CURRENT DECOMPOSITION BASED ON THE P-Q THEORY CONCEPTS

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REZUMAT. Lucrarea analizează posibilităţile de descompunere a curentului de sarcină deformat, în vederea utilizării ca şi curent de referinţă în filtrarea activă, pe baza teoriei p-q. Pe lângă descompunerea originală propusă de Akagi, este prezentată şi o descompunere modificată. Modelarea şi simularea în mediul Matlab/Simulink, pentru două studii de caz, permit interpretarea rezultatelor. Se arată că, pentru o compensare totală, obţinerea unui curent activ cu aceeaşi formă de undă şi în fază cu tensiunea nu este posibilă dacă în expresia curentului activ apare modulul fazorului spaţial al tensiunii.

Cuvinte cheie: teoria p-q, curent activ, tensiune nesinusoidală, puterea aparentă complexă.

ABSTRACT.This paper analyzes the possibilities to decompose the distorted load current based on the p-q theory, in order to be used by the reference compensating current calculation in active filters. Besides the original decomposition proposed by Akagi, a modified decomposition is presented. Modeling and simulation in Matlab/Simulink, for two case studies, allow us to analyze the results. It is shown that, when the total compensation is desired, obtaining an active current with the same shape and phase as the voltage is not possible if the active current expression contains the voltage space vector modulus.

Keywords: p-q theory, active current, non-sinusoidal voltage, complex apparent power.

1. INTRODUCTION

In practice non-sinusoidal conditions are frequently present because the use of static converter is widely spread. In these conditions there is a worldwide interest to improve the quality of the energy. Active filtering it is an efficient method to improve the shape of the current absorbed from the power grid and to improve the power factor. If total compensation is needed, the active filter allows the absorption from the grid of a current with the same shape and phase as the voltage. In these conditions, the proper current decomposition is justified for sinusoidal or non-sinusoidal voltage supply and as well as for any load type (resistive, nonlinear, symmetrical and asymmetrical).

This paper purpose is to analyze the current decomposition possibilities based on the classic p-q theory [6] and its development using some case studies.

2. P-Q THEORY

If $\underline{u}$ and $\underline{i}$ are the representative space vectors of the voltage and current, the instantaneous apparent complex power, with the real and imaginary parts, is [1], [5]:

$$\begin{aligned}\underline{s} &= p + jq = \frac{3}{2}\underline{u}\cdot\underline{i}^{*} = \\ &= \frac{3}{2}\left[u_d i_d + u_q i_q + j\left(-u_d i_q + u_q i_d\right)\right]\end{aligned} \qquad (1)$$

In the relation (1), real and imaginary parts of the instantaneous apparent complex power are:

$$p = \frac{3}{2}\left(u_d i_d + u_q i_q\right) \qquad (2)$$

$$q = \frac{3}{2}\left(-u_d i_q + u_q i_d\right) \qquad (3)$$

Real and imaginary parts of the complex instantaneous apparent power can be calculated as sum of DC components (P and Q) and AC components (p~ and q~):

$$p = P + p_{\sim}; \qquad (4)$$

$$q = Q + q_{\sim}. \qquad (5)$$

The expression of the instantaneous apparent complex power becomes:

$$\underline{s} = P + p_{\sim} + j\left(Q + q_{\sim}\right) \qquad (6)$$

Akagi has proposed the compensation for the AC components of the real and imaginary parts from the complex instantaneous apparent power, respectively calculate the reference currents of active filters with the following expression:

$$\underline{i} = i_d + j i_q = \frac{2}{3}\frac{1}{u_d^2 + u_q^2}\underline{u}\cdot\underline{s}^{*} \qquad (7)$$

Developing the scalar product the expression (7) becomes:

$$\underline{i} = i_d + j i_q = \frac{2}{3}\frac{1}{|\underline{u}|^2}\left[p u_d + q u_q + j\left(-q u_d + p u_q\right)\right] \qquad (8)$$

On this basis Akagi, Nabae and others defined [1]:

1. The instantaneous active current with components:

$$i_{ad} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2} p \qquad i_{aq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2} p \qquad (9)$$

2. The instantaneous reactive current with components:

$$i_{rd} = \frac{u_q}{|\underline{u}|^2} q \qquad i_{rq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2} q \qquad (10)$$

These names had been criticized by Czarnecki [4], who found some examples in which the current defined with (9) is not corresponding to active power, and the current defined with (10) is not corresponding to reactive power. This vagueness can be eliminated defining four components for the current [3], [7]. Now expression (8) becomes:

$$\underline{i} = \frac{2}{3}\frac{1}{|\underline{u}|^2}\left[\left(P + p_{\sim}\right)u_d + \left(Q + q_{\sim}\right)u_q + {} \right. \\ \left. + j\left(-\left(Q + q_{\sim}\right)u_d + \left(P + p_{\sim}\right)u_q\right)\right] \qquad (11)$$

Based on the previous relation the current can be defined:

1. The active current vector $\underline{i}_a$ with components:

$$i_{ad} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2} P \qquad i_{aq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2} P \qquad (12)$$

2. The reactive current vector $\underline{i}_r$ with components:

$$i_{rd} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2} Q \qquad i_{rq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2} Q \qquad (13)$$

3. The supplementary useless current vector for the AC real part $\underline{i}_{sp}$ with components:

$$i_{spd} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2} p_{\sim} \qquad i_{sqp} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2} p_{\sim} \qquad (14)$$

4. The supplementary useless current vector for the AC imaginary part $\underline{i}_{sq}$ with components:

$$i_{spd} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2} p_{\sim} \qquad i_{sqp} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2} p_{\sim} \qquad (15)$$

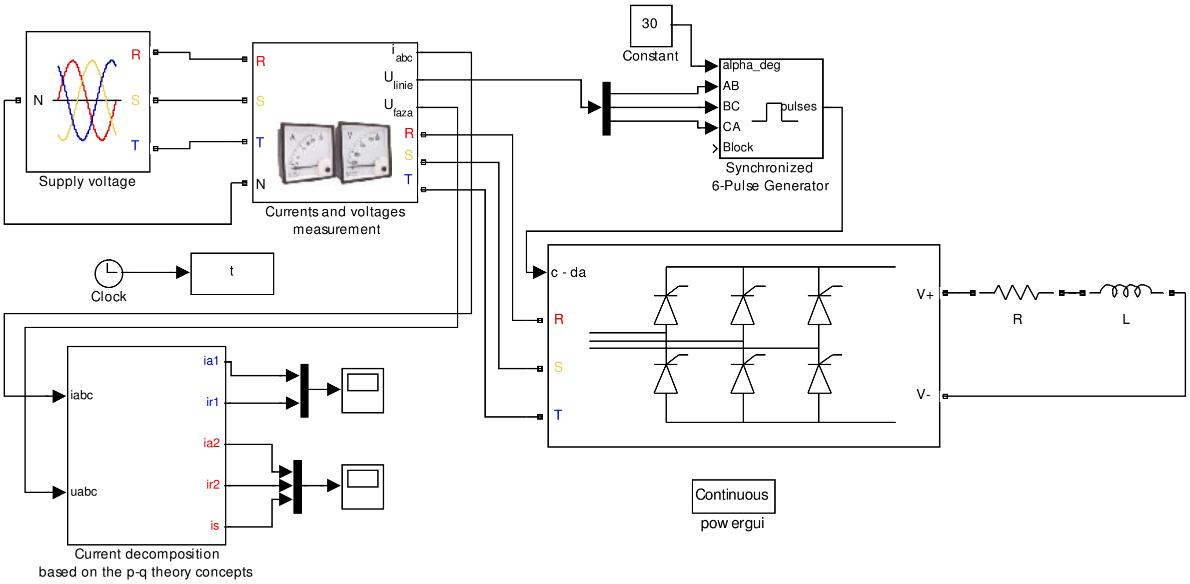

Fig.1. Simulink model of a symmetric nonlinear load formed by a three phase bridge rectifier with passive load RL

3. CASE STUDIES

Based on a Simulink model (figure 1), in which the current had been calculated with the relations given by the original Akagi's theory and the relations corresponding to the modified decomposition (rel. 12-15), two case studies were created for a three phase sinusoidal and non-sinusoidal voltage system.

These case studies will emphasize that the (12-15) relations lead to correct results if the voltage is sinusoidal [2]. The waveforms that will be presented for the two case studies have been obtained in Matlab/Simulink.

A. Sinusoidal voltage, symmetric nonlinear load

Let us consider a symmetric nonlinear load, formed by a three phase bridge rectifier with passive load RL, which absorbs a current of 30 A RMS.

In figure 2 shape of the voltage and phase current absorbed from the grid by the three phase bridge rectifier can be observed.

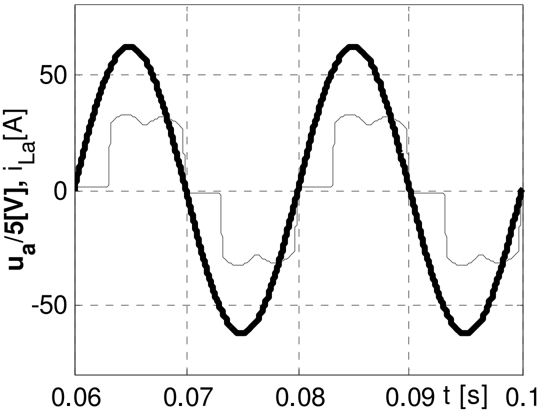

Fig. 2. The grid voltage and current waveforms for the three phase bridge rectifier

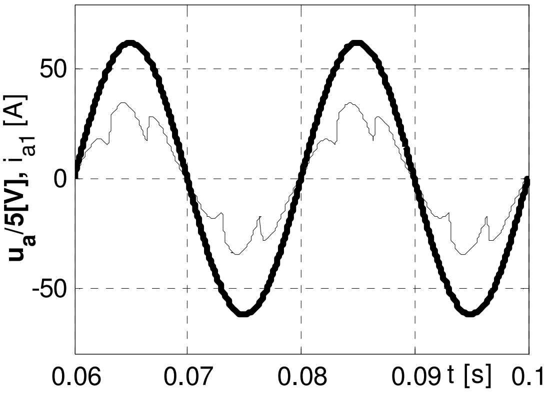

Fig. 3. The grid voltage and the Akagi's original active current waveforms

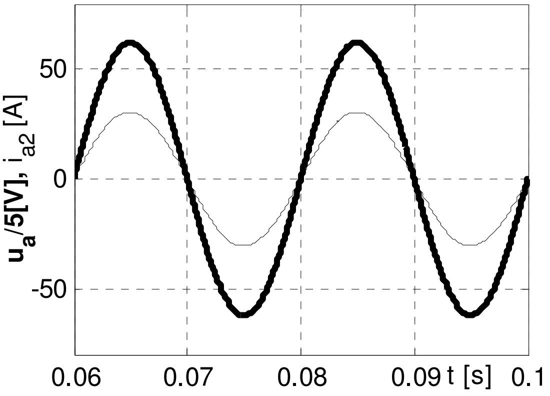

Fig. 4. The grid voltage and the active current (12) waveforms

In figures 3 and 4, the active current defined with the Akagi's original relations is non-sinusoidal and in the same phase as the voltage, and the active current defined with relation (12) is sinusoidal and in the same phase as voltage. Therefore, the original current proposed by Akagi contains besides the active component, a distorted component too.

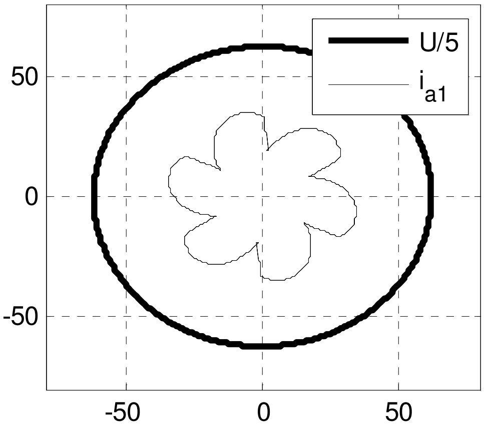

Fig. 5. The locus of the top of the grid voltage and of Akagi's original active current

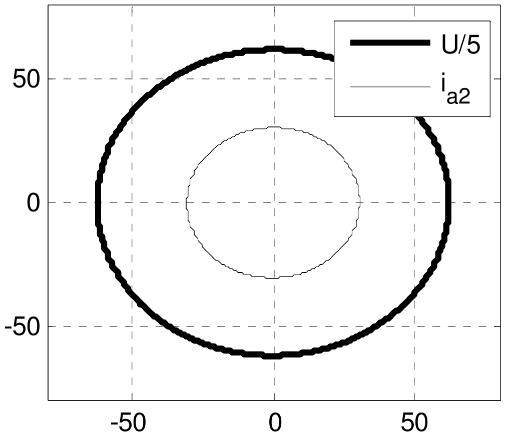

Fig. 6. The locus of the top of the grid voltage and of the active current defined with relation (12)

Also we can notice, in figures 5 and 6, that the geometrical locus of the active current space vector is a circle just in the definition (12) case.

In figures 7 and 8 it is shown the waveform of total supplementary current as well as the geometrical locus of the corresponding space vector.

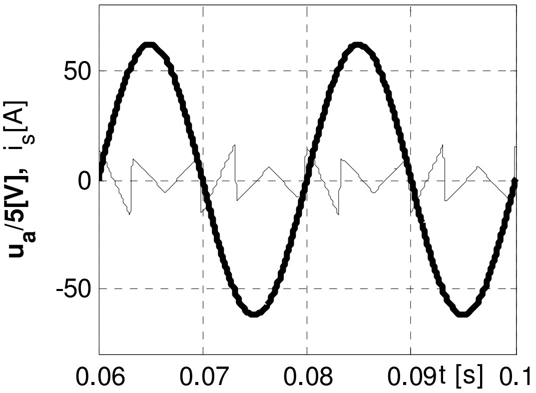

Fig. 7. The grid voltage and the supplementary current waveforms

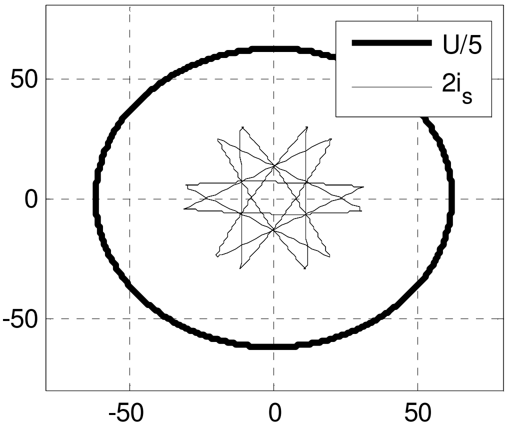

Fig. 8. The locus of the grid voltage and of the supplementary current space vector

In this case study we notice that the "active" name is not corresponding to the currents Akagi defined, because, in case of sinusoidal voltage, active power is transmitted only on the current fundamental. The current defined with relation (12) follows these properties.

B. Non-sinusoidal voltage and symmetric non linear load

In this case, the same nonlinear load is studied, but the 13th harmonic has been added to the grid voltage. To maintain the two cases equivalent, it was aimed to obtain the same load power to be transferred from the power grid.

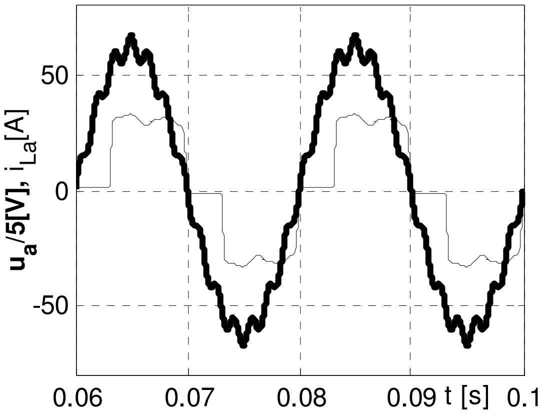

Fig. 9. The grid voltage and current waveforms for the three phase bridge rectifier

It can be noticed in figures 10 and 11 that in case of a non sinusoidal voltage supply, the waveform of the active current is different from the shape of the voltage supply, and the supplementary current (figures 14 and 15) is highly distorted.

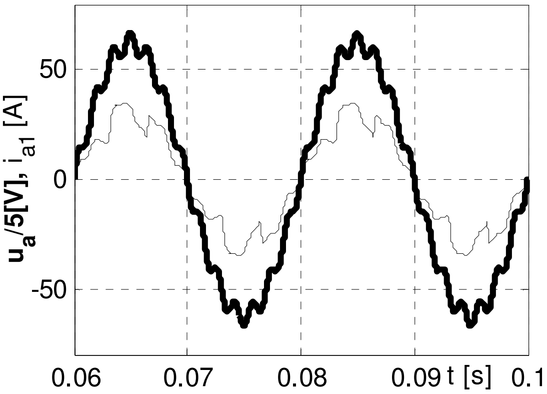

Fig. 10. The grid voltage and the Akagi's active current waveforms

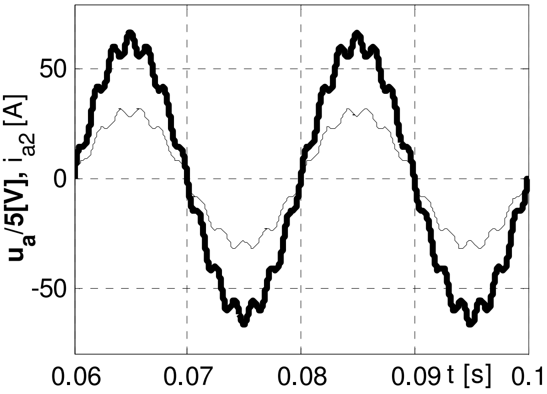

Fig. 11. The grid voltage and the active current defined with relation (12) waveforms

The fact that, in case of non-sinusoidal grid voltage, both presented theories lose their validity, also, can be demonstrated looking at the locus of the top of the active currents waveforms (figures 12 and 13).

It can be seen that the geometrical locus of the active current defined by relation (12) has no longer the same shape as the grid voltage.

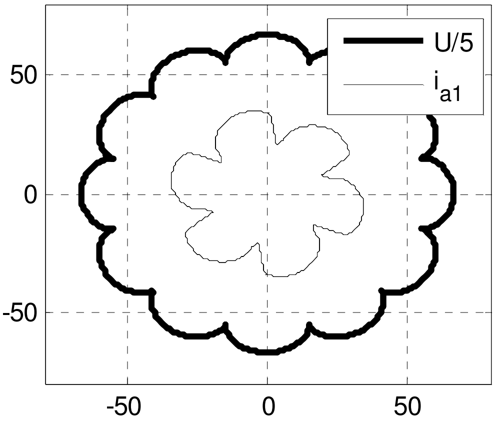

Fig. 12. The locus of the top of the grid voltage and of Akagi's original active current

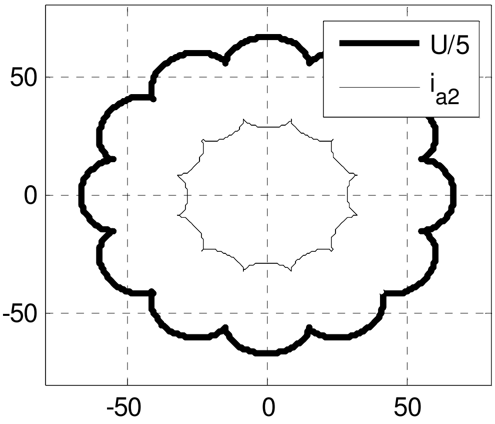

Fig. 13. The locus of the top of the grid voltage and of the active current defined with relation (12)

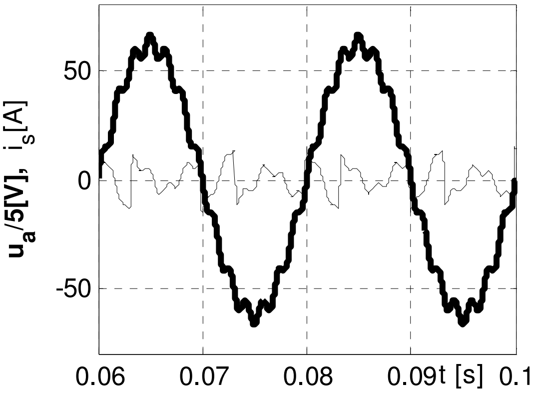

Fig. 14. The grid voltage and the supplementary current waveforms

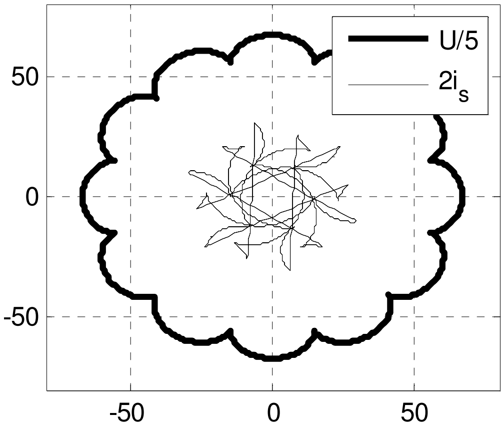

Fig. 15. The locus of the grid voltage and of the supplementary current space vector

4. CONCLUSIONS

✓ Considering the obtained results after the two case studies we can take some conclusions regarding the current components in a three phase system. The original active component defined by Akagi do not represent the active component of load current if the load is symmetric nonlinear.

✓ The active component defined with the relation (12) represents the active component of load current only if the voltage is sinusoidal. When the voltage is non-sinusoidal it is no longer valid the definition of active current with the expression (12).

✓ It is not possible to obtain an active current with the same shape as the voltage if the denominator from relation (12) depends on time.

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