V}^{m,r}_{G_2}$, respectively $\mathbf{P}^{m,r}_{G_1}(I^{m,r}_{G_1})$ with $\mathbf{P}^{m,r}_{G_2}(I^{m,r}_{G_2})$ to fit the elements of reduced vectors $\mathbf{I}^{m,r}_{G_1}$ and $\mathbf{I}^{m,r}_{G_2}$.

C2.5. Determination of the theoretical model parameters ($V^t_{oc,a}, I^t_{sc,a}, V^t_{MPP,a}, I^t_{MPP,a}, P^t_{MPP,a}$) *of PV panel and relative error calculation* is performed from the theoretical characteristics obtained in module *C2.4* and based on the properties defined in section 2, A. For each model parameters is defined and calculated the relative error related to the measured value. For example, the relative error of the open-circuit voltage is calculated as

$$\varepsilon_{V_{oc},a} = \frac{\left| V^m_{oc} - V^t_{oc,a} \right|}{V^m_{oc}} \cdot 100(\%) \quad (22)$$

4. EXPERIMENTAL AND THEORETICAL RESULTS

A. Experimental results. All the measurements are carried out by LabVIEW software application. Specific numerical example considers a PV panel with silicon monocrystalline cell Saturn [17]-[18]. The measured characteristics for one PV cell under two levels of irradiance are shown in Fig. 5 and Fig. 6. The



a) b)

Fig. 4. Structure of the computational procedure C1,a), and C2,b).

measured characteristics for PV panel composed of parallel (2 cells at 1000 W/m^2 and 1 cell at 600 W/m^2) and series (4 cells at 1000 W/m^2 and 1 cell at 600 W/m^2) connections of PV cells under two irradiance levels are shown in Fig. 7 respectively in Fig. 8.

B. Theoretical results processed by curve fitting method. Based on the measured data and the "*Raw Data and Fitted Curve*" procedure of LabVIEW, the theoretical polynomial functions for one PV cell under two levels of irradiance are obtained. The $I^t_{G1}(V)$ and

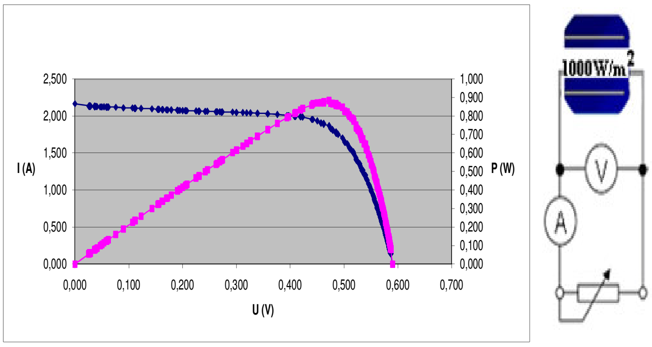


Fig. 5. *I-V* and *P-V* measured characteristics of one PV cell under G_1=G_0=1000 W/m^2.

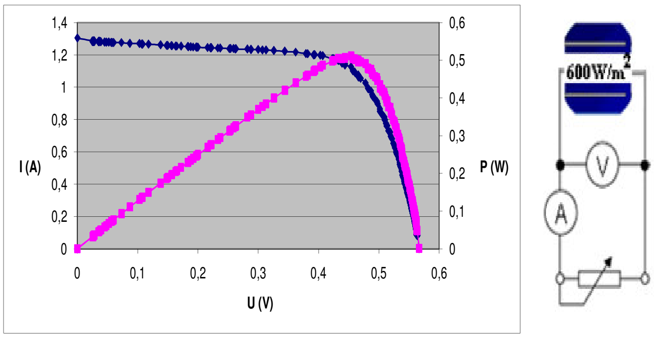


Fig. 6. *I-V* and *P-V* measured characteristics of one PV cell under G_2=600 W/m^2.

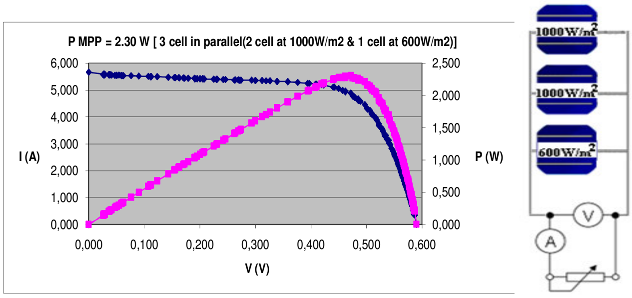


Fig. 7. *I-V* and *P-V* measured characteristics of PV panel: 2 cells under G_1=1000 W/m^2 in parallel with 1 cell under G_2=600 W/m^2.

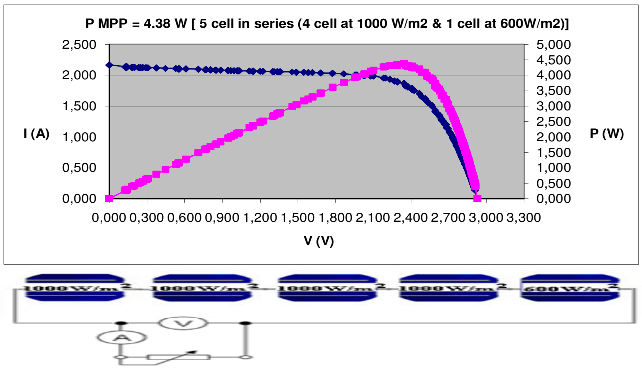


Fig. 8. *I-V* and *P-V* measured characteristics of PV panel: 4 cells under G_1=1000 W/m^2 in series with 1 cell under G_2=600 W/m^2.

$P^t_{G1}(V)$ characteristics of one PV cell with irradiance level G$_1$ = 1000 W/m^2 are approximated by a five order theoretical polynomial function, shown in Fig. 9, a

$$I^t_{G_1}(V) = 2{,}172 - 1{,}909\,V + 24{,}245\,V^2 - 149{,}902\,V^3 + 397{,}256\,V^4 - 374{,}891\,V^5 \quad (23)$$

and by an eight order theoretical polynomial function shown in Fig. 9, b

$$P^t_{G_1}(V) = 357{,}9\cdot10^{-3} + 1{,}721V + 14{,}763V^2 - 224{,}583V^3 + 1668V^4 - 6{,}721\cdot10^3V^5 + 14{,}853\cdot10^3V^6 - 16{,}764\cdot10^3V^7 + 7{,}426\cdot10^3V^8 \quad (24)$$

The $V^t_{G1}(I)$ and $P^t_{G1}(I)$ characteristics of one PV cell at G$_1$ = 1000 W/m^2 are approximated as

$$V^t_{G_1}(I) = 593{,}796\cdot10^{-3} - 141{,}037\cdot10^{-3}I + 0{,}987I^2 - 3{,}575I^3 + 6{,}099I^4 - 5{,}271I^5 + 2{,}224I^6 - 0{,}363I^7 \quad (25)$$

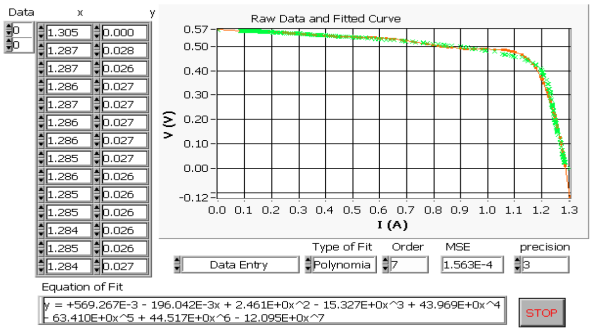
$$P^t_{G_1}(I) = 13{,}2\cdot10^{-3} + 217{,}11\cdot10^{-3}I + 2{,}972I^2 - 10{,}044I^3 + 16{,}33I^4 - 13{,}635I^5 + 5{,}603I^6 - 896{,}842\cdot I^7 \quad (26)$$

Analogue, the theoretical polynomial functions $I^t_{G2}(V)$, $P^t_{G2}(V)$, $V^t_{G2}(I)$, and $P^t_{G2}(I)$ which approximate the characteristics of one PV cell with irradiance level G_2 = 600 W/m^2 result as

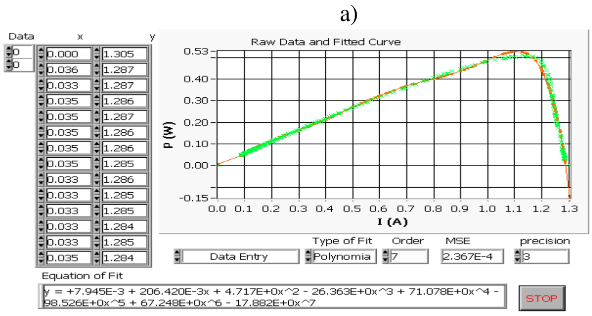
$$I^t_{G_2}(V) = 1,309 - 1,204V + 15,87V^2 - 101,803V^3 + \\ + 280,154V^4 - 274,715V^5 \quad (27)$$

$$P^t_{G_2}(V) = 1,291 \cdot 10^{-3} + 1,096V + 7,582V^2 - 124,124V^3 + \\ + 976,349V^4 - 4,184 \cdot 10^3 V^5 + 9,639 \cdot 10^3 V^6 - 11,406 \cdot 10^3 V^7 + 5,276 \cdot 10^3 V^8 \quad (28)$$

$$V^t_{G_2}(I) = 569,267 \cdot 10^{-3} - 196,042 \cdot 10^{-3} I + 2,461 I^2 - 15,327 I^3 + \\ + 43,969 I^4 - 63,41 I^5 + 44,517 I^6 - 12,095 I^7 \quad (29)$$



a)



b)

Fig. 9. Theoretical polynomial functions $I^t_{G1}(V)$ (a), and $P^t_{G1}(V)$ (b), of one PV cell under G_1=1000 W/m^2.

$$P^t_{G_2}(I) = 7,945 \cdot 10^{-3} + 206,42 \cdot 10^{-3} I + 4,717 I^2 - 26,363 I^3 + \\ + 71,078 I^4 - 98,526 I^5 + 67,248 I^6 - 17,882 \cdot V^7 \quad (30)$$

Using relations (15) for a PV panel consists of a parallel connection of two PV cells exposed to irradiance G_1=1000 W/m^2 and one PV cell exposed to G_2=600 W/m^2, the theoretical polynomial functions $I^t_e(V), P^t_e(V)$ result as

$$I^t_e(V) = 2I^t_{G_1}(V) + I^t_{G_2}(V) = 5,653 - 5,022V + 64,36V^2 - \\ - 401,607V^3 + 1074,666V^4 - 1024,497V^5 \quad (31)$$

$$P^t_e(V) = 2P^t_{G_1}(V) + P^t_{G_2}(V) = 8,448 \cdot 10^{-3} + 4,538V + 37,108V^2 - \\ - 573,290V^3 + 4312,349V^4 - 17,59 \cdot 10^3 V^5 + 39,345 \cdot 10^3 V^6 - \\ - 44,934 \cdot 10^3 V^7 + 20,128 \cdot 10^3 V^8 \quad (32)$$

Similarly, using relations (16) for a PV panel consists of a series connection composed of four PV cells exposed to irradiance G_1=1000 W/m^2, and one PV cell exposed to G_2=600 W/m^2, the theoretical polynomial functions $V^t_e(I), P^t_e(I)$ result as

$$V^t_e(I) = 4V^t_{G_1}(I) + V^t_{G_2}(I) = 2944,451 \cdot 10^{-3} - 760,191 \cdot 10^{-3} I + \\ + 6,409 I^2 - 29,627 I^3 + 68,365 I^4 - 84,494 I^5 + 53,413 I^6 - 13,547 I^7 \quad (33)$$

$$P^t_e(I) = 4P^t_{G_1}(I) + P^t_{G_2}(I) = 60,745 \cdot 10^{-3} + 107,86 \cdot 10^{-3} I + \\ + 16,605 I^2 - 66,359 I^3 + 136,398 I^4 - 153,066 I^5 + 89,66 I^6 - 21,466 I^7 \quad (34)$$

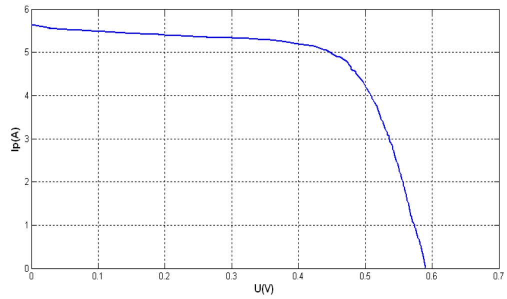
The polynomial functions (31)-(34) will be used in order to obtain the theoretical values of *model parameters* $V^t_{oc,e}, I^t_{sc,e}, V^t_{MPP,e}, I^t_{MPP,e}, P^t_{MPP,e}$) and for calculate of the relative errors. These values are presented in Table 1.

C. Theoretical results processed by analytical method. The initial vectors $\mathbf{V}^m_{G1}$, $\mathbf{I}^m_{G1}$,and $\mathbf{P}^m_{G1}(V^m_{G1})$ contain 310 elements, while the initial vectors $\mathbf{V}^m_{G2}$, $\mathbf{I}^m_{G2}$,and $\mathbf{P}^m_{G2}(V^m_{G2})$ contain 309 elements. At the end of the modules *C2.2* and *C2.3* the reduced vectors will have a total of 109 elements for parallel respectively 230 elements for series connection. The theoretical characteristics $I^t_a(V)$ and $P^t_a(V)$ - shown in Fig. 10, a and b - for PV panel consists of parallel connections of two PV cells exposed to G_1=1000 W/m^2 and one PV cell to G_2 = 600 W/m^2 are obtained using module *C2.4*. Similarly are obtained the theoretical characteristics $V^t_a(I)$ and $P^t_a(I)$ for series connection of four PV cells at G_1=1000 W/m^2, and one PV cell at G_2=600 W/m^2. From *C2.5* result the PV panel *model parameters* ($V^t_{oc,a}, I^t_{sc,a}, V^t_{MPP,a}, I^t_{MPP,a}, P^t_{MPP,a}$), and the relative errors, whose values are presented in Table 1.

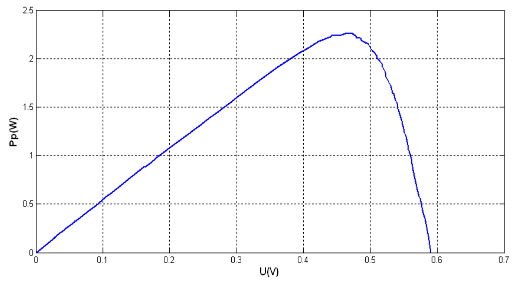
The numerical results of the theoretical and experimental values and of the relative errors for the *model parameters* of basic connections of PV cells are summarized in Table 1, and prove that the analytical method is more accurate than the curve fitting method.

Table 1. **Comparative study between theoretical and measured values of the model parameters of PV panel**

Model parameters	V_{oc} (V)		I_{sc} (A)		V_{MPP} (V)		I_{MPP} (A)		P_{MPP} (W)	
Connection	Parallel	Series	Parallel	Series	Parallel	Series	Parallel	Series	Parallel	Series
Measured values	0,59	2,927	5,644	2,166	0,472	2,344	4,891	1,87	2,863	4,383
Curve fitting method	0,585	2,944	5,653	2,314	0,467	2,53	4,623	1,99	2,309	3,35
Analytical method	0,592	2,932	5,639	2,279	0,477	2,418	4,95	1,952	2,987	4,951
Relative error of curve fitting method (ε_e %)	0,84	0,58	0,15	6,8	1,05	7,9	5,4	6,4	19,35	23,56
Relative error of analytical method (ε_a%)	0,33	0,17	0,08	5,2	1,05	3,15	1,2	4,38	4,33	12,95



a)



b)

Fig. 10. Theoretical characteristics $I^t_a(V)$ (a), and $P^t_a(V)$ (b), of parallel connections PV cells (two cells at G_1=1000 W/m^2 and one cell at G_2=600 W/m^2).

5. CONCLUSIONS

In this paper, the characteristics and parameters of PV cells/panels have been evaluated by using the curve fitting and analytical methods. Accordingly to Brune's conditions the approximation polynomial functions and theoretical characteristics are used to calculate the theoretical set of values for *model parameters* of series and parallel connections of PV cells under different levels of irradiance. The proposed numerical methods and computational procedure can be applied to any real case of PV cell and variation of irradiance level.

The developed models can facilitate the PV system study, design and analysis for different connections and different solar irradiation levels.

Acknowledgment. This research was financially partially supported by the project PREDEX-POSDRU/CPP107/DMI1.5/S/77497.

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