INVESTIGATION BY SIMULATION OF MOTOR AND GENERATOR MATHEMATICAL MODELS FOR AN ELECTRICALLY EXCITED SYNCHRONOUS MACHINE RUNNING IN GENERATOR MODE

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REZUMAT. În lucrare sunt prezentate ecuaţiile generale ale maşinii sincrone cu înfăşurare de excitaţie, din care sunt deduse modelul matematic de motor, respectiv de generator în vedrea simulării în regim de generator. Generatorul sincron este antrenat de către un motor de inducţie cu rotor în colivie şi controlat în cuplu şi debitează pe o rezistenţă trifazată pasivă rezistiv-inductivă. Sunt prezentate rezultatele simulării în cazul controlului scalar al generatorului în care reglarea amplitudinii tensiunii, respectiv a frecvenţei statorice sunt realizate independent în bucle separate.

Cuvinte cheie: generator sincron, model matematic de motor şi de generator, simulare, control scalar.

ABSTRACT. The paper presents the mathematical description of an electrically excited synchronous machine using generator and motor mathematical models. In simulation the synchronous machine is running in generator mode, it is driven by a squirrel-cage induction motor and generates energy on a passive resistive-inductive load. Simulation results are presented for a scalar controlled synchronous generator, in which the stator-voltage amplitude and frequency of the generator are independently controlled in two separated loops.

Keywords: synchronous generator, motor and generator mathematical model, input and output variables.

1. INTRODUCTION

The electrically excited synchronous machine (EE-SyM) is the main supplier with electrical energy of the national power grid. The control of the synchronous generators feeding the AC line is indispensable to maintain the frequency and voltage constant at their rated values. The operation of the power network is based on this condition as well as on the optimum running of the consumers. Considering that the total active and reactive power consumed in the network is fluctuating, carefully designed control strategies must be applied. Although, significant advances were made in the field of power switching devices, permanent magnet capabilities and dedicated processing units, the usage of permanent magnet synchronous generators (PM-SyG) in conjunction with back-to-back frequency converters is still limited by the low power level at which this devices can operate (MW per unit) [1]. Considering that the price of the magnetic material is high and still rising due to the various factors, the EE-SyM becomes a solution even in isolated low power generation applications such as isolated and inaccessible communities, remote research facilities or remote vacation houses.

The main advantage of the EE-SyM is that it can operate at any arbitrary power factor such as unity power factor or leading power factor due to the independent control of the electrical excitation. Considering this characteristic, the EE-SyM is used for reactive power generation in order to balance the grid. These advantages however are balanced by a series of serious disadvantages.

The EE-SyM, in response to even small load peaks, tends to produce oscillations which are only poorly damped. Analyzing the fundamental properties of the EE-SyM, a drive unit which provides a steady driving torque at fixed speed must be used to overcome this problem. In renewable energy applications, such as wind turbines, coupling an EE-SyM directly to a fixed-frequency grid, it forces the generator to run at constant speed. On the other hand, the wind turbine rotor wants to follow the variations in wind speed. In between there is the mechanical drive train of the wind turbine. High dynamic loads on the mechanical components and severe fluctuations in the electrical power output are the consequences. Reduction of the dynamic loads can only be achieved by allowing the wind rotor speed a degree of freedom from the grid frequency. This can be achieved by using a continuous variable transmission (CVT) gearbox [2].The mathematical modeling and simulation of electric generators have a key role in developing new and improved control strategies, considering the more and more stringent codes applied in power generation and transmission.

The synchronous machine with salient poles and damper windings gives the most general mathematical model (MaMo), from which particular models may be obtained, such as in case of the non-salient pole machines or in the case of the machines without damper windings. In this paper, the MaMo of the salient pole EE-SyM with damper windings without taking into account the saturation is considered [3]. Using the same simulation conditions, a comparison between the motor MaMo (M-MaMo) and the generator MaMo (G-MaMo) of the EE-SyM behavior is presented. The classical control principle of the EE-SyM running in generator mode is well known: the frequency and amplitude of the stator voltage are controlled by adjusting the active and reactive power of the generator. The MaMo of the entire setup is considered which include the squirrel-cage induction motor (SqC-IM), its frequency converter, the excitation chopper and the load.

2. MATHEMATICAL MODELS

In a synchronous machine the average speed is directly proportional to the frequency of the AC supply to that it is connected. The MaMo of the EE-SyM based on the two reaction theory and on the space-phasor theory is valid for both, steady state and dynamic operation [3], [4]. Considering the rotor quantities referred to the stator and the dθ-qθ reference frame, where θ is the electrical angle of the rotor position, differential equations with constant parameters are obtained, which describe the behavior of the machine under any operation conditions. The G-MaMo is described by the following voltage-, flux- and motion equations, and electromagnetic torque expression [4]:

$$-u_{sd\theta} = R_s i_{sd\theta} + \frac{\mathrm{d}\Psi_{sd\theta}}{\mathrm{d}t} - \omega\Psi_{sq\theta}; \tag{1}$$

$$-u_{sq\theta} = R_s i_{sq\theta} + \frac{\mathrm{d}\Psi_{sq\theta}}{\mathrm{d}t} + \omega\Psi_{sd\theta}; \tag{2}$$

$$u_{Ad} = R_{Ad} i_{Ad} + \frac{\mathrm{d}\Psi_{Ad}}{\mathrm{d}t} = 0; \tag{3}$$

$$u_{Aq} = R_{Aq} + \frac{\mathrm{d}\Psi_{Aq}}{\mathrm{d}t} = 0; \tag{4}$$

$$u_e = R_e i_e + \frac{\mathrm{d}\Psi_e}{\mathrm{d}t}; \tag{5}$$

$$\Psi_{sd\theta} = L_{sd} i_{sd\theta} + L_{md}(i_{Ad} + i_e); \tag{6}$$

$$\Psi_{sq\theta} = L_{sq} i_{sq\theta} + L_{mq} i_{Aq}; \tag{7}$$

$$\Psi_{Ad} = L_{Ad} i_{Ad} + L_{md}(i_e + i_{sd\theta}); \tag{8}$$

$$\Psi_{Aq} = L_{Aq} i_{Aq} + L_{mq} i_{sq\theta}; \tag{9}$$

$$\Psi_e = L_e i_e + L_{md}(i_{Ad} + i_{sd\theta}); \tag{10}$$

$$m_e^{SM} = \frac{3}{2} \cdot z_p \cdot (\Psi_{sd\theta} \cdot i_{sq\theta} - \Psi_{sq\theta} \cdot i_{sd\theta}); \tag{11}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{z_p}{J_{tot}} \cdot (m_e^{IM} + m_e^{SM}), \tag{12}$$

where

R_s, R_{Ad}, R_{Aq} and R_e are the stator, direct damper, quadrature damper and excitation resistances and

L_{sd}, L_{sq}, L_{Ad}, L_{Aq} and L_e – direct and quadrature stator, direct and quadrature damper and excitation inductances;

L_{md} and L_{mq} – mutual direct and quadrature stator inductances;

$i_{sd\theta}$ and $i_{sq\theta}$ – direct and quadrature stator currents;

i_{Ad} and i_{Aq} – direct and quadrature damper currents;

i_e – excitation current;

$\Psi_{sd\theta}$ and $\Psi_{sq\theta}$ – direct and quadrature stator fluxes;

Ψ_{Ad} and Ψ_{Aq} – direct and quadrature damper fluxes;

Ψ_e – excitation circuit resultant flux;

$u_{sd\theta}$ and $u_{sq\theta}$ – direct and quadrature stator voltage;

u_e – excitation voltage;

ω – electrical angular speed of the rotor;

z_p – number of pole pairs;

m_e^{SM} and m_e^{IM} – electromagnetic torque of the EE-SyG and SqC-IM.

The motion equation is a common mechanical one which uses the torques of the EE-SyG and SqC-IM and the common rotor speed. In this case the angular speed is an input variable for the MaMo of the both electrical machines.

In the case of the G-MaMo the input variables are $i_{sd\theta}$, $i_{sq\theta}$, u_e and ω and the output ones are $u_{sd\theta}$, $u_{sq\theta}$ and m_e^{SM}. It can be observed that the MaMo consists of a system of eleven equations with the same number of unknown variables. Considering (8) and (10) the expressions of i_{Ad} and i_e are obtained, expressed with the Ψ_{Ad} and Ψ_e. In the next step we replace these expressions in (3) and (5) and obtain through some basic mathematical manipulations a system of two differential equations with Ψ_{Ad} and Ψ_e unknown variables. Furthermore using (9) and (4) with a similar approach Ψ_{Aq} and i_{Aq} are computed. By grouping (1), (2), (6) and (7) and considering the same procedure, $u_{sd\theta}$

and $u_{sq\theta}$ are found. In the last step, considering (11) the electromagnetic torque is computed.

For the M-MaMo almost the same system of equations is used, the difference appears only in equations (1), (2) and (12), which are replaced by:

$$u_{sd\theta} = R_s i_{sd\theta} + \frac{\mathrm{d}\Psi_{sd\theta}}{\mathrm{d}t} - \omega \Psi_{sq\theta}; \tag{13}$$

$$u_{uq\theta} = R_s i_{sq\theta} + \frac{\mathrm{d}\Psi_{sq\theta}}{\mathrm{d}t} + \omega \Psi_{sd\theta}; \tag{14}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{z_p}{J_{tot}} \cdot (m_e^{IM} - m_e^{SM}). \tag{15}$$

In this case $u_{sd\theta}$, $u_{sq\theta}$, u_e and ω are considered inputs variables, while $i_{sd\theta}$,$i_{sq\theta}$ and m_e will be computed by the model. Using similar mathematical manipulations as in previous case the system of equations is computed. The simplified block diagrams of the models are presented in Fig. 1 and Fig. 2.

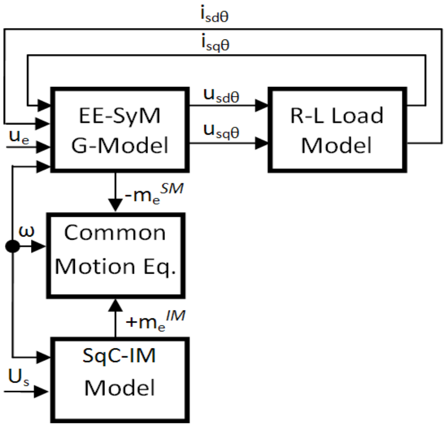

Fig. 1. Simplified block diagram realized with generator MaMo for simulation of the EE-SyM in generator operation.

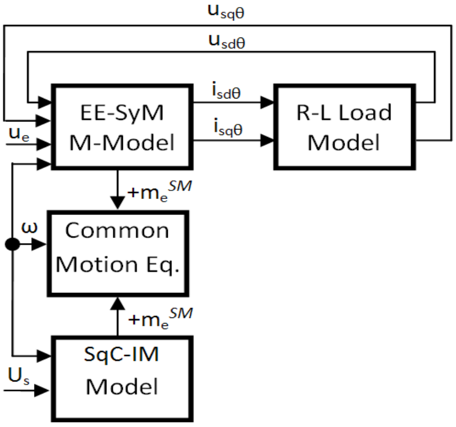

Fig. 2. Simplified block diagram realized with motor MaMo for simulation of the EE-SyM in generator operation.

The flux-MaMo of the SqC-IM using space phasors and fixed stator-oriented coordinates was deduced based on the general equations [5], and it is described by the following state-space equations:

$$\frac{\mathrm{d}\underline{\Psi}_s}{\mathrm{d}t} = \underline{u}_s - \frac{R_s}{\sigma \cdot L_s} \cdot \underline{\Psi}_s + \frac{R_s}{\sigma \cdot (1+\sigma_r) \cdot L_s} \cdot \underline{\Psi}_r; \tag{16}$$

$$\frac{\mathrm{d}\underline{\Psi}_r}{\mathrm{d}t} = \left(\frac{-R_r}{\sigma \cdot L_r} + j \cdot \omega_r \right) \cdot \underline{\Psi}_r + \frac{R_r}{\sigma \cdot (1+\sigma_s) \cdot L_r} \cdot \underline{\Psi}_r; \tag{17}$$

$$m_e = \frac{3}{2} \cdot z_p \cdot \frac{L_m}{\sigma \cdot L_s \cdot L_r} \cdot \mathrm{Im}\left(\underline{\Psi}_s \cdot \underline{\Psi}_r^{\;*} \right), \tag{18}$$

where Im indicates the imaginary part, * the complex conjugate value, and

R_s and R_r – stator and rotor resistances;
L_s, L_r and L_m – stator, rotor and mutual inductances;
σ_s, σ_r, σ – stator-, rotor- and resultant leakage coefficients;
ω_r – electrical angular speed of the rotor;
z_p – number of pole-pairs;
m_e – electromagnetic torque;
$\underline{u}_s$ – stator-voltage phasor;
$\underline{\Psi}_s$, $\underline{\Psi}_r$ – stator- and rotor-flux space phasors.

The *R-L* load is modeled as a first order lag.

3. CONTROL STRATEGY

In order to validate the presented models, a scalar control procedure was used in order to control the EE-SyM stator-voltage in both cases. Two independent control loops were designed: one for the frequency control and the other for control the amplitude of the stator voltage. In Fig. 3 the PI stator-voltage-controller generates the reference of the excitation voltage which in this case commands an ideal DC-to-DC chopper, while the stator-frequency controller generates the reference torque for the control of the SqC-IM. In addition, an indirect field-oriented control with space-vector pulse-width modulation and an ideal frequency converter (with ideal switching devices) is used to properly control the SqC-IM, as is shown in Fig. 3.

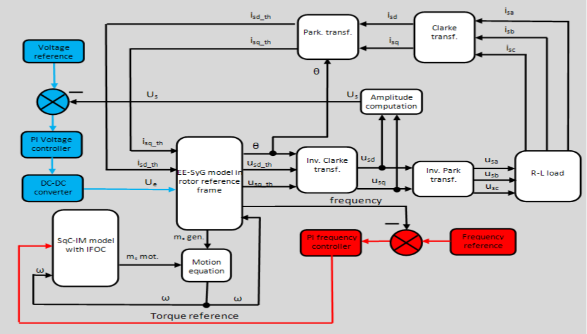

Fig. 3. Scalar control strategy of the EE-SyM running in generator mode.

4. SIMULATIONS

In order to analyze the above described modeling and control methods, simulations were performed using MATLAB-Simulink® environment. Each model was submitted to the same simulation conditions in order to emphasize better the behavior of the system.

The models included two major sub-systems: in the first one the control structure and the SV-PWM block [6] were modeled using discrete sample time (0.0002 s), while the hardware components of the experimental setup (PWM-modulator, VSC, chopper, EE-SyM, SqC-IM, R-L load) were simulated in continuous sample time. With this approach the simulation model has a similar behavior to a real experimental setup.

The simulation models were developed for a 1.5 kW, 1415 rpm SqC-IM and 1 kW 1500 rpm EE-SyM (limited by the laboratory test bench).

Simulation starts with 230 V reference for the voltage amplitude and 1500 rpm (50 Hz) frequency reference. After 1 s the amplitude reference is set to 200 V, and at 2 s simulation time the frequency reference is set to 1200 rpm (40 Hz).

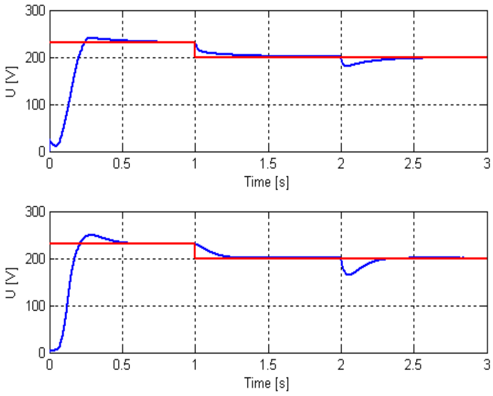

Fig. 4. Step response on the control loop of the stator-voltage amplitude simulated with the G- and M-MaMo, respectively.

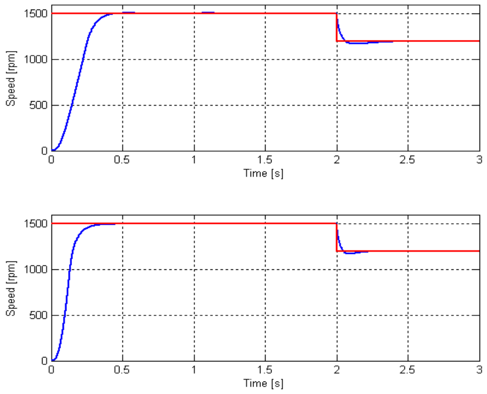

Fig. 5. Step response on the control loop of the stator-frequency simulated with the G- and M-MaMo, respectively.

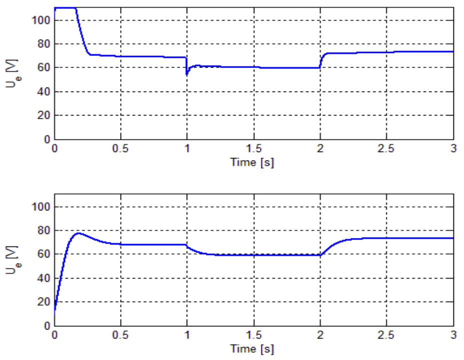

Fig. 6. Excitation voltage simulated with the G- and M-MaMo, respectively.

In Fig. 4-6 it can be observed that the system has good static and transient behavior, and also, there are small differences between the two approaches despite the identical conditions in which the simulations were made. A cause for these differences could be the different mathematical computation steps applied in the two cases. A high settling time in both cases is observed but that is not an issue for the goals of this paper.

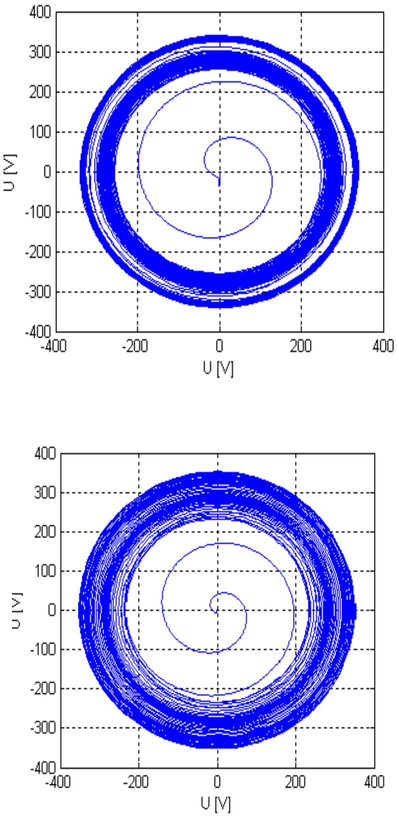

Fig. 7. Space-phasor diagram of the stator voltage of the EE-SyM simulated with the G- and M-MaMo, respectively.

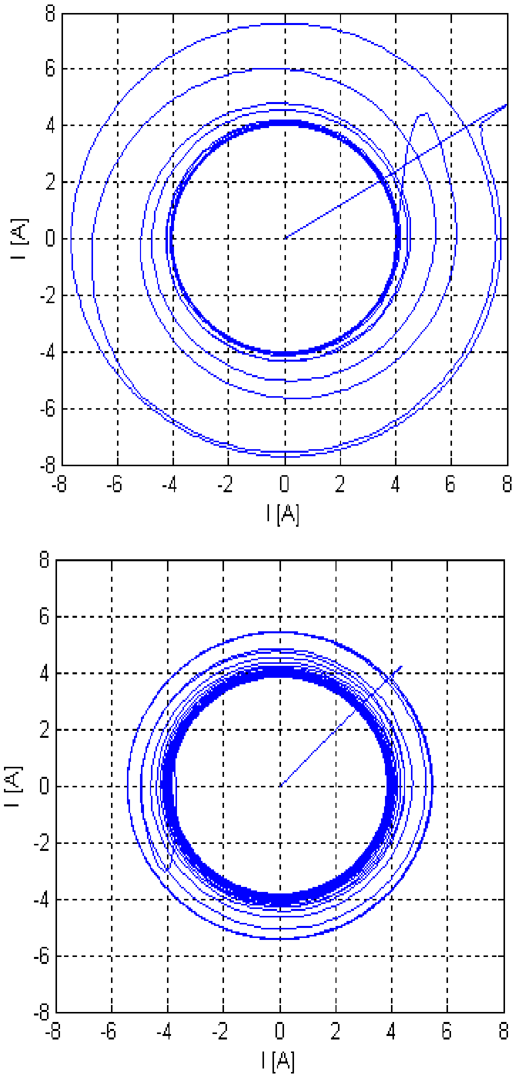

Fig. 8. Space-phasor diagram of the stator current of the SqC-IM simulated with the G- and M-MaMo, respectively.

Fig. 7 and Fig. 8 show the space-phasor diagram for the stator voltage of the EE-SyG and stator current of the SqC-IM. Both models behave well in transient and steady state, but small differences may be observed despite the identical conditions in which the simulations were performed.

5. CONCLUSIONS

In this paper the mathematical description of an electrically excited synchronous machine using generator and motor mathematical models is presented. The models were validated by simulations and good results were obtained for transient and steady state operation. Simulation results are presented for a scalar control of the synchronous generator, in which the stator-voltage amplitude and frequency of the generator are independently controlled. In order to improve the step response of the system a vector-control strategy may be used for the electrically excited synchronous generator [7].

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