NUMERICAL TECHNIQUES APPLIED IN *PEEC* METHOD IMPLEMENTATION

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REZUMAT. Metoda elementului de circuit partial echivalent (PEEC) este o tehnica de modelare integrala utilizata pentru rezolvarea problemelor de camp si circuit atat in domeniul timp cat si in domeniul frecventa. Metoda se bazeaza pe conversia ecuatiei integrale de camp electric in elemente de circuit partiale. In domeniul de frecventa s-au evaluat erorile introduse atat de calculul elementelor de circuit partiale cat si de inversarea matricii *Analizei Nodale Modificate.* In domeniul timp rezultatele numerice au fost comparate cu cele din PSpice. A rezultat o buna concordanta intre valori.

Cuvinte cheie: metoda PEEC, elemente partiale, modelare numerica.

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ABSTRACT. Partial equivalent element circuit (PEEC) is a full wave technique used for solving mixed electromagnetic field and circuit problems in both time and frequency domain. The method is based on Electric Field Integral Equation (EFIE) conversion to partial circuit elements. The errors introduced by simplified formulas and different numerical techniques in partial element calculation and in the inversion of the *Modified Nodal Analysis* (MNA) sparse matrix, in frequency domain, have been evaluated. The results in time domain were compared with the PSpice solver. Good agreement was found.

Keywords: PEEC Method, partial elements, numerical modeling.

1. INTRODUCTION

Electromagnetic behavior analysis is performed by solving numerically or analytically Maxwell equations. Numerical methods are used for general problems, meanwhile the analytical solutions are applicable to more simple ones. Numerical methods give approximate solutions to Maxwell equations but are computationally demanding in terms of time and memory consumption. Classical methods for numerical EM modeling are finite difference methods (FDM), moments method (Mom), finite element method (FEM) and the PEEC method. Instabilities associated with integral equations techniques in time domain are well-known. These problems appear due to the approximations in the numerical techniques used or due to the discretization of the formulation. Basic PEEC method has been extended to non-orthogonal cells and magnetic materials [1]. Various areas of electromagnetic (EM) modeling like transmission lines, radio frequency applications [2], antenna analysis or lightning protection systems have been successfully modeled using PEEC.

2. PEEC METHOD FORMULATION

The starting point in a PEEC model is writing the total electric field in terms of magnetic vector potential and electric scalar potential, $\boldsymbol{A}$ and V, respectively:

$$\boldsymbol{E}^i(\boldsymbol{r},\omega)=\frac{\boldsymbol{J}(\boldsymbol{r},\omega)}{\sigma}+j\omega\boldsymbol{A}(\boldsymbol{r},\omega)+\nabla V(\boldsymbol{r},\omega) \quad (1)$$

where $\boldsymbol{E}^i$ is a potential applied incident electric field. If the observation point, $\boldsymbol{r}$, is on the conductor surface then $\boldsymbol{J}$ is the current density within that conductor having conductivity σ. EFIE is derived from the definitions of electromagnetic potentials at the observation point, $\boldsymbol{r}$:

$$\boldsymbol{A}(\boldsymbol{r},\omega)=\sum_{n=1}^{K}\mu_0\cdot\int_{v_n}G(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{J}(\boldsymbol{r}',\omega)dv_n \quad (2)$$

The summation in (2) accounts for the contribution of all K conductors, μ_0 is permeability of free space and $\boldsymbol{r}'$ is the source point.

When free space is considered, Green function in frequency domain has the form:

$$G(\boldsymbol{r},\boldsymbol{r}') = \frac{e^{-j\omega\tau}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|} \tag{3}$$

τ accounts for the time delay of the retarded electric and magnetic coupling:

$$\tau = \frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c} \tag{4}$$

3. EQUIVALENT CIRCUIT REPRESENTATION

From the definitions of EM potentials the current and charge densities are discretized by using pulse basis functions for the conductors and dielectrics, if necessary. Rearranging equation (1) in terms of weighting functions, a Garlekin type solution results and the weighted volume integrals over the cells can be interpreted as Kirchhoff's voltage law.

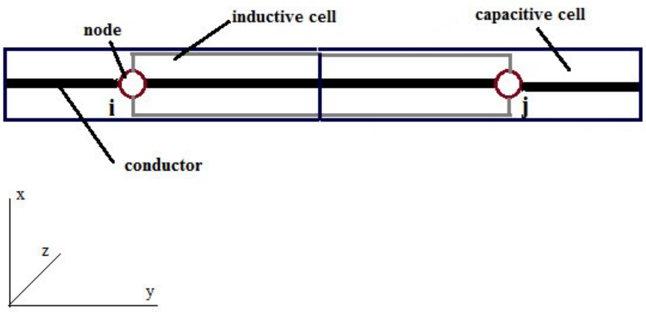

Fig. 1 A discretized conductor in one inductive cell and two capacitive cells (dark blue) shifted half a cell with respect to the inductive cell (light grey)

If PEEC method is applied to a conductor (Figure 1), one inductive cell in the current direction and two capacitive cells were obtained. Partial inductance definition in[3] has been the foundation of PEEC method. The equivalent electric circuit for the cell is given in Figure 2. The partial inductance is defined as:

$$Lp_{k,m} = \frac{\mu_0}{4\pi}\frac{1}{a_k a_m}\int_{v_k}\int_{v_m}\frac{1}{|r_k - r_k|}dv_m dv_k \tag{6}$$

If $k=m$ the integral is taken over the same cell and become the partial self inductance. Partial mutual inductance represents the magnetic field coupling from all other cells of the conductor in terms of voltage sources, V_L in series with the inductor.

$$V_{Lm}(\omega) = e^{-j\omega\tau_L}\sum_{\forall n,k\neq m}\frac{Lp_{km}}{Lp_{mm}}\cdot V_{Ln}(\omega) \tag{7}$$

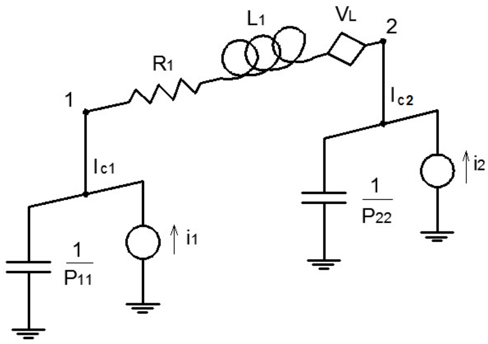

Fig. 2 Corresponding equivalent circuit for the basic PEEC cell

The capacitive coupling is realized by one partial self pseudo-capacitance to each node.

$$C_k = \frac{1}{p_{kk}} \tag{8}$$

To account for the retarded electric field coupling, current controlled sources were defined.

$$I_k(\omega) = e^{-j\omega\tau_c}\sum_{k\neq m}\frac{p_{km}}{p_{kk}}\cdot I_{Cm}(\omega) \tag{9}$$

DC resistances between the nodes are defined as:

$$R_\gamma = \frac{l_\gamma}{\sigma\cdot a_\gamma} \tag{10}$$

where a and l represent the volume cell cross section normal to the current direction γ and the length in current direction, respectively.

Many 3D problems can be reduced to 2D or 1D in order to reduce the complexity and computing time of very large problems. Retardation is important when large domains are discretized. A transmission line cell can be modeled using 1D representation as seen in Figure.1, where current is assumed to flow only in the y direction.

The size of the volume and surface cells, for which the partial inductances and partial potential coefficients are to be calculated, depends most of the selected model frequency range and the shape of the conductors. Due to the available analytical formulas rectangular cells are most frequently used. A few approaches based on the filament formula for non-orthogonal cells discretization and cylindrical elements have been proposed [4]. Partial inductances matrices model interactions between a very large number of objects. They are time and memory consuming. A method to reduce these problems was proposed in [5] using a QR decomposition algorithm.

4. PARTIAL ELEMENT COMPUTATIONS

Poor meshing of geometry and large aspect ratios can cause inaccuracies and wrong values in partial elements calculation. A proper choice of a cell dimension is given in [6]. At high frequencies the current is flowing only on the surface of the metals. In most cases they are considered as perfect conductors.

In Figure 3 a strip of copper having a length of 20 mm and a cross section of 0.8 x 2 mm is shown. Current flow is considered only in the y direction.

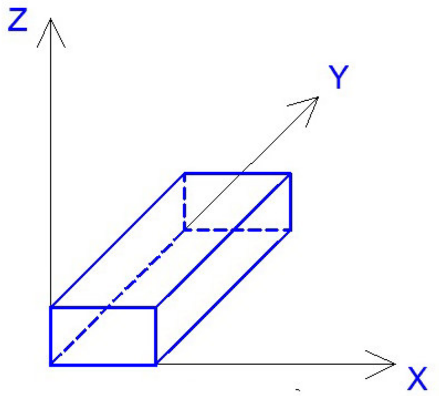

Fig.3 Copper strip model

A correct representation of the real waveforms, implies that the dimension of a cell should not exceed one twentieh of the shortest wavelentgth. A few discretizations have been investigated. First, four volume cells in *y* direction were chosen for the numerical analysis. The upper frequency in this case is 3 GHz. Partial inductances were calculated using analytical formulas and Simpson's integration technique. The differences have been small and an error less than 10^{-7} has been obtained. The capacitive coupling between the nodes is given by the short circuit capacitance matrix, $\boldsymbol{C_S}$**,** as detailed in [6]. When the coefficients of potential were correctly determined, $\boldsymbol{C_S}=\boldsymbol{P}^{-1}$ is positive semidefinite (all its off-diagonals have negative values). Formula (15) in [7] was used for computing the self and partial coefficients of potential. The results were verified with Simpson's formula and a relative error of 10^{-8} was found.

To simplify the calculations, in further analysis the same copper strip is divided in two volume and three surface cells. A volume cell has a length of 15 mm and the cross section of 0.8 x 2 mm. In this case the upper frequency was 1GHz. This two cell model is used in the next section when MNA matrix is investigated.

Partial inductance matrix was calculated in the same way as the case of five cells and is given by:

$$L = \begin{bmatrix} 8.75484 & 2.0075 \\ 2.0075 & 8.75484 \end{bmatrix} nH$$

Formula (15) from [3] have been used for the partial self coefficients of potential and satisfactory values resulted. This values have been verified with Simpson's formula and the results were found to be in good agreement. Zero thickness formula given by Ruehli in [3], accordingly adapted for the coefficients of potential, did not follow the same results. The correct version of $\boldsymbol{C_S}$ matrix is given below:

$$\boldsymbol{C_S} = \begin{bmatrix} 0.168773 & -0.046533 & -0.002857 \\ -0.046533 & 0.282732 & -0.046533 \\ -0.002857 & -0.046533 & 0.168773 \end{bmatrix} [pF]$$

5. MATRIX FORMULATION

Admittance Method and *Modified Nodal Analysis* (MNA) were used for the solution of PEEC in time and frequency domain. The effect of the partial elements accuracy calculation on the matrix is very important. Frequency domain admittance matrix is obtained from system equation given in (11):

$$\begin{bmatrix} -\boldsymbol{A} & -(\boldsymbol{R}+j\omega\boldsymbol{L}) \\ j\omega\boldsymbol{P}^{-1}+\boldsymbol{Y}_L & \cdot -\boldsymbol{A}^T \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{V}_n \\ \boldsymbol{I}_b \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_S \\ \boldsymbol{I}_S \end{bmatrix} \qquad (11)$$

where: $\boldsymbol{A}$ - connectivity matrix
$\boldsymbol{Z} = \boldsymbol{R} + j\omega\boldsymbol{L}$ - impedance matrix
$\boldsymbol{P}$ - partial coefficient of potential matrix
$\boldsymbol{V_n}$ - capacitive node potentials vector
$\boldsymbol{I_b}$ - branch currents vector
$\boldsymbol{Y_L}$ - lumped components admittance matrix
$\boldsymbol{V_s}$, $\boldsymbol{I_s}$ – voltage and current source excitations vectors

The admittance matrix for the PEEC model is then:

$$\boldsymbol{Y} = \left(\boldsymbol{A}^T(\boldsymbol{R}+j\omega\boldsymbol{L})^{-1}\right)\boldsymbol{A} + j\omega\boldsymbol{P}^{-1} + \boldsymbol{Y}_L \qquad (12)$$

MNA formulation is derived according to [8] from the Admittance matrix using the two following matrix properties:

$$j\omega\boldsymbol{P}^{-1} = j\omega\boldsymbol{F}\boldsymbol{S}^{-1} \qquad (13)$$

$$j\omega\boldsymbol{F}\boldsymbol{S}^{-1} = j\omega\boldsymbol{S}^{-1^T}\boldsymbol{F} \qquad (14)$$

where $S = \frac{P_{i,j}}{P_{j,j}}, i \neq j$ and $F = \frac{1}{P_{j,j}}$

5.1 NUMERICAL METHODS APPLIED FOR THE MATRIX SOLUTION

The equivalent circuit for the two cell conductor chosen in the previous section can be noticed in Figure 4. A sinusoidal current source, I_S, was applied at node 1. MNA matrix is sparse and therefore it has a large condition number for high frequencies. By increasing the frequency spectrum, the condition number is decreasing. This dependency is captured in Figure 5.

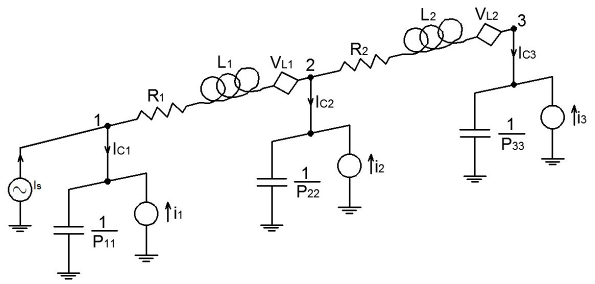

Fig.4 Equivalent circuit for two PEEC

In DC, the PEEC problem becomes a resistive circuit. For a correct representation of the electromagnetic problem, at high frequencies, resistors, capacitors and inductors must be included in the analysis. Starting from 1 MHz the influence of ***R*** matrix is very small and the circuit can be reduced to (L,P,τ)PEEC model. In this paper, for an easy comparison with Spice results in time domain, the delay is also neglected. A frequency of 100 MHz was chosen for the computations.

The matrix system in (11) can be rewritten as a linear system of equations, $C*x=b$. The inversion of the sparse matrix, ***C,*** can introduce errors in the solution.

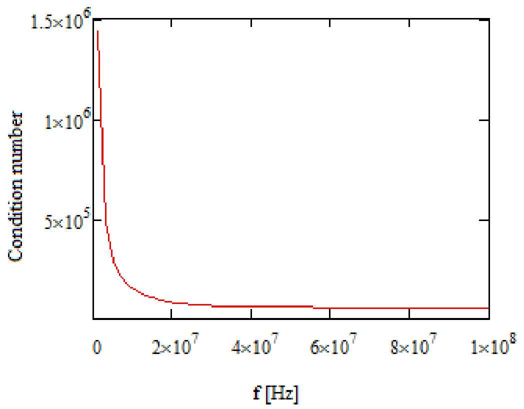

Fig.5 Condition number versus frequency MNA matrix

In order to evoid that, different inversion methods are proposed.

At first, a simple *LU decomposition* algorithm was applied to the ***C*** matrix. The condition number did not improve and an absolute error of 10.1 was found with classic inversion. Next, a linear system was assumed and a Gauss triangularization method, [9] with a correction factor was considered. An significant difference in the solution resulted.

Finally, the PEEC model was considered as a non-linear system of equations. Simple *Newton* method applied to the system proved not to be convergent and therefore the results are not presented. A second analysis involved the *Conjugate Gradient* (CG) technique. The rate of convergence for the potential in node three when CG method was used is presented in Figure 6. A predefined function for matrix inversion in Mathcad, called *lsolve* was also used as a comparison term.

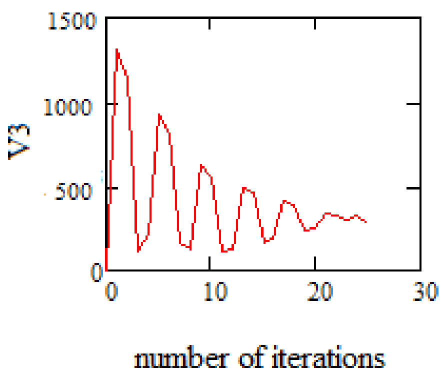

Fig.6 Convergence process for Nonlinear Conjugate Gradient Method

5.2 RESULTS

In order to quantify the above discussed methods, the relative errors of the solutions are summarized in Table 1.

Table 1.

Relative errors in the numerical methods

Method	LU decomposition	Gauss	*lsolve*	CG
relative error	$1.34273 \cdot 10^{-4}$	1.36	$1.727 \cdot 10^{-6}$	$5.71 \cdot 10^{-6}$

It can be noticed that the Gauss method was the less accurate method for solving of MNA matrix. The *Conjugate Gradient* method proved to be the most convergent and the most accurate method when a linear system of equation is considered.

Time domain equations can obtained by applying an inverse Laplace transformation or directly by solving the *Delay Differential Equations* system. In this paper the first method was considered. The current through the first inductor obtained in Mathcad was compared with the PSpice solution. A good agreement has been found as it can be seen in Figure 7.

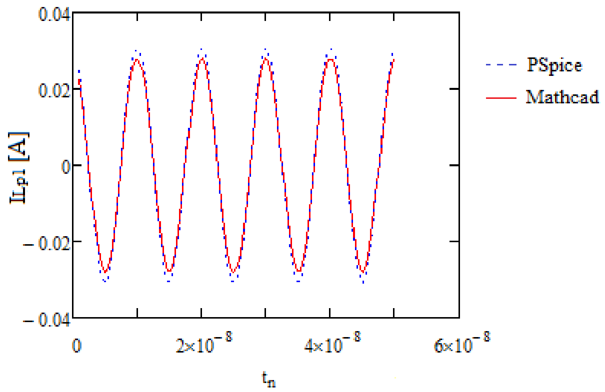

Fig.7 Comparison of the current through Lp11

6. CONCLUSIONS

The errors introduced by simplified formulas and different numerical techniques in partial element calculation have been evaluated. For the inversion of the sparse MNA matrix in frequency domain, a few numerical methods were applied. *Conjugate Gradient* method proved to be more convergent and also the best fit for the system of non-linear equations.

Time domain results presented in comparison with PSpice solver validated the proposed model.

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