VOLTAGE CONTROLLED OPTIMAL ELECTRICAL DRIVE SYSTEM WITH VARIABLE TORQUE

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REZUMAT. Se prezintă o problemă de control optimal din punct de vedere energetic al unui sistem de actionare electrică. Se are în vedere o variație treaptă a cuplului rezistent în intervalul de optimizare. Criteriul de performanță ține cont doar pierderile Joule, deoarece acestea depăşesc în mod semnificativ alte pierderi în perioada tranzitorie. Lucrarea se referă la o structură cu control în tensiune a sistemului de acționare. Studiul se prezintă în domeniu de timp discret.

Cuvinte cheie: actionare electrică, cuplu rezistent variabil, control optimal liniar pătratic discret.

ABSTRACT. The optimal control from the energetic point of view of the transient state of electrical drive systems is presented. A step variation of the load torque in the optimization interval is considered. The performance criteria consider only Joule losses, since they significantly overcome other ones in the transient state. The paper refers to a structure with voltage control of the drive. The study is performed in discrete time domain.

Keywords: electrical drives, variable load torque, discrete linear quadratic optimal control.

1. INTRODUCTION

The optimal control [1], [2] of the electrical drive systems represents an important way for energy saving and there are numerous studies dedicated to this problem, for different types of motors, control strategies, criteria, or used methods (for instance, we mention [3], [4], [5], [6], [7], but many other papers can be indicated). Moreover, the optimization is appreciated as a main direction of the developing of the electrical drive systems in the future [8].

It should be also noticed that the optimal control is useful not only for energy saving, but in many cases offers the possibility to reduce the motor rated power and therefore the weight and volume. Indeed, the motor rated power is chosen from heat consideration and the optimal control leads just to a diminished Joule losses. Moreover, the decrease of the weight of certain sub-ensemble leads in certain applications to the decrease of the energy consumption of the all plant.

The number of applications is nowadays small in spite of the mentioned advantages. An unjustified reluctance may be the complexity of the algorithms, so that simple implementable solutions for optimal control problems are necessary. Following certain previous results of the authors, the present paper studies a case of an optimal drive system for a variable load torque.

The control of an electrical drive must be chosen so that to obtain a small energy losses and an acceptable behaviour of the system. The demands for different applications are not the same and certain differences in the adopted optimization criteria can occur. For instance, the goal for the steady state operation is the minimization of the global power losses. The criterion for the transient period refers to the energy (not power) losses and it usually take into account only the cooper losses, since they significantly overcame all other losses, because of very great values of currents.

Different other considerations for various applications of the optimal control of the electrical drives are indicated below:

- The structure of the optimal drive system may be with current or with voltage control, depending on the type of the power electronic converter. The first one can be easier implemented, but different other considerations can influence the choice of the structure.
- It is possible in many cases to adopt one or two control variables. The last variant is useful for optimal control if the drive operates in many situations with a reduced load. However, the most used case refers to a system with one control variable.
- In order to ensure small energy losses and an acceptable behaviour, the optimization criterion contains different components. Also, the formulated optimal problem can be with free or fixed final time, or with free or fixed final states variables.

The authors have presented in some previous paper different variants for optimal control problems for transient period of the electrical drives. For instance, the voltage control for the drives with constant load torque was studied in [6] and [9]. The current control variant was presented in [10] for constant load torque and in [11] for a variable one.

The paper deals with optimal control of a voltage controlled drive system with variable load torque. Of course, a general algorithm for a variable torque can be established, but the implementation is significantly simpler for a constant load torque. Therefore, a suboptimal solution can be obtained if the variation of the load torque in the transient period is approximated with a step function. Such situations are frequently met in the electrical drive systems, when the no-load or small load torque is succeeded by a great one. Examples for such operation are the rolling mills, or cutting processes.

The studied problem refers to one with free end point and with fixed final time and it is performed in discrete time domain.

Only the case of one control variable is considered. The results are valid for different motor types, because the mathematical model is the same with adequate assumptions.

2. PROBLEM FORMULATION

An electrical drive system with the state equation

$$\dot{x}(t) = A_c x(t) + B_c u(t) + W_c m(t), \quad x(0) = x^0, \quad (1)$$

is considered, where $x(t) = [\omega(t) \;\; i(t)]'$ is the state vector (prime denotes the transposition), $m(t)$ is the disturbance variable (the load torque) and $\omega(t)$ is the rotor speed. If a brushed d.c. motor is used, i and u are the rotor current and voltage, respectively (the flux is constant). For synchronous and asynchronous motors, i and u correspond to the q current and voltage components and similar equations may be adopted with adequate assumptions (mainly, the i_d component is constant or very small). The matrices in (1) are in the form

$$A_c = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \; B_c = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}, \; W_c = \begin{bmatrix} w_1 \\ 0 \end{bmatrix}, \quad (2)$$

depending on the constant parameters of the drive system.

The aim of the optimal control of the electrical drive system is to obtain small energy consumption and steady state and transient errors as smaller as possible for the state variables.

Since the optimal control will be computed in a discrete manner, an adequate discrete time model for (1) must be obtained. The corresponding matrices in this case are computed as

$$A = e^{A_c T}, \; B = \int_0^T e^{A_c T} dt B_c, \;\; W = \int_0^T e^{A_c T} dt W_c, \quad (3)$$

where T is the sampling period. The discrete model of the drive system is

$$x(k+1) = Ax(k) + Bu(k) + Wm(k). \quad (4)$$

The discrete state was denoted $x(k) = x(kT)$, $k \in Z$ and similar notations were used for all other variables.

In order to achieve the mentioned goals, it is adopted the performance index

$$\begin{aligned} J = & \frac{1}{2} s_1 [\omega(k_f) - \omega_d]^2 + \\ & + \frac{1}{2} \sum_{k=0}^{k_f - 1} q_1 [\omega(k) - \omega_d]^2 + q_2 i^2(k) + p_0 u^2(k), \end{aligned} \quad (5)$$

which penalizes great values of the control variable and the transient and steady state errors (ω_d is the desired speed). Also, the cooper energy losses are penalized by the second term in sum. In (5), k_f corresponds to the final time and is fixed. The performance index (5) can be written in the general form

$$\begin{aligned} J = & \frac{1}{2} [x_d - x(k_f)]' S [x_d - x(k_f)] + \\ & + \frac{1}{2} \sum_{k=0}^{k_f - 1} \left\{ [x(k) - x_d]' Q [x(k) - x_d] + u^T(k) P u(k) \right\}, \end{aligned} \quad (6)$$

with

$$S = diag(s_1, 0) \geq 0, \quad Q = diag(q_1, q_2) \geq 0, \quad P = p_0 > 0. \; (7)$$

and $x_d = [\omega_d \;\; 0]'$.

The above-formulated problem is a discrete linear-quadratic (LQ) optimal problem with finite final time. The solution is well known [1], [2] and it gives the optimal control as a state feedback:

$$u(k) = \tilde{F}(k) x(k) + \tilde{u}_c(k), \quad (8)$$

where the component $\tilde{u}_c(k)$ expresses the influence of the exogenous variables x_d and m. The feedback matrix $\tilde{F}(k)$ is obtained on the basis of the solution $\tilde{R}(k)$ of a difference matrix Riccati equation. The optimal controller is time variant. This fact leads implementation difficulties; moreover, the solution to the difference Riccati equation is computed in inverse time, starting from the final condition

$$\tilde{R}(k_f) = S \; . \qquad (9)$$

In order to avoid these difficulties, a new procedure to design the optimal controller using only invariant blocks was indicated in [9] for discrete optimal drive systems.

2. OPTIMAL CONTROLLER DESIGN

The main idea is to consider the adjoint vector [1] from the Hamiltonian of the problem in the form

$$p(k) = Rx(k) + v(k), \quad p(k), v(k) \in R^n \, , \qquad (10)$$

where R is a constant matrix (and not variable, as in classical procedures) and $v(k)$ is a supplementary vector. This vector contains terms depending on the exogenous variables x_d and m and a corrective component, which compensate the fact that R is a constant and not a time-variant matrix.

Using the canonical equations for the Hamiltonian of the problem in the discrete case, yields:

$$u(k) = -P^{-1}B^{'}p(k+1) \, , \qquad (11)$$

$$p(k) = Qx(k) - Qx_d + A'\,p(k+1). \qquad (12)$$

One obtains from (4), (11) and (12) that R is the solution to the algebraic equation

$$R - A'R(I + NR)^{-1}A - Q = 0, \qquad (13)$$

with $N = BP^{-1}B^{'}$, and $v(k)$ satisfies the recurrence

$$Zv(k+1) = v(k) + e(k), \qquad (14)$$

where

$$Z = A' - A'RLN, \qquad (15)$$

$$e(k) = Qx_d - A'RLm(k) \, , \qquad (16)$$

$$L = (I + NR)^{-1}. \qquad (17)$$

(I is the identity matrix).

Equation (13) is identical with one that appears in the discrete LQ problem with infinite final time.

From (10) and from the transversality condition [1], it results the final value for the vector $v(k)$:

$$v(k_f) = (S - R)x(k_f) - Sx_d \, . \qquad (18)$$

This terminal condition can not be used in real-time computing, because $x(k_f)$ is known only at the end of the optimization process. This fact implies to express $x(k_f)$ in terms of $x(0)$, which is known at the start of the optimization. For this purpose, taking into account (10), (11) and (12), the equations (1) and (14) are rewritten as

$$z(k+1) = Gz(k) + w(k), \qquad (19)$$

where

$$z(k) = \begin{bmatrix} x(k) \\ v(k) \end{bmatrix},$$
$$w(k) = \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} = \begin{bmatrix} -LNZ^{-1}Qx_d & (I + LNZ^{-1}A'R)Lm(k) \\ -Z^{-1}Qx_d & -Z^{-1}A'RLm(k) \end{bmatrix} \qquad (20)$$

and

$$G = \begin{bmatrix} G_{11} & G_{12} \\ 0 & G_{22} \end{bmatrix} = \begin{bmatrix} LA & -LNZ^{-1} \\ 0 & Z^{-1} \end{bmatrix}. \qquad (21)$$

The form of the matrix G is advantageous and it is not difficult to verify that

$$G^{-1} = \begin{bmatrix} \bar{G}_{11} & \bar{G}_{12} \\ 0 & \bar{G}_{22} \end{bmatrix} = \begin{bmatrix} A^{-1}L^{-1} & A^{-1}N \\ 0 & Z \end{bmatrix} \qquad (22)$$

and
$$G^j = \begin{bmatrix} G_{11}^j & G_{12\,j} \\ 0 & G_{22}^j \end{bmatrix} \qquad (23)$$

with
$$G_{12\,j} = \sum_{i=1}^{j} G_{11}^{j-i} G_{12} G_{22}^{i-1} \qquad (24)$$

(similar relations can be written for G^{-1}).

The solution to the discrete linear equation (19) is

$$z(k) = G^{k-k_f} z(k_f) - \sum_{j=1}^{k_f-k} (G^{-1})^j w(k_f - j) \, . \qquad (25)$$

Using (18) and the properties of matrix G, one can compute the components of the vector z for every moment:

$$x(k_f) = M^{-1}[x(0) + g] \qquad (26)$$

and

$$v(k) = \bar{G}_{22}^{k_f-k}\left[(S - R)M^{-1}(x(0) + g) - Sx_d - h_k\right]. \qquad (27)$$

In the last relations,

$$M = \bar{G}_{11}^{k_f} + \bar{G}_{12k_f}(S - R) \, , \qquad (28)$$

$$g = \sum_{i=1}^{k_f}\left[\bar{G}_{11}^{j} w_1(k_f - j) + \bar{G}_{12\,j} w_2(k_f - j)\right] + \bar{G}_{12k_f} Sx_d \, , \qquad (29)$$

$$h_k = h(k) = \sum_{j=1}^{k_f-k} \bar{G}_{22}^{j} w_2(k_f - j) = \sum_{j=1}^{k_f-k} Z^j w_2(k_f - j). \qquad (30)$$

The above results allow the computation of the optimal control, which can be written as

$$u(k) = u_f(k) + u_c(k), \tag{31}$$

where $u_f(k)$ is the feedback component and $u_c(k)$ is a corrective one. These components can be computed from (10) and (11), after replacing of $x(k+1)$ and $v(k+1)$ in terms on $x(k)$ and $v(k)$ from (19). One obtains

$$u_f(k) = -P^{-1}B^{'}RLAx(k) \tag{32}$$

and

$$u_c(k) = -P^{-1}B^T(I - RLN)Z^{-1}v(k), \tag{33}$$

with $v(k)$ given by (27).

One can remark from (27) and (33) that $u_c(t)$ contains a corrective component

$$\hat{u}_c = -P^{-1}B^T(I - RLN)Z^{k_f-k-1}(S - R)M^{-1}x(0),$$

which depends on the initial state $x(0)$ and a component depending on the exogenous variables x_d and $m(k)$:

$$\tilde{u}_c = -P^{-1}B^T(A^{'})^{-1}\{Z^{k_f-k}[(S-R)M^{-1}g - Sx_d - h_k] + Qx_d\},$$

with g and h_k given by (29) and (30). This last component can be computed only if $m(k)$ is beforehand known on the interval [0, k_f], because (29) and (30) can be computed only in this case. The problem can also be solved in the case when it is known the shape of $m(k)$, $k = 1,2,...,k_f$ and its magnitude is measured or estimated at the beginning of the optimization interval. The simplest case (but frequently met in electrical drives) $m(k)$ = m = *constant* was discussed in [9]. The paper deals with the case when the load torque has a step variation during the transient period, from a small (no-loaded) operation to another great value. Such situations are frequently met in different applications and, in many situations, it is possible to know beforehand the two values of the torque and the switching moment.

We shall suppose that the load torque is m_1= *const.* for $k = 1,2,...,k_\theta - 1$ and m_2=*const.* for $k = k_\theta,...,k_f$. Consequently, the components of the vector w given by (20) will have the components w_{11}, w_{12} and w_{21}, w_{22} on the mentioned intervals. The vector g given by (29) is computed with

$$\begin{aligned} g = &\sum_{i=1}^{k_\theta-1}\left[\bar{G}_{11}^{j}w_{11}(k_f - j) + \bar{G}_{12j}w_{21}(k_f - j)\right] + \\ &+\sum_{i=k_\theta}^{k_f}\left[\bar{G}_{11}^{j}w_{12}(k_f - j) + \bar{G}_{12j}w_{22}(k_f - j)\right] + \bar{G}_{12k_f}Sx_d. \end{aligned} \tag{34}$$

In order to compute $h(k)$, one transforms the formula (30) noting $j = i + 1 - k$:

$$h(k) = \sum_{i=k}^{k_f-1} Z^{i+1-k}w_2(i). \tag{35}$$

For the mentioned two value of the load torque, one obtains:

$$\begin{aligned} &h(k) = \sum_{i=k}^{k_\theta-1} Z^{i+1-k}w_{21} + \sum_{i=k_\theta}^{k_f-1} Z^{i+1-k}w_{22}, for\ 0 < k < k_\theta, \\ &h(k) = \sum_{i=k}^{k_f-1} Z^{i+1-k}w_{22}, \quad for\ k \ge k_\theta. \end{aligned} \tag{36}$$

The corrective component $u_c(k)$ of the control vector will be computed using the relations (34) and (36).

Remark 1: The procedure indicated above can be extended for a drive system with a more general form of the variation of the load torque. In this case, it is possible to approximate this variation with a step function on several subintervals and the problem is solved in a similar manner.

Remark 2: The above presented relations are quite complicated, but the most part of the computing is performed off-line, in the stage of the controller design. This stage implies to establish the solution to the discrete Riccati equation (13) (the command *lqrd* from Matlab can be used), and to compute the matrices Z, L, G, G^{-1}, M with (15), (17), (21), (22), (28), the constant vector (34) and the constant matrices from (32) and (33).

The real-time computing implies to establish a usual feedback component $u_f(k)$ given by (32) and the corrective component $u_c(k)$, given by (33), which depends on $v(k)$, given by (27). One can remark in (27) that there are two variable elements: the matrix $\Gamma_k = \bar{G}_{22}^{k_f-k} = Z^{k_f-k}$ and the vector h_k given by (30) or (35). The both elements can be iteratively computed:

$$\Gamma_k = \Gamma_{k-1}Z^{-1},\ \Gamma_0 = Z^{k_f},\quad k = 1,2,...,k_f \tag{37}$$

For the second variant element, one has to consider the relationships (36) on subintervals:

$$\begin{aligned} &h_k = h_{k-1} - w_{21} + h_f, \quad for\ 0 < k < k_\theta, \\ &h_k = h_{k-1} - w_{22}, \quad for\ k \ge k_\theta, \end{aligned} \tag{38}$$

with

$$h_f = \sum_{i=k_\theta}^{k_f-1} Z^{i+1-k}w_{22} \text{ and } h(0) = \sum_{i=k}^{k_\theta-1} Z^{i+1}w_{21} + \sum_{i=k_\theta}^{k_f-1} Z^{i+1}w_{22}.$$

Remark 2: The adopted optimal control ensures that the value $x(k_f)$ at the final moment k_f will be in the proximity of the desired value x_d. The behaviour after the moment k_f is not reflected by this control. Therefore, one must adopt another control law in order to obtain a convenient behaviour of the drive system for $k > k_f$. A possibility in this direction can be found in [9].

3. SIMULATION RESULTS

A drive system with the matrices in (1)

$$A_c = \begin{bmatrix} 0 & 20 \\ -3.6 & 19.4 \end{bmatrix},\ B_c = \begin{bmatrix} 0 \\ 80 \end{bmatrix},\ W_c = \begin{bmatrix} -35 \\ 0 \end{bmatrix}$$

was considered. Theses data correspond to a drive system with a d.c. motor with rated data U=110V and I=3.3A. The sampling period is T=0.002s and the final time is 0.3s. The desired speed is ω_d=25 rad/s.

Fig. 1 shows the behaviour of the drive system for a step variation of the load torque from 0.2 Nm to 0.8 Nm (the rated value) at the moment t=0.1s.

Since the optimal control $u(t)$ is computed based on a beforehand computed „mean" value of the load torque (see (34) and (35)), the optimal variation of the voltage and of the current are not affected by the step variation of the load torque. Only a small variation of the acceleration can be observed at the moment t=0.1s.

Fig. 2 and 3 indicate the effect of an erroneous estimation of the load torque and of the switching moment, respectively.

A comparison between the behaviour of the cases of correct estimation (continuous curves) and erroneous estimation (doted curves) is indicated in both cases.

In fig. 2, the result of estimation is considered the values 0.2 and 0.6 Nm (instead of real values 0.2 and 0.8). In Fig. 3, the switching moment was anticipated at 0.2s instead of 0.1s. The variations of currents, voltages, velocities, energy losses and performance indices were compared for different estimation errors.

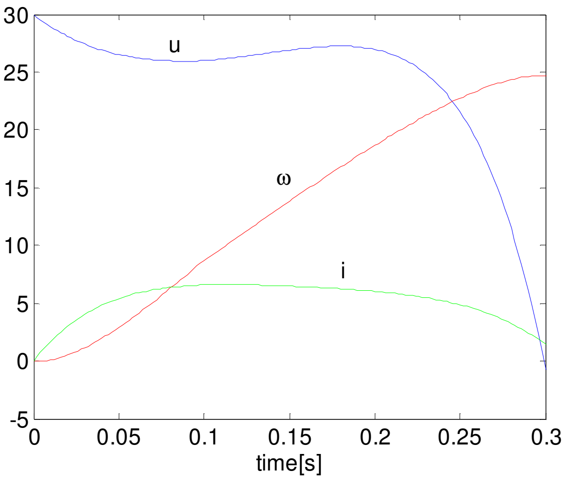

Fig. 1. Behaviour of the optimal system

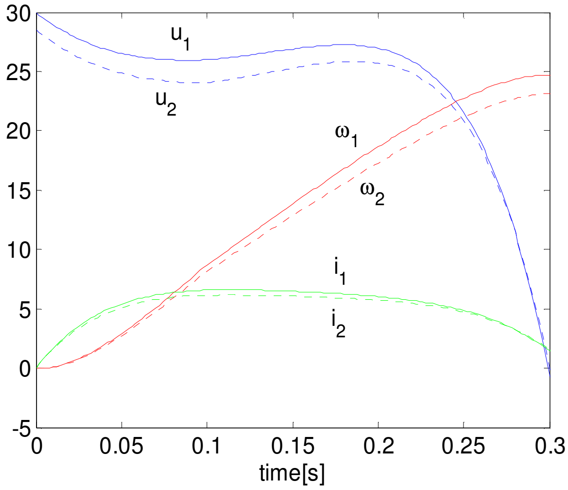

Fig.2. Behaviour in the case of an erroneous estimation of the load torque

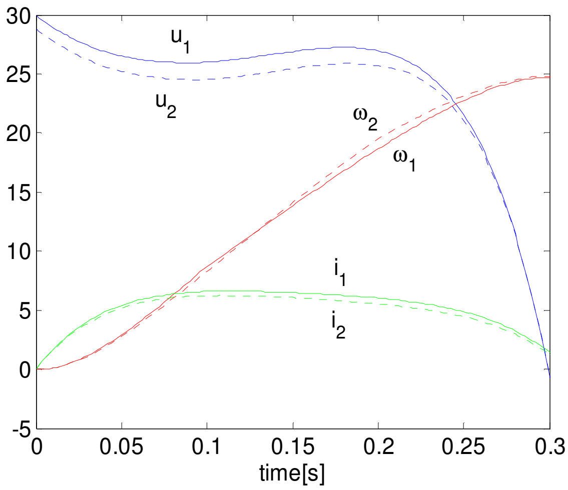

Fig. 3. Behaviour in the case of erroneous anticipation of the switching moment

The amplitude and sign of the mentioned variations are influenced by the sign and size of estimation errors. In any case, the variations of the mentioned entities around its optimal values are acceptable for reasonable size of estimation errors.

The analysis of the energy losses shows a small decrease in comparison with conventional cascade structure when the mean load torque is closely related to the rated one. But the reduction of the energy losses is up to 25% in the case of small mean load torque.

5. CONCLUSIONS

✓ A new method for the discrete optimal control for an electrical drive system with variable load torque is presented. The proposed algorithm can be easier implemented then the classical procedure for the LQ optimal control.

✓ The adopted criterion ensures a good behavior for the system and a significant decrease of the energy consumption, especially in the transient states that appears at the changing of the imposed value of speed and the mean load torque is small in comparison with the rated one.

✓ The described optimal control is useful especially for the medium and high power electrical drives with frequent changing of the speed.

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