

# THE CONCEPT AND THEORETICAL STUDY OF MICRO HYDROPOWER PLANT WITH GRAVITATIONAL VORTEX AND TURBINE WITH RAPIDITY STEPS

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**REZUMAT.** Pentru un nou concept de microhidrocentrală cu vârtej gravitațional și turbină în trepte de rapiditate s-a modelat curgerea apei prin canalul conic în lipsa și în prezența treptelor. În primul caz, liniile de curent au forma unor elice conice. Viteza axială și radială ale apei nu sunt nule doar în stratul limită. În cazul al doilea structura curgerii se modifică cu formarea de turbioane în fiecare spațiu dintre palete. Viteza apei crește rapid pe măsura apropierii de orificiul de scurgere și depinde puternic de debitul de apă.

**Cuvinte cheie:** microhidroenergetică, vârtej gravitațional, turbină hidraulică, rapiditate

**ABSTRACT.** For a new concept of micro hydropower plant with gravitational vortex and turbine with rapidity steps has been modeled the water flow through the conical channel in the absence and the presence of steps. In the first case, the power lines appear as conical propellers. Axial and radial velocity of the water are not zero only in the boundary layer. In the second case the structure of the water flow is changes with the formation of swirls every area of the blades. The water velocity increases rapidly as proximity to the drain hole and strongly depends on water flow.

**Keywords:** micro hydropower, gravitational vortex, hydraulic turbine, rapidity

## 1. INTRODUCTION

In micro or nano hydro power plants (micro HPP or nano HPP) combines on the one hand, the advantages of high power HPP with the possibility of a decentralized energy supply [1],[2]. This type of power plants has not many disadvantages of high power plants such as expensive transmission and environmental problems [3],[4]. In addition, the implementation of small hydro power lead to use of decentralized electricity and creates favorable conditions for the development of economic zones, based mainly on providing its own resources.

Worldwide has been installed thousands of micro HPP. The modern hydropower technologies are very advanced [5]. In the past 40 years have been essential modernized of hydraulic turbines constructions, primarily in order to increase efficiency. The essential reduction of the small hydro turbine size leads to considerable increase in capital expenditures per unit of installed power. On the other hand the micro hydropower environmental impact is so small that often speaks about the small hydro power as about some nonexistent [6],[7],[8].

In most cases micro HPP are connected to mains power (energy network). These have no dams and works located in natural stream water until the river will dry up. In this case we use hydro power plants with battery power.

Safety equipment requirements of micro HPP operation are not as strict as in the case of large power plants. Capital expenditures for the improvement of this micro HPP in the absence of these dams and use of local building materials are reduced. Initial investments in micro HPP construction is between 1800 and \$ 8800 U.S. per 1 kW of installed capacity (for water falls between 2.3 and 13.5 m) and 1000 to \$ 3000 U.S. to 1 kW. At the same time, operating costs are low. Between 40 and 50% of total investment returns to the equipments.

## 2. THE MICRO HYDROPOWER PLANT CONCEPT

The micro hydropower plant proposed [10] developed a new technology for converting hydraulic energy into mechanical work of small rivers due to

local restructuring of water flow in spiral flow through a conical channel as a gravitational vortex [11]. In the vortex thus formed, at the different heights of the conical channel are positioned steps of rotor blades with different diameters. The step rapidity increases with the height of the conical channel in the axial flow direction.

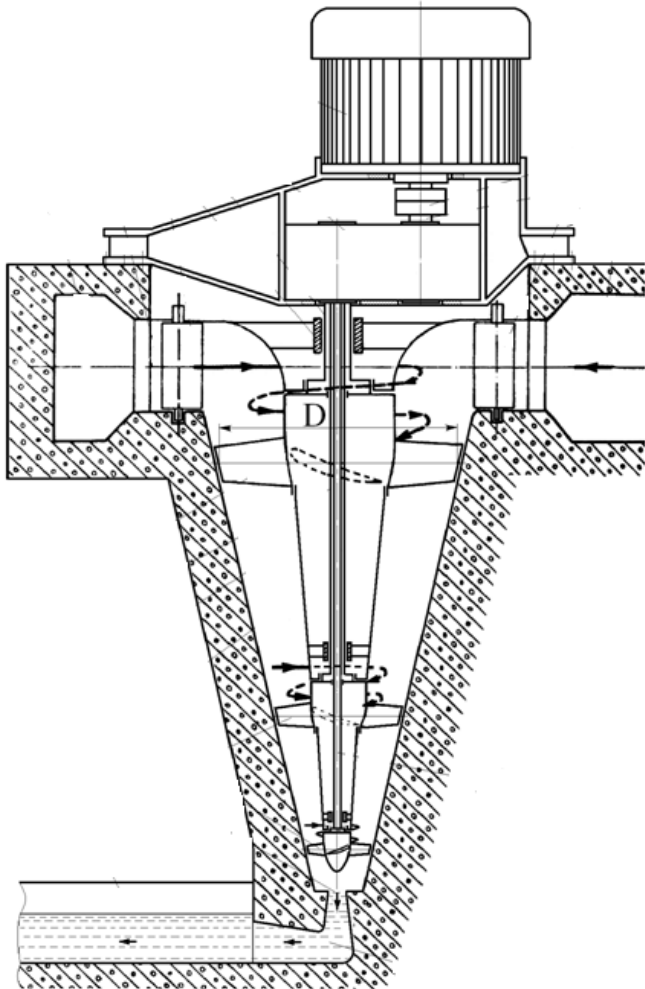


Fig. 1. Section through the small hydropower plant using gravitational vortex

It is known that in the case of a high power HPP, increasing output by 1% hydro turbine increase the power of several MW. Under the new concept solves the problem of increasing the efficiency of hydraulic flow energy conversion into electricity by using a hydraulic turbine with rapidity steps. Thus, the small hydropower plant from Figure 1, consists of a foundation ashore in the form of a cylindrical concrete spiral cases with tangential inlet connection, which is deflected a part of the river. The spiral chamber is equipped with a fixed Fink blade unit director with adjustable tilt. Under the spiral chamber is placed a conical channel for the formation of the water flow as a gravitational vortex. In the bottom of the conical

channel is made a central drainage hole, positioned over a channel of escape. Between the central drainage hole of the chamber and the running channel is connected to a draft tube in the elbow.

In the center of the spiral chamber and the conical channel is positioned the hydraulic turbine with vertical shaft, kinematic connected by a mechanism of angular velocity reduction and multiplication gear with electric generator. The hydraulic rotor is performed in stages with characteristic diameters of mobile blades which decreases along the entrance to the chamber drain hole directly proportional to decreasing cross-sectional diameter of the chamber in which are located. Each step of the rotor is mounted on its axis of rotation. The axis are assembled telescopic and are coupled to respective driving gears of the mechanism of reduction and multiplication. The types of blades used in each step are chosen based on its rapidity. For water flow in the gravitational vortex the most indicated are the helical turbine stages with the hydrofoil profile.

Can be used Francis turbine steps with the form according to rapidity. Due to large variation of the peripheral velocity  $u = \omega \cdot r$  along the blade, is necessary to twist the blade.

The mechanism of rpm reduction and multiplication gear consist of a number of pairs of gears engaged with each other, equal to the number of steps of the turbine, which, depending on turbine gear rpm, some pairs are used to reduce angular velocity and others to multiply up to the nominal velocity of the electric generator. So that if one angular velocity of step turbine exceeds the rated velocity of the electric generator, its axis of rotation of the driving wheel is coupled to the pair that reduce this velocity to nominal angular velocity value of the electric generator. If one angular velocity of step turbine is below the rated velocity of the electric generator, its axis of rotation of the driving wheel is coupled to the pair that multiply the velocity to nominal value of electric generator angular velocity.

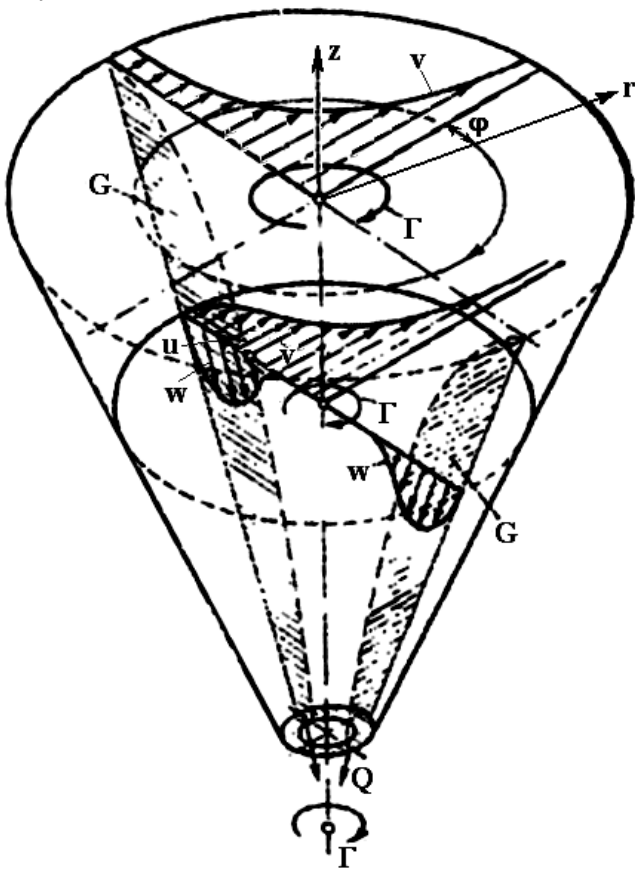
### 3. THE WATER FLOW IN THE CONICAL CHANNEL WITHOUT TURBINE ROTOR

We consider the flow of incompressible fluid with the density  $\rho$  through a conical channel (Figure 2) with the swirl intensity  $\Gamma = 2\pi r v$  and the water flow  $Q$ . Geometrical dimensions of the conical channel are given.

We have to determine the distribution of the radial component  $u$ , the circular (tangential)  $v$  and axial  $w$  and the fluid pressure distribution axes.

# THE CONCEPT AND THEORETICAL STUDY OF MICRO HYDROPOWER PLANT WITH GRAVITATIONAL VORTEX AND TURBINE WITH RAPIDITY STEPS

For simulation of this flow we will rely on models of Binnie, Haris [12] and Taylor [13], using the cylindrical coordinate system  $r, \varphi, z$ . The plane  $z=0$  will pass through the fluid outlet section at the top of conical channel. We consider that remote from the channel wall (outside the hydrodynamic boundary layer  $G$ ) the fluid rotates as a solid body with angular velocity  $\omega$ . Due to axial symmetry, all derivatives on  $\varphi$  in the Navier-Stokes equation is canceled. To solution obtained in this case will be the exact solution of the Navier-Stokes equations, given that the terms on these equations, which would be lost when crossing the boundary layer equation following simplifying assumptions it would go away.



**Fig. 2.** Representation of vortex flow of incompressible fluid through a conical chamber:

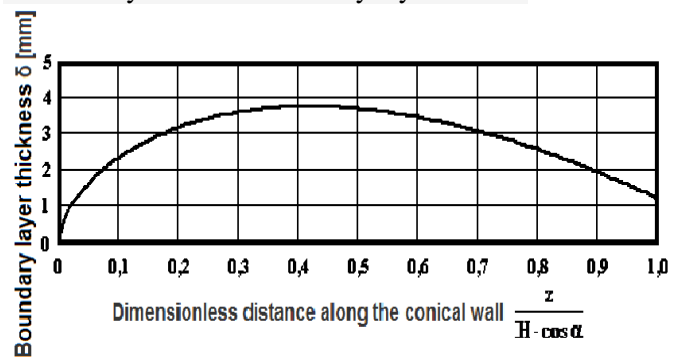
$G$ -hydrodynamic boundary layer flow conical chamber wall secondary oriented to the top of the cone;  $\Gamma$ -swirl intensity;  $Q$ -water flow through the top of the cone;  $z, r, \varphi$ -cylindrical coordinate system;  $u, v, w$ -components of radial, circular and axial velocity

At large distance from the conical channel wall the flow occurs without friction. For this reason for the radial pressure gradient we can write:

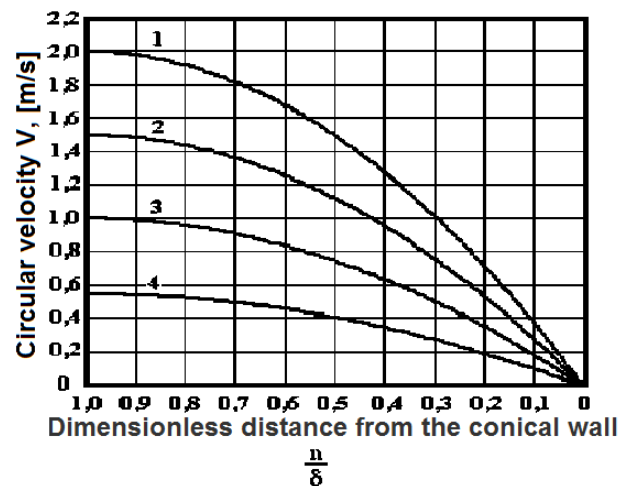
$$\frac{1}{\rho} \cdot \frac{\partial p}{\partial r} = r\omega^2 \quad (1)$$

Solving the problem in the boundary layer approximation [14], we admit that the pressure gradient in accordance with the equation (1) exists in the liquid layer near the wall, where it shows the phenomenon of friction.

Following low simulation software Mathcad 7 was found that water flow space is separated into two areas. The first area includes within hydrodynamic boundary layer adhering to the conical wall. The second area is the fluid flow outside the boundary layer. The zones are bounded by the outer boundary layer  $\delta$  limit.



**Fig. 3.** Hydrodynamic boundary layer profile to conical wall



**Fig. 4.** Circular velocity distribution of the hydrodynamic boundary layer depending of the dimensionless distance from the conical wall:  $z/(H \cos \alpha)$ : 1 – 0; 2 – 0,2; 3 – 0,5; 4 – 1

Figure 3 presents the dependence of boundary layer thickness  $\delta$  of the dimensionless distance  $z/(H \cos \alpha)$  along the conical wall at the top of the conical chamber. It is noted that the boundary layer thickness is zero edge entry section, reaches a maximum along the conical wall and the edge is non-zero output, which ensures a water evacuation from the conical chamber with the flow  $Q$ .

Inside the boundary layer the circular velocity of the liquid decreases from the border of the boundary layer until zero value on the wall (Figure 4).

The radial velocity,  $u$ , is zero on the conical wall and the outer border of the boundary layer  $\delta$ , reaching a maximum inside the boundary layer (Figure 5).

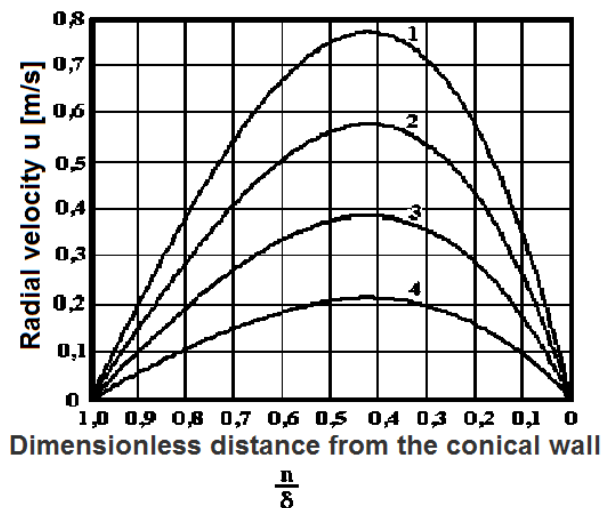


Fig. 5. Radial velocity distribution in the hydrodynamic boundary layer depending on the dimensionless distance from the conical wall  $z/(H \cos \alpha)$ : 1 - 0; 2 - 0,2; 3 - 0,5; 4 - 1.

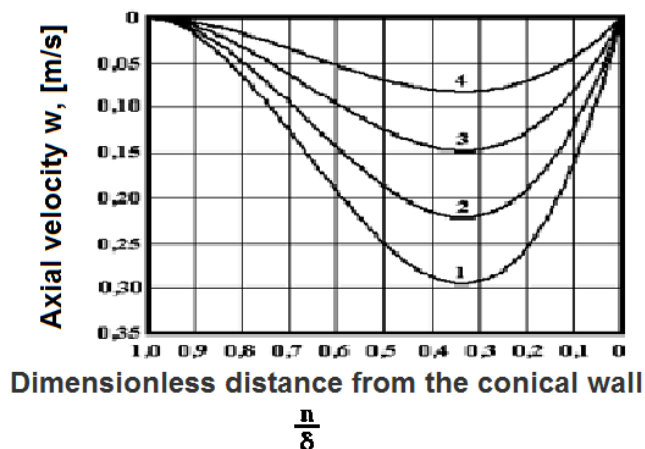


Fig. 6. Axial velocity distribution in the hydrodynamic boundary layer depending on the dimensionless distance from the conical wall  $z/(H \cos \alpha)$ : 1 - 0; 2 - 0,2; 3 - 0,5; 4 - 1.

The axial velocity,  $w$ , along the conical wall is also zero on the wall and the outer border boundary layer, reaching a maximum in the interior (Figure 6).

The profiles of the circular velocity,  $v$ , and the axial velocity,  $w$ , in the boundary layer are Pohlhausen type [6] that has been experimentally validated. Thus, the circular velocity,  $v$ , corresponds to the gravitational vortex strength  $\Gamma$ , radial velocity,  $u$ , bring fluid in the boundary layer and the axial velocity,  $w$ , ensures the

motion along the conical wall and the water evacuation with the flow  $Q$ .

Outside the boundary layer the velocities  $u$  and  $w$  are zero. The distribution of circular velocity  $v$  is directly proportional to circulation  $\Gamma$ , but inversely proportional to the radius. (Figure 7).

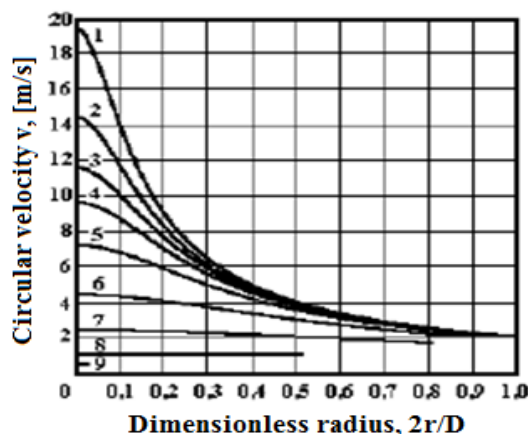


Fig. 7. Circular velocity distribution in the hydrodynamic boundary layer depending on the dimensionless radius:  $z/(H \cos \alpha)$ : 1 - 0; 2 - 0,01; 3 - 0,02; 4 - 0,03; 5 - 0,05; 6 - 0,1; 7 - 0,2; 8 - 0,5; 9 - 1.

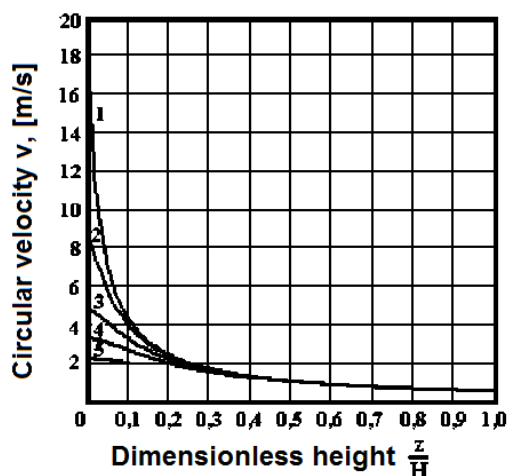


Fig. 8. Circular velocity distribution in the hydrodynamic boundary layer depending on the dimensionless height  $z/(H \cos \alpha)$ : 1 - 0; 2 - 0,1; 3 - 0,5; 4 - 0,8; 5 - 1.

Circular velocity decreases with distance from the top of the conical chamber (Figure 8). Thus, outside the boundary layer the flow is potential. The fluid behaves like a solid body in rotation.

Cavern shape formed by the free surface of water in the form of vortex rotational motion with angular velocity  $\omega$  gravity in a conical channel, converging in the force of gravity  $g$ , the height  $H$ , radius  $R$  and the angle of the upper section taper  $2\alpha$  (Figure 9) is given by:

$$z = \frac{\omega^2 r^2}{2g} \quad (2)$$

which is the equation of a parabolic with the origin of coordinates in the lower part of the free surface. Equation (2) is obtained by solving the Euler equation of rotational motion of incompressible fluid with free surface boundary condition  $p = \text{const}$ . Clearly, with increasing angular velocity vortex gravity cave (cavern) space increases.

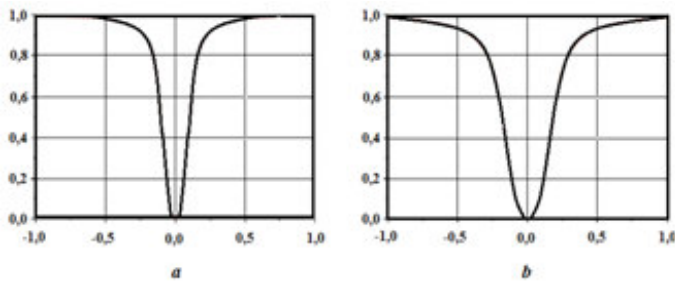


Fig.9. Cave (cavern) formed by the free surface of water in motion in the gravitational vortex:

a -  $\omega = 40 \text{ rad/s}$ ;  $H = 150 \text{ mm}$ ;  $R = 60 \text{ mm}$ ;  $2\alpha = 38^\circ$ ;  
 b -  $\omega = 117 \text{ rad/s}$ ;  $H = 150 \text{ mm}$ ;  $R = 60 \text{ mm}$ ;  $2\alpha = 38^\circ$

To determine the temporal and spatial distribution of circular velocity  $v(r, t)$  and angular velocity  $\omega(r, t)$  of water in the gravitational vortex was solved differential equation describing no stationary diffusion of the vortex:

$$\frac{\partial \omega}{\partial t} = \nu \cdot \left[ \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \omega}{\partial r} \right] \quad v(r, t) = \omega(r, t) \cdot r \quad (3)$$

with initial condition  $t = 0: v(r, 0) = 0$  and asymptotic condition  $t = \infty, r = R: v(r, \infty) = \Gamma / (2\pi R)$ .

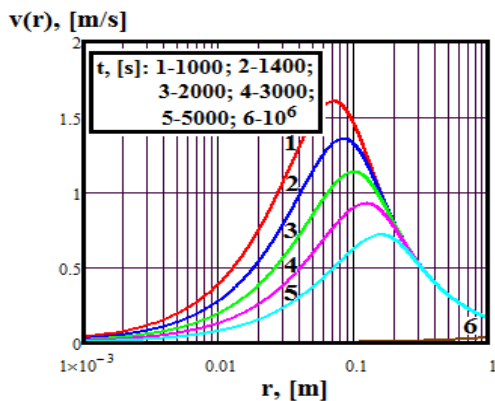


Fig.10. Temporal and spatial distribution of the circular velocity of gravitational swirl.

The solution of this problem resulting angular velocity,

$$\omega(r, t) = \frac{\Gamma}{4\pi\nu \cdot t} \cdot \exp\left(-\frac{r^2}{4\nu \cdot t}\right) + \frac{\Gamma}{\pi R^2} \quad (4)$$

and circular velocity (tangential) of the gravitational vortex,

$$v(r, t) = \frac{\Gamma}{2\pi \cdot r} \cdot \left[ 1 - \exp\left(-\frac{r^2}{4\nu \cdot t}\right) \right] + \frac{\Gamma}{2\pi R^2} \cdot r \quad (5)$$

relations that have been drawn in Figures 10 and 11.

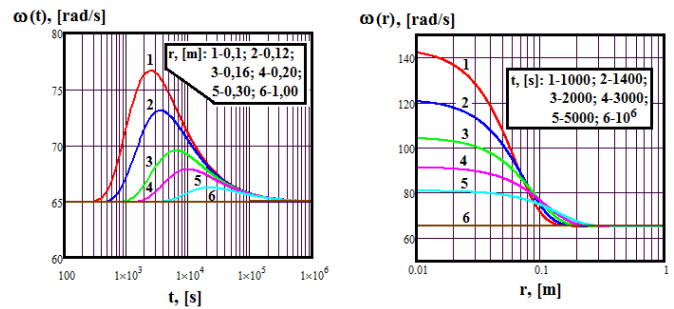


Fig. 11. Temporal  $\omega(t)$  and spatial  $\omega(r)$  distribution of the angular velocity of gravitational swirl

#### 4. THE WATER FLOW IN CONICAL CHANNEL IN THE PRESENCE OF THE TURBINE ROTOR

In the presence of the turbine rotor in steps, the flow structure changes qualitatively. Rotor blades require boundary conditions of water velocities equal to the velocities of their surfaces, speed is not known a priori, but the settlement is determined depending on when torque resistance that opposes the rotation shaft and

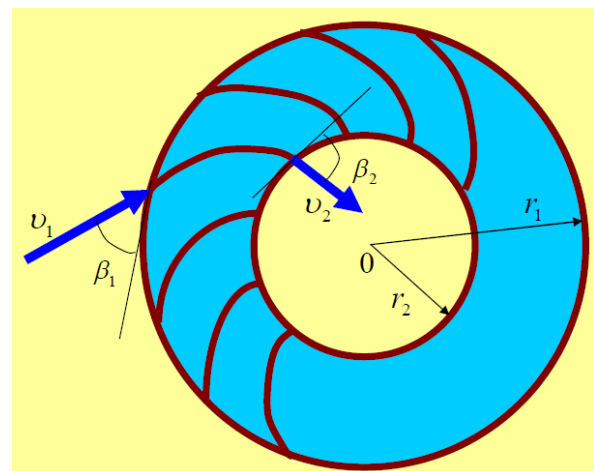


Fig.12. Explanatory drawing for the Euler turbine equation

velocity diagrams into and out of blades, due to absolute velocity of the water entry into blades, blade profile, entry and exit angles of the blades and blade fluid (Figure 12).

To calculate torque exerted by the fluid flow through the blades, Euler proposes the following equation known as turbine equation:

$$M_{rot} = \dot{m} \cdot (v_1 \cdot r_1 \cdot \cos \beta_1 - v_2 \cdot r_2 \cdot \cos \beta_2), \quad (6)$$

where  $\dot{m} = Q \cdot \rho$  is the mass flow of fluid flowing through all parts of the blades, [kg/s];  $\rho$  - water density, [kg/m<sup>3</sup>];  $v$  - absolute velocity of water, [m/s].

There are two ways of consecutive acceleration of the water flow: acceleration under the action of gravity required to operate the wicket gate and acceleration resulting from gravitational vortex formation. In the first case the velocity will be determined by the relation  $v(\Delta H) = [2g\Delta H / (\xi + 1 + \lambda \cdot (\Delta H / D))]^{0.5}$ , in the second case - the formula  $v(\Delta H) = v(\Delta H/n) \cdot (A_0/A_c)$ , where  $\Delta H$  is available drop of water [m]  $\xi$  - local loss coefficient;  $\lambda$  - coefficient of linear losses of Darcy,  $D$  - diameter of the channel, [m];  $n$  - coefficient of reduction of available water fall;  $A_0/A_c$  - contraction coefficient of the water in the gravitational vortex.

In Figure 13 are presented in comparison the graphics  $v(\Delta H)$ , for these two cases resulting the advantage from the vortex gravitational acceleration for recovering energy from small waterfalls.

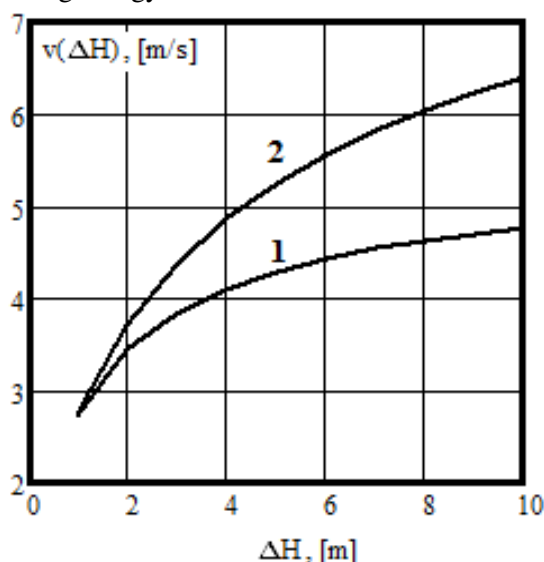


Fig.13. Comparison of flow rates of water under the acceleration of the gravity (1) and as the result of the gravitational vortex formation (2):  $D = 0,12$  m;  $\xi = 1$ ;  $\lambda = 0,08$ ;  $n = 3,2$ ;  $A_0/A_c = 1,65$

The cavern height formed by the vortex decisively influences the extraction efficiency of the available power of the water. If the height of the cavern extends downstream of the turbine stage, output stage will be very small and will work under close idle mode. Increased resistance torque for rotation of the shaft, cavern height is reduced and the limit will be extended until the next section after stage turbine. Further increase in resistance torque rotor blades stages will stop.

To simulate water flow through the conical channel in the presence of rotor blades stages was implemented the gravitational vortex turbine 3D CAD project in software SolidWorks at experimental model scale. Simulation has been developed in Comsol software. In Figure 14 is presented the velocity field and flow lines  $Q = 0.5$  l / s and vortex intensity  $\Gamma = 0.3$  m<sup>2</sup> / s. It notes the formation of swirls every area of the blades. Water velocity increases rapidly as it approaches the drain hole and strongly depends on water flow  $Q$ .

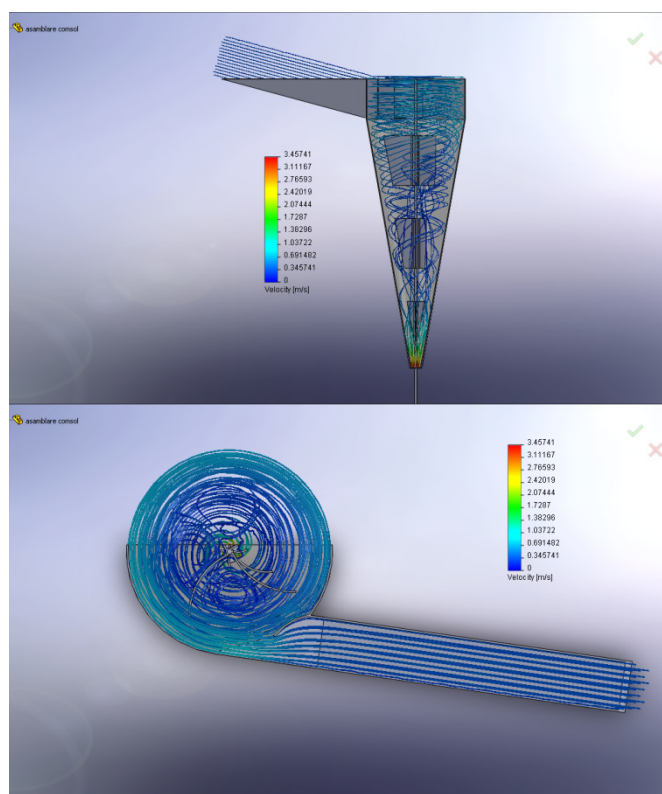


Fig.14. Structure of water flow through the conical channel in the presence of the rotor blades stages

## 5. CONCLUSIONS

✓ It was developed a new hydro power plants with gravitational vortex and turbine with rapidity steps for

# THE CONCEPT AND THEORETICAL STUDY OF MICRO HYDROPOWER PLANT WITH GRAVITATIONAL VORTEX AND TURBINE WITH RAPIDITY STEPS

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hydraulic energy recovery of rivers with falls and with small flows. For water flow in the gravitational vortex the most indicated are helical turbine steps with hydrofoil profile. Can be used Francis turbine steps with the form according to rapidity. Due to large variation along the peripheral velocity of the blade is necessary twisting the blade.

✓ It was simulated the water flow through the conical channel in the presence and absence of the rotor blades steps. In the first case, according to the vortex intensity, the free surface of the water forms a cavern (cave, cavity) that extends to the drain hole. Flow lines are shaped conical propellers. Axial and radial velocities of the water are not zero only in the boundary layer. Circular and axial velocity profiles in the boundary layer are of Pohlhausen. Outside the boundary layer water acts as a perfect fluid and a solid body in rotation. Under non-stationary diffusion of the swirl, angular rotation velocity, default circular velocity; reach maximum values for which the increase of the radius and time degenerates, reaching asymptotic stationary. In the second case the structure of the water flow is changes with the formation of swirls every area of the blades. Increased efficiency is possible by keeping the flow regime in which the caverns extending to the output section of the step.

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