SYSTEMIC APPROACH TO DYNAMICS OF THE HYBRID ELECTRIC VEHICLES

Eng. **George-Cosmin BIVOL**
Military Technical Academy from Bucharest.

REZUMAT: Lucrarea subliniază principalele avantaje ale vehiculelor electrice hibride ca o soluţie tehnică actuală precum şi o soluţie de perspectivă la folosirea combustibililor fosili în sistemul de transport auto. Lucrarea prezintă de asemenea principalele caracteristici ale diferitelor soluţii constructive de vehicule electrice hibride. De asemenea sunt prezentate principalele elemente care definesc configuraţia vehiculelor electrice hibride, modelul matematic utilizat, precum si descrierile matematice ale componentei mecanice şi electrice. Modelul matematic propus se aplica folosind MATLAB.

Cuvinte cheie: auto, vehicul electric hibrid (VEH), dinamica vehiculului, sistem de propulsie hibrid.

ABSTRACT: The paper highlights the main advantages of the hybrid electric vehicles, as a present technical solution and a perspective solution to the use of fossil fuels in the auto transportation system. The paper also presents the main particularities for different configuration solutions of the electric hybrid vehicles. The main elements that define the configuration architecture of the electric hybrid vehicles are highlighted as well as the used mathematical model and the Mathematical descriptions of the mechanical part and of the electric part. We apply the proposed mathematical model using Matlab.

Keywords: automotive, hybrid electric vehicle (HEV), vehicle's dynamics, hybrid powertrain.

1. INTRODUCTION

New solutions were developed in automotive engineering due to the fact that the number of vehicles is fast increasing, new and more drastic pollution emissions standards are imposed, the necessity to diminish the consumption of oil…all these led to the development and the emerge of new energetic solutions in the automotive industry; speaking of the solution which is the forefront of the industry we look at the hybrid electric vehicles which combine electro-mechanical and electro-chemical systems of great complexity [4].

2. VEHICLE'S DYNAMICS MODELING

In figure 1, the series configuration of a hybrid electric vehicle is presented, where there is no mechanical connection between the thermal engine and the wheels; in the figure we used the following notations: F – fuel; TE – thermal engine; EG – electric generator; EM – electric motor; GB – gear box; BP – battery pack; EI – electronic interface.

The thermal engine must ensure the energy conversion… that of the fuel's into mechanical energy. Further on, the electric generator transforms the mechanical energy into electricity, thus allowing the battery to be recharged so it can power up the electric motor which ensures the vehicle's movement.

As we can see, there are several energy conversions and that is why the efficiency of a system like this one is very low [3].

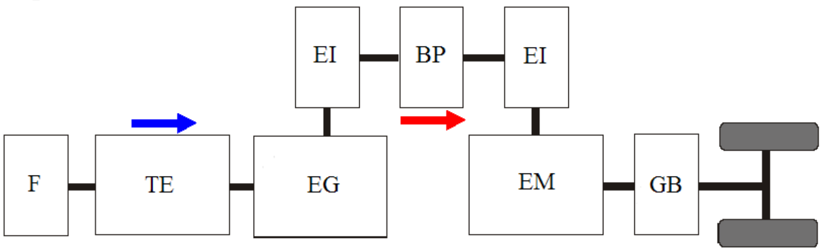

Fig. 1. The series configuration of a hybrid electric vehicle

The most simple series configuration example is represented by the pure electric vehicle, which obviously has the advantage that has no polluting emissions of any kind. So, a clear advantage consists of the fact that it ensures low polluting emissions because in order to move the vehicle, we act on it electrically.

Another advantage consists in the fact that the electric and internal combustion engines can be individually mounted in any convenient positions; this ensures the advantage of distributing the weigh evenly on the axels or as we consider to be fit.

The name of series configuration comes from the fact that the power flow of the two power sources are positioned in series. The mathematical expression described in the first relation (1) represents the differential equation which describes the hybrid vehicle's dynamic behavior with series topology. It is a result following the application of the d'Alambert principle, similar as the classical vehicle [1].

$$\frac{\mathrm{d}v_s}{\mathrm{d}t} = \frac{1}{r_r m_a}\left(M_{EM} i_t - M_r - M_b\right) \quad (1)$$

where: v_s – the speed with which the vehicle is moving (the s letter comes from the word series), r_r – rolling radius, m_a – vehicle's mass, M_b – braking torque, M_r – required torque needed to overcome the resisting forces when moving forward, M_{EM} – the torque generated by the electric motor, i_t – transmission ratio.

We can see in figure 2 the sketch for parallel configuration of an electric hybrid vehicle, on which we can see the fact that there is a mechanical connection between the internal combustion engine and the electric one. The name comes from the fact that the power flows if the two sources are parallel. The solution has a better efficiency than the series configuration because there are fewer energy conversions [3].

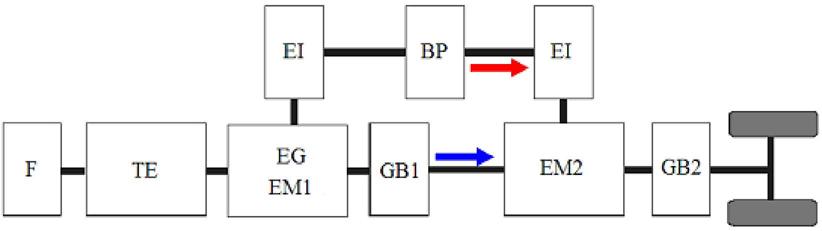

Fig. 2. The sketch for parallel configuration of an electric hybrid vehicle

We can see from this sketch that the vehicle can move in four distinct ways: relying only on it's electric power; relying only in it's internal combustion engine and at the same time the combustion engine is charging the battery that powers the electric motor; and at the same time the two engines can power up the car at the same time.

This is where it derived the version of dual transmission functioning method, which exists on most hybrid electric vehicles.

The second mathematical expression represents the differential equation which describes the parallel hybrid vehicle's dynamics. It also results from applying the d'Alambert principle.

$$\frac{\mathrm{d}v_p}{\mathrm{d}t} = \frac{1}{r_r m_a}\left[\left(M_{TE} + M_{EM1}\right) i_{t_1} i_{t_2} + M_{EM2} i_{t_2} - M_r - M_b\right] \quad (2)$$

The used notations are similar with the previous ones, M_{TE} being the torque developed by the thermal engine.

Figure 3 presents a sequential configuration of a hybrid electric vehicle. It is a combination of both parallel and series configurations. Mounting a clutch mechanism C on the lower branch from the parallel configuration, it is possible to change between parallel into series configuration.

Engaging or disengaging the clutch, thus changing from one sequence onto another, is being done by the on board computer in such a way that the power losses are minimized. From this reason this solution it is named combined configuration; thus we can say that the vehicle's dynamics is described either by the firs mathematical relation (1) either by the second one (2) depending on the sequence that the vehicle finds its self.

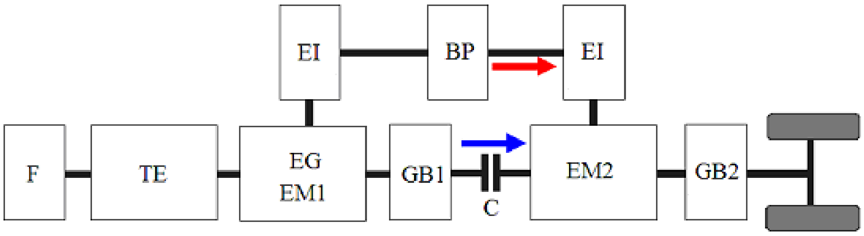

Fig. 3. Sequential configuration of a hybrid electric vehicle (a combination of both parallel and series configurations).

Figure 4 presents the configuration with "power (torque) split", on which a planetary mechanism PM is present which connects the internal combustion engine with two electric motors. The electric motor EM2 is a traction engine and is connected to the ring of the mechanism, as you can see in figure 4. The electric motor EM1 is a generator and is connected to the sun. Finally, the internal combustion engine is connected to the planet carrier thus offering the possibility of turning it off and moving the vehicle based only on the power generated by the EM2 [2, 3].

Most of the cases the vehicle functions when there is a power flow coming from the internal combustion engine, like the parallel configuration, but also from the electric motor EM2, as the case for series configuration. The proportion between these two power flows depends on the revolution's speed. The name of this solution comes from the fact that the planetary geared mechanism ensures a power (torque) split; at the planetary transmission the following notations were made: C – carrier; S – sun; R – ring.

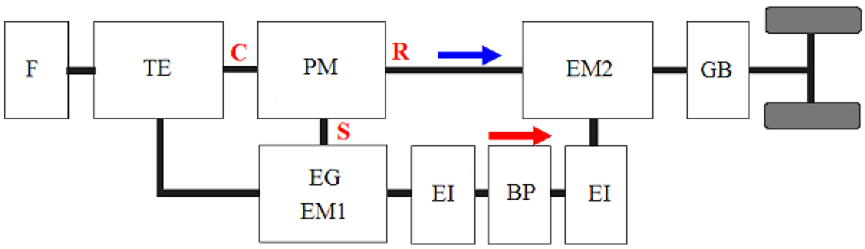

Fig. 4. The configuration with "power (torque) split".

Based on figure 5, the mechanism's dynamics and kinematics relations are established.

So, the mechanism's kinematics is described by the third relation (3):

$$\omega_R r_R + \omega_S r_S = \omega_C (r_R + r_S) \tag{3}$$

And it's dynamics is described by the differential equations no. 4:

$$J_R \frac{\mathrm{d}\omega_R}{\mathrm{d}t} = Fr_R - M_R;\ J_C \frac{\mathrm{d}\omega_C}{\mathrm{d}t} = M_C - Fr_R - Fr_S;\ J_S \frac{\mathrm{d}\omega_S}{\mathrm{d}t} = Fr_S - M_S \tag{4}$$

Based on figure 5b we also have the following:

$$M_C - M_R - M_S = 0;\ M_C\omega_C - M_R\omega_R - M_S\omega_S = 0 \tag{5}$$

From (3) and (5) we get:

$$M_R = \frac{\omega_C - \omega_S}{\omega_R - \omega_S} M_C;\ M_S = \frac{\omega_R - \omega_C}{\omega_R - \omega_S} M_C \tag{6}$$

Relations number (6) shows how the mechanism splits the torque. It is from here that the name of "torque split configuration". These expressions give us the torque values on the sun gear and on the ring gear considering the torque of the planet carrier; according to figure 5a, the expressions described in relations number (6) offers the torque values for the electric motors depending on the torque provided by the internal combustion engine.

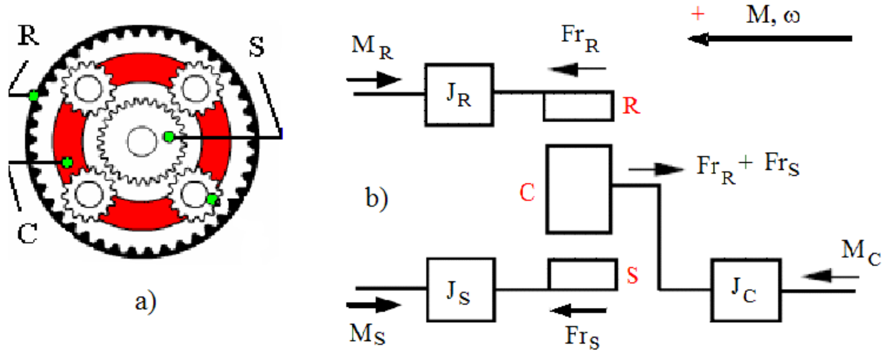

Fig. 5. The sketch for dynamics and kinematics relations

In order to establish the vehicle's dynamic equations, we use the sketch from figure 6 (MG: motor-generator) where, just as the case of figure 5b, we can see the components, the forces, torques and specific angular rotational speeds [3, 5].

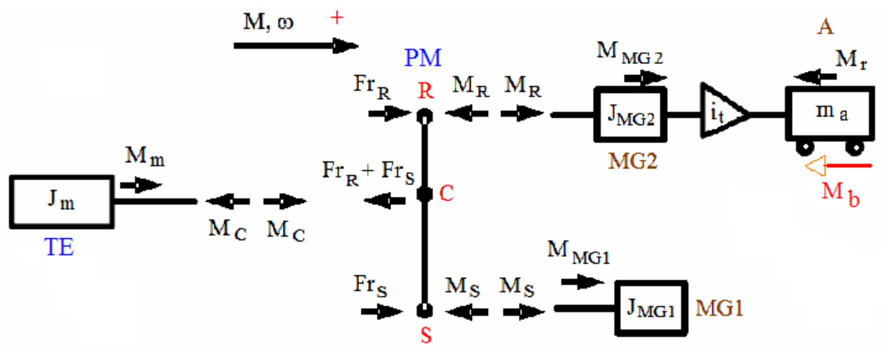

Fig. 6. The sketch for dynamics relations

So the moving law of the vehicle is being deducted from the relations described in the mathematical expressions number (7):

$$\left(J_{MG2}+\frac{m_a r_r^2}{i_t^2}\right)\frac{\mathrm{d}\omega_R}{\mathrm{d}t}=M_R+M_{MG2}-\frac{1}{i_t}\left(M_b+M_r\right);\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{r_r}{i_t}\frac{\mathrm{d}\omega_R}{\mathrm{d}t} \quad (7)$$

…as well as from the mathematical expressions number (4); we finally achieve equation number (8), which describes the vehicle's dynamics:

$$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{r_r}{J_R i_t^2+J_{MG2}i_t^2+m_a r_r^2}\left(Fr_R i_t+M_{MG2}i_t-M_r-M_b\right) \quad (8)$$

Just as the classic dynamic equation [1], in the given relations, inertial torques intervene, constructional elements of the vehicle, external torques; in this case inner torques and forces intervene, for example those from the electric motors and the inner F forces of the planetary gear mechanism.

For a hybrid electric vehicle, on the given expressions, relation number (9) is added which gives us the time variation of battery charging degree, know in the technical literature as SOC – State of Charge [5].

$$\frac{\mathrm{d}SOC}{\mathrm{d}t}=-\frac{V_b-\sqrt{V_b^2-4\left(M_{MG1}\omega_{MG1}\eta_{MG1}^k\eta_{c1}^k+M_{MG2}\omega_{MG2}\eta_{MG2}^k\eta_{c2}^k\right)}}{2R_bQ_m} \quad (9)$$

Figure 7 presents the specific battery parameters, which are found in the mathematical expression number (9); besides that we have battery power P_b and maximum battery capacity Q_m.

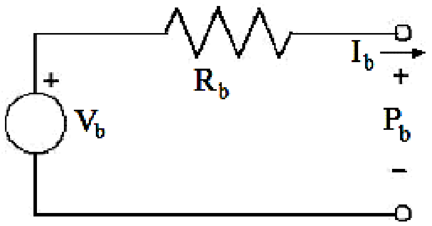

Fig. 7. Specific battery parameters

The mathematical expression (9) represents the power supplied to the MG:

$$P_{MG} = M_{MG}\omega_{MG}\eta_{MG}^{k} \tag{10}$$

where: M_{MG} and ω_{MG} represent the torque and respectively the rotational speed. If the velocity and torque of the MG have the same signs (i.e., either positive or both negative), the power is positive, which means that the motor is consuming energy. Similarly, if the signs for velocity and torque differ (i.e., one positive, the other one negative), the MG is generating energy. In the mathematical expression (10), k is the sign for the power flow direction. When MG is consuming energy, k=-1 and the power flows inward, from the battery to the MG. When the MG is generating energy, k=1 and the power flows outward, from the MG to the battery.

The efficiency η_{MG} accounts for the energy lost from both the MG (MG1 and MG2) and other accessories, including the power converter and controller.

The mathematical expression (9), η_c represents the power converter efficiency, and k is the sign of the power flow direction as explained above. When the battery is discharged, k=-1 and the power flows away from the battery. When the battery is charged, k=1 and the power flows to the battery.

3. VEHICLE'S DYNAMICS SIMULATION

In order to model and simulate the hybrid electric vehicle's movement we can find various software to do just that. For example, figure 8 presents the modeling scheme of Matlab which offers the possibility to model an electric hybrid vehicle dynamic behavior, using real data gathered from the Toyota Prius [6].

As we can see, the scheme includes 5 subsystems: vehicle 1, thermal engine 2; electric subsystem 3, planetary mechanism 4 and energy management system 5 which is the actual system that has the control for the vehicle's movement.

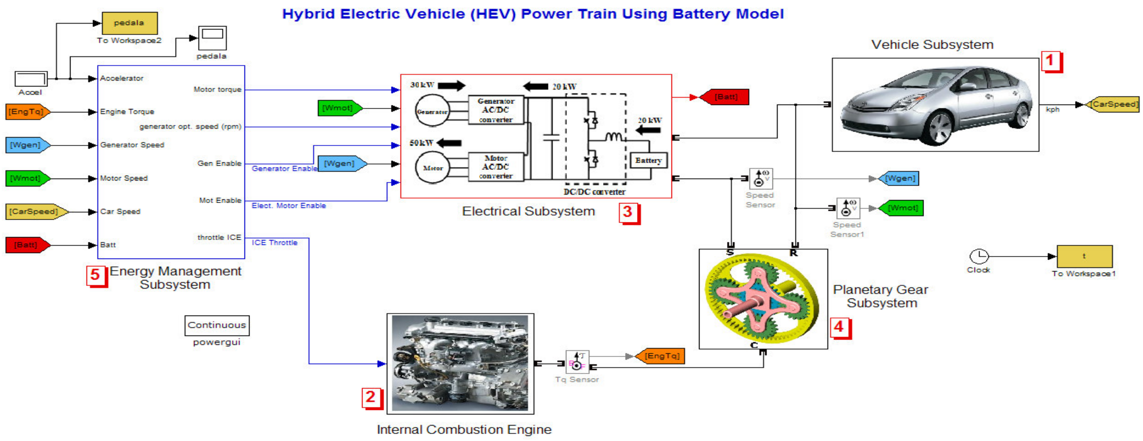

Fig. 8. The modelling scheme of Matlab

Each of previously mentioned subsystems is situated at different stages and depths, other being detailed, as we can see in figure 9 the electric subsystem.

As we can see here, the vehicle has amongst its components, high voltage battery pack 6, direct current electric converter 7, electric machine MG1 of 30 kW which most of the times functions as a generator 8 and the electric machine MG2 of 50kW which most of the times functions as an electric motor 9.

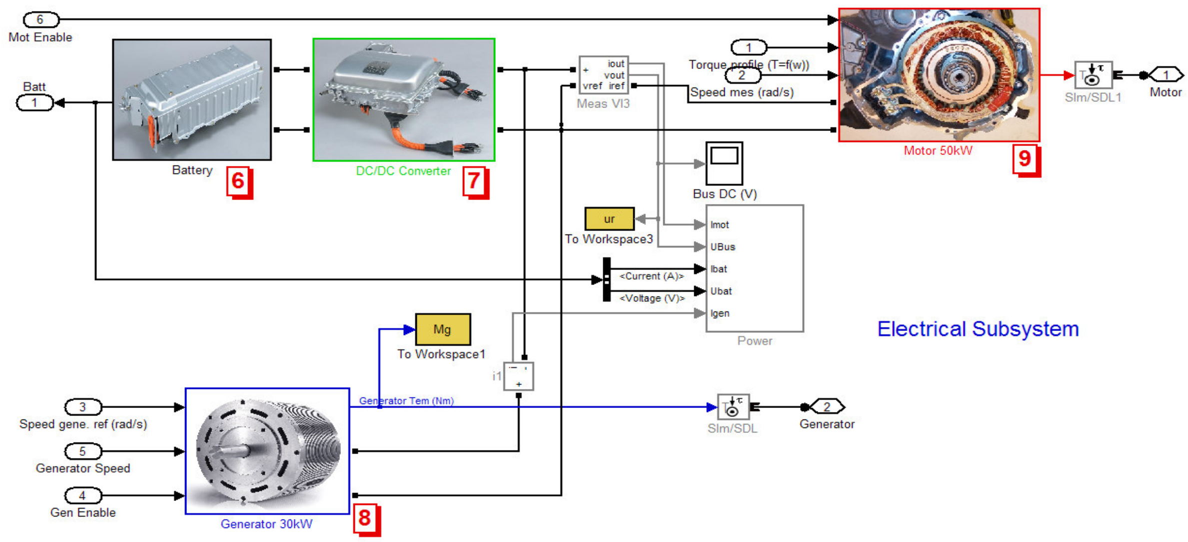

Fig. 9. The electric subsystem

The scheme ensures the calculus of a large volume of functional parameters: moving speed; rotational speeds, torque and output power of thermal engine, electric machines and elements of the planetary mechanism; forces, torques and power on the wheel; battery power, network voltage and current intensity etc. Just the same the model offers the possibility for a systemic approach of dynamics because it takes into consideration a certain terrain, the driver's inputs and the vehicle's characteristics.

Figure 10 presents some functional parameters which were calculated with the help of the presented Matlab model scheme. The vehicle's movement simulation includes a complete cycle: starting, accelerating, and charging the high voltage battery pack and energy recuperating braking.

The graphs from figure 10 were intentionally presented here in order to explain the hybrid electric vehicle dynamic phenomenology.

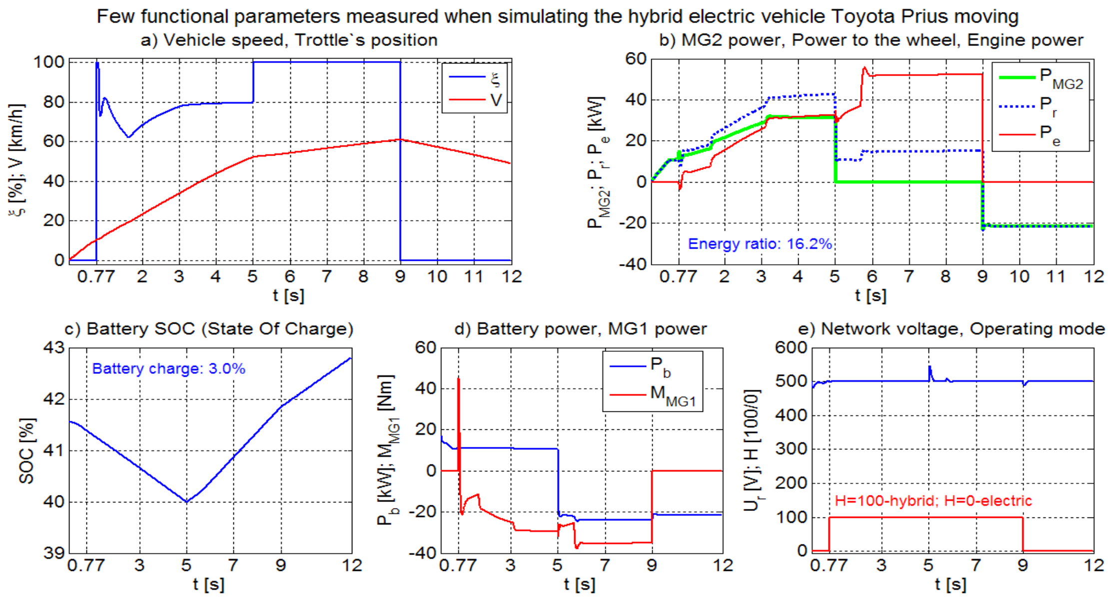

Fig. 10. The hybrid electric vehicle dynamics

As we can see in figure 10a, during the movement the driver's input was taken into account, reason for which the gas pedal acting was simulated, thus the throttle's position ξ. Also we can see that when the vehicle starts its initial speed *V* is zero, so we are in the situation when the vehicle starts from a dead stop.

After pressing the gas pedal at time ***t*=0 seconds** and just as long the vehicle's speed is under 10 km/h, a condition introduced by the control algorithm, the vehicle moves using only electric power, as we can see in figure 10e where H=0. Figure 10b shows that the wheel power is equal with the power provide by the electric machine MG2 which functions as an electric motor because it has a positive output reading, and in figure 10d the battery power is dropping. So at the beginning, the vehicle is propelled with the help of the electric motor MG2 which is powered by the battery, and the thermal engine is still turned off, fact which can be seen in figure 10b where it's power is null.

At the time ***t*=0.77 seconds**, we have reached the speed of 10 km/h and therefore hybrid mode is engaged, as we can see from figure 10e where H=100. At this point the thermal engine is started, as we can see from figure 10b where it's power becomes suddenly negative, because it consumes energy to be started; this thermal engine ignition is also confirmed by MG1's torque which has a sudden increase to a positive value as we can see in figure 10d, thus MG1 functions as an electric motor ensuring the thermal engine's ignition. After the thermal engine is turned on, the vehicle is being propelled with this engine's output power and also the power provided by the MG2 electric motor, both their outputs being positive as in figure 10b; MG2 powers itself from the battery, and the battery's state of charge is dropping, as we can see from figure 10c. But MG2 also consumes power provided by the MG1, this last electric machine functions as an electric generator because it's torque is negative as we can see from figure 10d.

At ***t*= 5 seconds** the battery's state of charge reached the imposed limit of 40% imposed by the control algorithm as we can see in figure 10c; following that moment the battery must be recharged. Thus, MG2 electric motor's output power becomes null, as we can see in figure 10b so it doesn't propel the vehicle forward any longer. A part from the thermal engine's output power is used to move the vehicle and the other part powers up the MG1 generator which charges the battery.

This power split can be observed in figure 10b, where the thermal engine's output power becomes greater than the power measured at the wheel; the fact that MG1 is working as a generator can be seen also in figure 10d where its torque is negative. The fact that the battery is charging can be seen in figure 10c where its state of charge is increasing and also in figure 10d where the battery power is negative.

Starting with **t=9 seconds** and until the end, energy recuperating braking is being simulated. To this purpose the driver completely releases the gas pedal, thus the throttle's position becomes null as in figure 10a. Starting with this point, the thermal engine is turned off, it's output power becoming zero as in figure 10b and therefore a pure electric mode is engaged as we can see in figure 10e where H=0.

MG2 electric machine becomes a generator, it's output power becoming negative as in figure 10b and equal in value with the wheel measured power, which also is negative and which means that the vehicle is propelling the wheels due to its inertia and not vice versa. MG2 generator is powered by the vehicle's wheels and therefore transforms the movement's mechanical energy into electric energy used to charge the battery as we can see in figure 10c where battery's SOC is increasing and from figure 10d where the battery's power is negative. At the same time *t*=9 seconds, MG1 electric generator's torque becomes null as in figure 10d, so it's power is null which means that it does no longer charge the battery as it used to until now.

Other observations can be made regarding the hybrid vehicle's electric subsystem. For example in figure 10e we can see that the network voltage varies around the value of 500 V, with small oscillations at those specific moments that we have mentioned earlier. From figure 10b, we can see that through energy recovering braking throughout *t*=9÷12 seconds, 16.2% of the consumed energy was recuperated from the start of the braking, meaning we recupereted the energy consumed by the vehicle since the begining until it reached second 9. From figure 10c we can see that at the end of the vehicle's movement, the battery has a state of sharge higher with 3% compared whith it's SOC at the begining of the simulation. Also we can see in figure 10a that throughout the simulation, the vehicle's speed increased from it's initial value of zero until 60.7 km/h which was reached at *t*=9 seconds, than dropping to the final value of 48.7 km/h due to the fact that the driver didn't act on the gas peddal.

Finally, we have to mention another aspect. In conditions that the driver completely pressed the gas pedal at the beginning and didn't completely released before 9 seconds passed, we can see that the throttle position didn't respect what the driver was commanding. We can see in figure 10a that during t=0.77÷5 seconds the throttle's position recorded values below its maximum. This aspect confirms the intervention of electronic control which adapts to the moving conditions and to the restrictions imposed by the adapted control algorithms.

4. CONCLUSIONS

In a similar manner the mathematical model can be established for any hybrid electric vehicle configuration, let's say in the case of a transmission with two working ways (dual-mode power split power train).

BIBLIOGRAPHY

[1] **Copae I., Lespezeanu I., Cazacu C.,** *Dinamica autovehiculelor*. Ed. ERICOM, București, 2006

[2] **Jinming Liu** – *Modeling, Configuration and Control Optimization of Power-Split Hybrid Vehicles*, University of Michigan, 2007

[3] **Jonasson Karin** – *Analysing Hybrid Drive System Topologies*, Lund University, 2002

[4] **Livinț Gh., Gaiginschi R.,** ș.a. – *Vehicule electrice hibride*, Ed. Venus, Iași, 2006

[5] **Zhou Y.L.**– *Modeling and Simulation of Hybrid Electric Vehicles*, University of Victoria, 2007

[6] *** MathWorks, Simulink: Hybrid Electric Vehicle (HEV) Power Train Using Battery Model: "power_HEV_powertrain.mdl"

About the author

Eng. **George BIVOL,** PhD
Military Technical Academy from Bucharest.
email: geobi78@yahoo.com

He graduated the Military Technical Academy from Bucharest in 2002 as mechanical engineer. He worked as logistic officer in charge with the vehicles of a Logistic Battalion for two years and as advisor for an Afghan logistic battalion staff for six months. At the present he works as legal adviser and he is PhD in systemic study of vehicle's dynamics.