P-Q THEORY AND TOTAL ACTIVE FILTERING COMPENSATION UNDER DISTORTED SUPPLY VOLTAGES

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REZUMAT. Lucrarea este concentrată pe curentul de compensare al unui filtru activ de putere trifazat, în condiții de tensiune deformată, pentru a compensa atât puterea reactivă, cât şi distorsiunea armonică a curentului de sarcină. Este propusă o expresie simplă pentru amplitudinea curentului sinusoidal care se doreşte a fi absorbit din rețea după compensare, bazată pe conceptele teoriei p-q. Rezultatele simulărilor, urmate de validarea experimentală, arată eficiența strategiei de compensare într-un sistem de filtrare activă paralel.

Cuvinte cheie: filtre active de putere, teoria p-q, curent de compensat, compensare totală, tensiune nesinusoidală.

ABSTRACT. The paper is focused on the compensating current of a three-phase active power filter which operates under distorted voltages in order to compensate both reactive power and harmonic distortion of the load current. A simple expression of the desired magnitude of the sinusoidal supply current is proposed based on the p-q theory concepts. The simulation results followed by experimental validation show the effectiveness of this total compensation strategy in a shunt active filtering system.

Keywords: active power filter, p-q theory, compensating current, total compensation, nonsinusoidal voltage

1. INTRODUCTION

The harmonic pollution in power systems accompanied by the nonsinusoidal operating regimes is today one of the most important secondary indicators of power quality, which is influenced by the nonlinear loads operation. In this context, the increasing amount of nonlinear consumers, particularly based on static converters, have been determined the continuous search for performant harmonic mitigation solutions in order to meet the requirements of specific power quality standards, such as IEEE 519-1992 and IEC 61000.

Obviously, the best solution to avoid the harmonic pollution is its prevention by selecting the modern so-called clean equipments in the energy conversion stage or by adopting the simple solution to isolate the equipments which could cause harmonic distortion. However, with existing power systems, the installation of equipments able to mitigate harmonics remains the only solution.

Although the passive filtering is a less expensive solution and partially solves the harmonic pollution problem, the inability of adaptation to changes in the power system makes this solution to be less efficient [1].

On the other hand, the active power filters (APF) make use of the control possibilities of voltage source inverters and can perform any compensation objective. Thus, the shunt active power filters (SAPF) is able to inject an appropriate compensating current in the point of common coupling (PCC) in order to improve the supply current waveform both in terms of its shape and phase (Fig. 1).

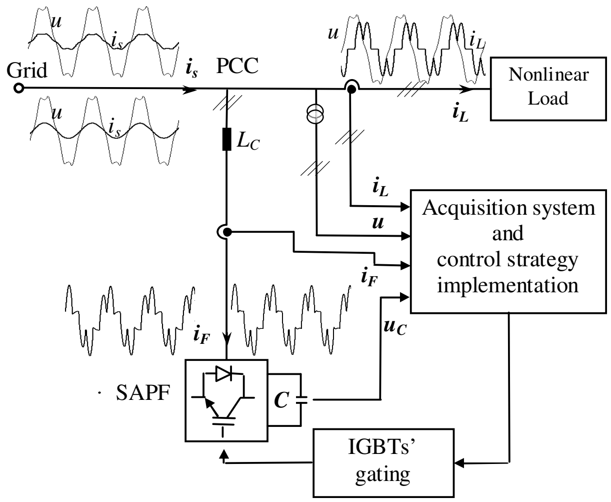

Fig. 1. Active compensation through SAPF.

Clearly, any SAPF's control strategy takes into account, in a more or less obvious manner, the power flow in the power system. In this context, the p-q theory of instantaneous reactive power introduced and developed by Akagi [2], [3] was brought into actuality to provide the mathematical foundation in the control of static converters involved in the power quality improvement. Furthermore, the p-q theory is about to become a means of identifying and analyzing the properties of powers in circuits with nonsinusoidal voltages and currents [4], [5].

Thus, based on the components of the instantaneous powers introduced by the p-q theory, different component of the current can be expressed and used in the control strategy of an SAPF.

In this paper, after introducing the p-q theory concepts in section 2, section 3 is devoted to the generation of the sinusoidal reference supply currents when total compensation is expected under distorted voltage conditions. Then, the performance of compensating current generation system is analyzed in section 4 through simulation under Matlab/Simulink environment and confirmed in section 5 by experimental results. Finally, some concluding remarks are drawn.

2. THE P-Q THEORY CONCEPTS

The first version of the p-q theory known as the instantaneous reactive power theory for three-phase circuits was published in 1984 in a prestigious international journal by professor Akagi and his coauthors Kanazawa and Nabae [2].

The first step was to introduce the instantaneous space vectors ($\underline{u}$ and $\underline{i}$) by transforming the three-phase systems of voltages (u_a, u_b, u_c) and currents (i_a, i_b, i_c) into two-phases orthogonal stationary reference frames (u_α, u_β) and (i_α, i_β). Note that the power invariant transformation was used in defining the transformation matrix.

Then, the conventional instantaneous power (p) was defined as

$$p = u_\alpha \cdot i_\alpha + u_\beta \cdot i_\beta = u_a \cdot i_a + u_b \cdot i_b + u_c \cdot i_c \tag{1}$$

and the instantaneous imaginary power (q) was defined as the amplitude of a reactive power vector ($\underline{q}$), which is perpendicular to the α-β plane and expressed as

$$\underline{q} = \underline{u}_\alpha \times \underline{i}_\beta + \underline{u}_\beta \times \underline{i}_\alpha . \tag{2}$$

Thus, the original matriceal form of p and q definitions is

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} u_\alpha & u_\beta \\ -u_\beta & u_\alpha \end{bmatrix} \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} . \tag{3}$$

Another way to introduce p and q is to consider them as the real and imaginary parts of the instantaneous complex power ($\underline{s}$). For the original adopted power invariant *a-b-c* to α-β transformation,

$$\underline{s} = \underline{u} \cdot \underline{i}^* = p + jq , \tag{4}$$

where

$$\underline{i}^* = i_\alpha - j i_\beta . \tag{5}$$

This way, the obtained expression of q is

$$q = u_\beta \cdot i_\alpha - u_\alpha \cdot i_\beta . \tag{6}$$

which is the minus of expression of q in (3).

Note that both expressions (3) and (6) of q were used in the implementation of the active compensators control.

The specific use of the p-q theory for compensation reasons is to express the current components as a function of voltage α-β components and the instantaneous powers p and q. If both p and q are considered as a sum of average components (P and Q) and oscillating components ($p_\sim$ and $q_\sim$), any decomposition of the current can be expressed.

If the non-power invariant transformation *a-b-c* to α-β is adopted in order to preserve the magnitude of the instantaneous three-phase quantities as the space vector modulus, expression (4) becomes

$$\underline{s} = \frac{3}{2} \cdot \underline{u} \cdot \underline{i}^* = p + jq \tag{7}$$

and the current space vector can be expressed as

$$\underline{i} = \frac{2}{3} \cdot \frac{\underline{u}}{|u|^2} \cdot \underline{s}^* = \frac{2}{3} \cdot \frac{\underline{u}}{|u|^2} \cdot \left(P + p_\sim - jQ - jq_\sim \right), \tag{8}$$

where

$$|u|^2 = u_\alpha^2 + u_\beta^2 . \tag{9}$$

In shunt active filtering systems, expression (8) can be used either to calculate the desired compensating current

$$\underline{i}_{F\_ref} = \frac{2}{3} \cdot \frac{\underline{u}}{|u|^2} \cdot \underline{s}_F^* . \tag{10}$$

on the basis on instantaneous complex power to be compensated, or to calculate the desired supply current after compensation,

$$\underline{i}_{s\_ref} = \frac{2}{3} \cdot \frac{\underline{u}}{|u|^2} \cdot \underline{s}_{sup\,ply}^* , \tag{11}$$

from which the load current must be subtracted in order to obtain the reference compensating current.

3. TOTAL COMPENSATION STRATEGY

When the shunt active filtering system fulfills the task of total compensation, only the active power (P) must be drawn from the power supply by the assembly consisting of SAPF and the nonlinear load. It means that

$$\underline{s}^{*}_{sup\,ply} = P\,. \tag{12}$$

The components of the distorted current to be compensated correspond to the instantaneous powers $p_{\sim}$ and q in (8).

If the supply voltage is sinusoidal, a sinusoidal supply current will be obtained under unity power factor condition which is the active component of the load current,

$$\underline{i}_{s\_ref} = \frac{2}{3}\cdot\frac{\underline{u}}{|\underline{u}|^{2}}\cdot P\,. \tag{13}$$

Indeed, as the modulus of the voltage space vector is constant, the supply current space vector is proportional to the voltage space vector

$$\underline{i}_{s\_ref} = \underline{i}_{sa} = k_{s}\cdot\underline{u}\,, \tag{14}$$

where

$$k_{s} = \frac{2}{3}\cdot\frac{P}{|\underline{u}|^{2}}\,. \tag{15}$$

However, when the supply voltages are distorted, the modulus of the voltage space vector is time dependent (Fig. 2) and the calculation of the desired supply current by (13) leads to a nonsinusoidal waveform of this current which has a different distortion level compared to the voltage distortion.

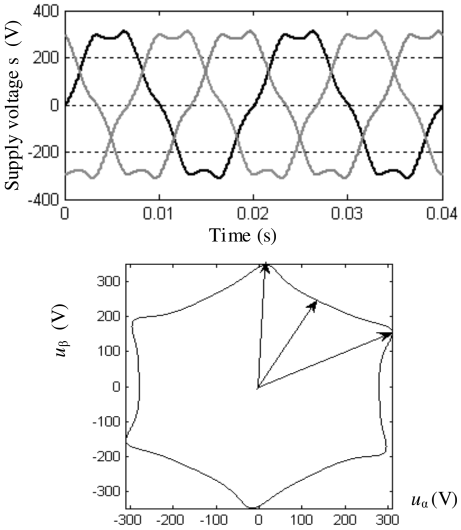

Fig. 2. Example of distorted voltages and associated space vector locus.

In this case, to handle the supply current waveform and keep its harmonic distortion factor below the limits required by the power quality standards, either the unity power factor compensation strategy or perfect harmonic cancelation compensation strategy can be adopted [6], [7].

Especially for the operation under significant distorted voltage conditions, the perfect harmonic cancelation which means a sinusoidal supply current and unity displacement power factor after compensation is preferred [8], [9].

To this end,

$$\underline{i}_{s\_ref} = k_{ns}\cdot\underline{u}_{1}\,, \tag{16}$$

where $\underline{u}_1$ is the fundamental voltage space vector and k_{ns} is a proportionality constant which can be determinate from the condition that the supply active power is equal to the load active power, i.e.

$$P = \frac{1}{T}\int_{t-T}^{t} Re\{\underline{s}\}dt = \frac{1}{T}\int_{t-T}^{t} Re\left\{\frac{3}{2}\cdot\underline{u}\cdot\underline{i}^{*}_{s\_ref}\right\}dt\,. \tag{17}$$

After replacing (16) into (17),

$$P = \frac{3K_{ns}}{2}\cdot\frac{1}{T}\int_{t-T}^{t} Re\left\{\underline{u}\cdot\underline{u}_{1}^{*}\right\}dt\,. \tag{18}$$

Thus, the obtained expression for k_{ns} is

$$K_{ns} = \frac{2}{3}\cdot\frac{P}{\frac{1}{T}\int_{t-T}^{t} Re\left\{\underline{u}\cdot\underline{u}_{1}^{*}\right\}dt}\,. \tag{19}$$

If the synchronous d-q reference frame, with d axis in the direction of the voltage space vector, is chosen, then:

$$u_{d} = |\underline{u}|;\quad u_{q} = 0;$$

Moreover, as u_{1d} is very close to the average value of u_d (Fig. 3), the following assumption can be made:

$$u_{1d} \approx \frac{1}{T}\int_{t-T}^{t} u_{d}\,dt = constant\,. \tag{20}$$

In these conditions, expression (19) becomes

$$K_{ns} = \frac{2}{3}\cdot\frac{P}{\frac{1}{T}\int_{t-T}^{t}\left(u_{d}\cdot u_{1d} + u_{q}\cdot u_{1q}\right)dt} = \frac{2}{3}\cdot\frac{P}{u_{1d}^{2}}\,. \tag{21}$$

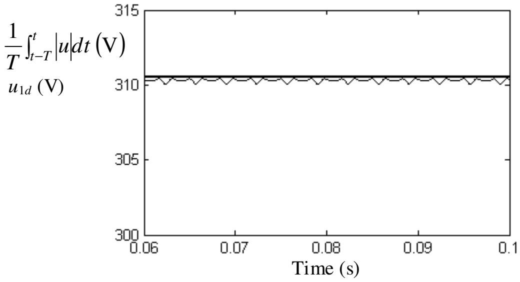

Fig. 3. d-axis component of the fundamental voltage space vector and average value of $|\underline{u}|$ for the example given in Fig. 2.

Thus, the expression of the reference supply current becomes:

$$\underline{i}_{s\_ref} = \frac{2}{3}\cdot\frac{P}{u_{1d}^{2}}\cdot\underline{u}_{1}\,, \tag{22}$$

and its modulus is:

$$\left|\underline{i}_{s\_ref}\right| = \frac{2}{3}\cdot\frac{P}{u_{1d}^{2}}\cdot\sqrt{u_{1d}^{2}+u_{1q}^{2}} = \frac{2}{3}\cdot\frac{P}{u_{1d}}\cdot\sqrt{1+\left(\frac{u_{1q}}{u_{1d}}\right)^{2}}\,. \tag{23}$$

As confirmed by Fig. 4 for the distorted voltage in Fig. 2, u_{1q} is very small compared with u_{1d} and, all the more, u_{1q}^2 compared with u_{1d}^2 ($\left(u_{1q}/u_{1d}\right)^2 < 2.5\cdot10^{-3}$ for the example taken into consideration).

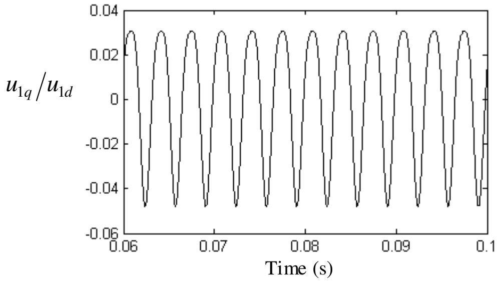

Fig. 4. Time evolution of u_{1q}/u_{1d} for the example given in Fig. 2.

Therefore, following the above assumptions, a simple expression can be used to calculate the modulus of the desired supply current vector, which is the magnitude of the supply current after compensation, i.e.

$$\left|\underline{i}_{s\_ref}\right| = \frac{2}{3}\cdot\frac{P}{\frac{1}{T}\int_{t-T}^{t}\left|\underline{u}\right|dt}. \qquad (24)$$

As the active power does not depend on the chosen coordinate system, the calculation in (24) can be performed under α-β stationary reference frame in accordance with p-q theory concepts (Fig. 5).

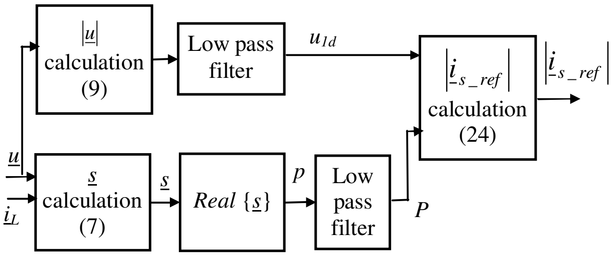

Fig. 5. Block diagram for the magnitude of the desired supply current calculation.

The sinusoidal shape of the reference supply currents is provided by the phase-locked loop (PLL) circuit which is already used in the control system in order to cope for losses in the power circuit by keeping the DC-capacitor voltage at its reference value [10].

4. PERFORMANCE OF COMPENSATING CURRENT GENERATION

The block diagram in Fig. 5 has been implemented under Matlab/Simulink environment and then integrated into a specific model to verify the performance of the reference current calculation component of the SAPF's control system (Fig. 6). The nonlinear load is a three-phase controlled rectifier and the power supply provides a balanced system of distorted voltages of different total harmonic distortion (*THD*) factors. The waveforms and harmonic spectra in Fig. 7 prove the capability of the control system to generate sinusoidal reference supply currents in the case of a load current of $THD_I \approx 40\%$ and three distortion voltage degrees (THD_U = 10.12%, THD_U = 5.83% and THD_U = 4.91%) under the same fundamental phase voltage of 220 V. There will be no reactive power flow, the rms value of the supply current is reduced by more than three times and the total power factor becomes very close to the unity value (0.995 for the highest THD_U and 0.998 for the lowest THD_U).

For comparison reasons, Fig. 8 illustrates the performance of the original p-q theory implementation when total compensation is desired. Although the power factor is very significantly increased (0.98 for the highest THD_U), the supply reference current distortion is comparable with that of supply voltage and the standards recommendations can not be met.

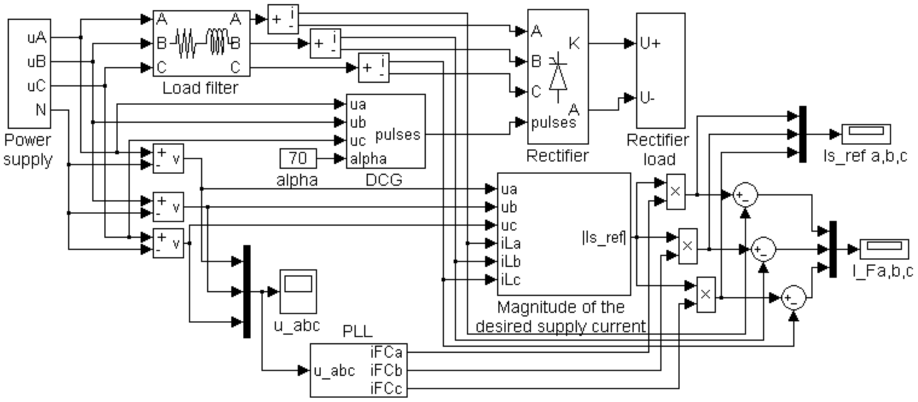

Fig. 6. Simulink model for the compensating current generation.

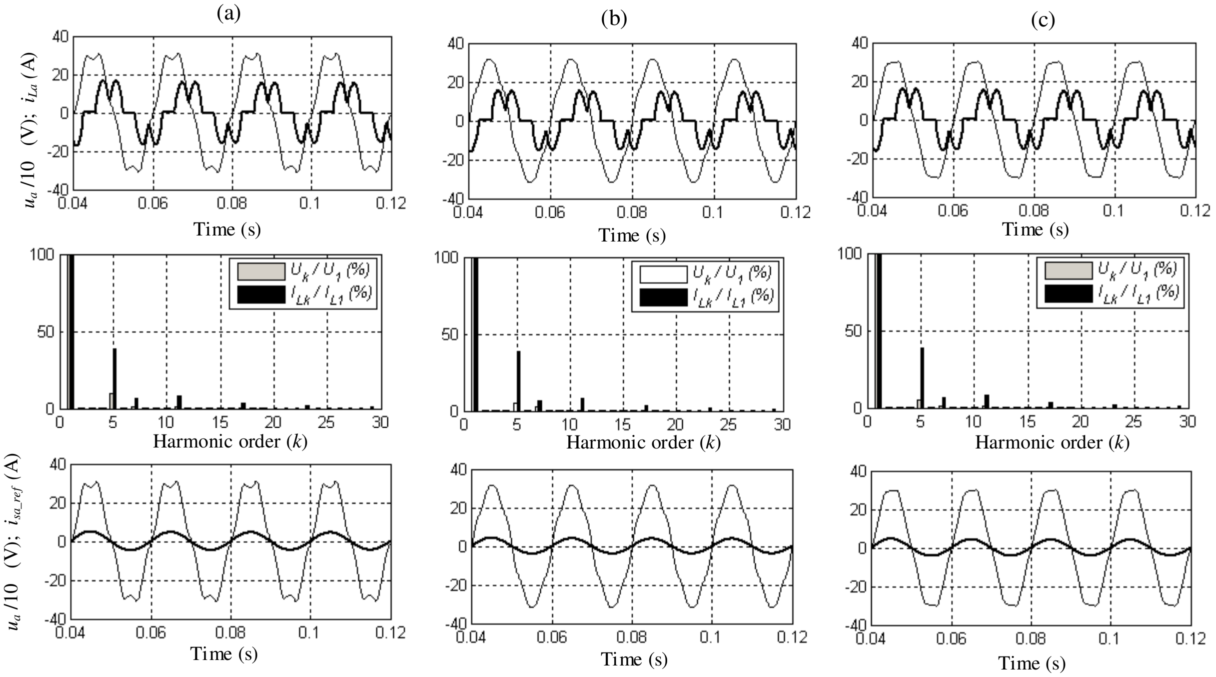

Fig. 7. Distorted load current, distorted supply voltage and sinusoidal reference supply current for three voltage distortion levels: (a) THD_U =10.12%; (b) THD_U =5.83%; (c) THD_U =4.91%.

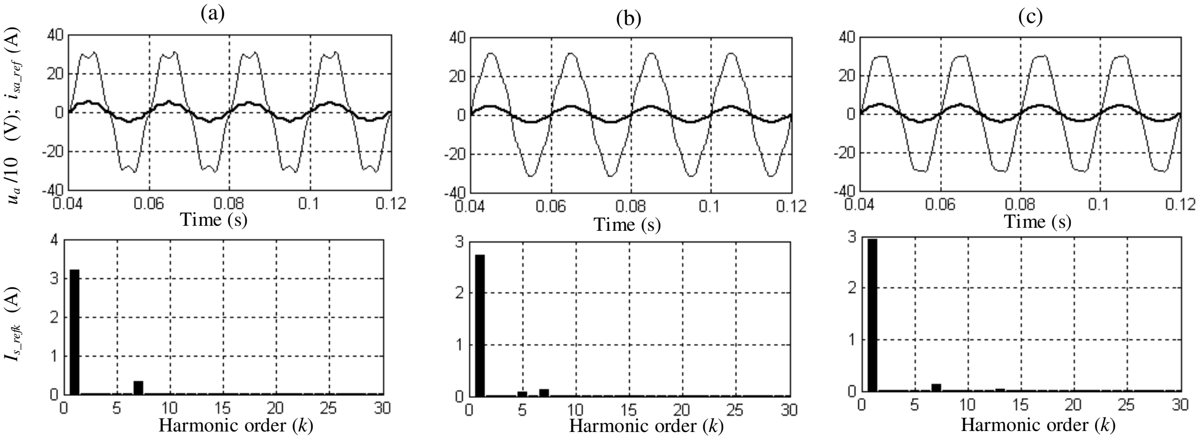

Fig. 8. Distorted supply voltage and original Akagi's reference supply current for three voltage distortion levels: (a) THD_U =10.12%; (b) THD_U =5.83%; (c) THD_U =4.91%.

5. EXPERIMENTAL RESULTS

An experimental setup which consists of a voltage source inverter of 15 kVA and a control system based on a dSPACE 1103 board allowed to implement the total compensation strategy. The current tracking is guaranteed by a three-phase hysteresis band controller, whereas the compensating current dynamics is ensured by a coupling inductor of 4.4 mH. The highly distorted current of THD_I =108% is drawn by the nonlinear load under the nonideal supply voltage of THD_U =3% (Fig. 9a).

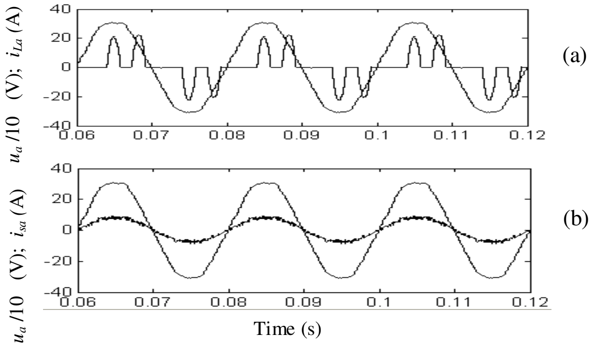

Fig. 9. Experimental supply voltage, load current and supply current.

When the shunt active power filter absorbs the compensated current in accordance with the adopted compensation strategy, the supply current tracks a sinusoidal shape and a unity displacement power factor is obtained (Fig. 9b). Although there is a remaining current distortion of about 9% due especially to the high switching frequency of the power devices, a good efficiency of the active filtering system is achieved.

6. CONCLUSIONS

With the proposed generation of the sinusoidal reference supply current, a good performance of the total active filtering system is achieved irrespective of the supply distortion level. As the main advantage of this algorithm is the reduction of computing time, new developments are needful to illustrate the influence of the sampling time increasing.

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