WAVE ITERATIVE PROCESS VALIDATION FOR 3D SPHERICAL METALLIC OBSTACLE SCATTERING

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REZUMAT. Lucrarea prezintă o aplicaţie a Metodei Iterative Bazată pe Conceptul de Undă (WIP) în cazul studiului difracţiei unei unde electromagnetice plane pe suprafaţa unui obstacol metalic 3D. Aplicaţia este realizată în scopul validării Metodei Iterative în cazul unei probleme clasice de difracţie. Se tratează Cazul unui obstacol difractant sferic. Se studiază convergenţele modală şi iterativă. Se evaluează densitatea de curent de pe suprafaţa sferei, rezultatele obţinute fiind comparate cu soluţia analitică exactă disponibilă pentru acest caz particular.

Cuvinte cheie: difracţie, undă plană, conceptul de undă, sferă metalică

ABSTRACT. The paper presents an application of the Wave Iterative Process (WIP) in the case of the study of the scattering of an electromagnetic plane wave by a metallic 3D obstacle. The application is made in order to validate the original Iterative Method for a classical metallic obstacle diffraction problem. The case of a spherical scatterer is treated. Modal and iterative convergences are studied. Current density is calculated and compared with the exact solution available for this particular case.

Keywords: scattering, plane wave, wave concept, metallic sphere

1. INTRODUCTION

Recent years have witnessed a great development of the necessary tools able to numerically model, simulate the performance of, and design complex electromagnetic systems. An important class of competing computational electromagnetics approaches is based on the use of iterative methods in solving scattering problems [1], due to the simplicity and high efficiency of these methods.

An important class of numerical methods is based on the analogy between the behaviour of the voltages and currents in an interconnected transmission line network and the behaviour of the electromagnetic fields in a propagation environement. The Transmission Line Matrix [2] (TLM) method formulation was further developed as the FDTLM [3], a symmetrical node being derived from the Maxwell equations, using centred differencing and averaging [4].

The integral formulation of the TLM method is defined in the time domain, spectral domain being also used in the ATLM method [5].

In the same way the integral field equations methods EFIE [6] and MFIE [7] are related to the Finite-Difference Time-Domain FDTD [8] techniques, the TLM method evolved into the Wave Iterative Process (WIP) method, introduced at first by [9].

Among the most recent and the most efficient iterative methods, the Wave Iterative Process (WIP) was developed at first as an instrument for the study of in-guide and planar circuits scattering problems [10]. Due to its combined spectral – real domain formulation issued from the description of boundary conditions and modal behavior and due to its applicability to all range of geometrical dimensions of the scattering obstacle, the Wave Iterative Process is highly recommended for its extension to free space electro-magnetic diffraction studies.

Already successfully tested in many circuit designs [11]-[13], the WCIP has a great potential of development to solving complex shaped structures and multiple obstacle electromagnetic diffraction problems. The method was validated for several 2D scattering structrures [14], 3D obstacles being taken into study.

The paper presents a brief review of the principles of the WCIP, wave definitions and scattering operators. The WCIP is then applied to the case of a plane wave incidence upon a 3D metallic spherical obstacle, TE and TM modal functions bases being considered. Spherical coordinates are used to express initial waves of the iterative process. Modal and iterative convergences of the WIP method are investigated and the results given by WCIP are compared with the exact solution available for this particular case.

2. THEORY

The Wave Iterative Process:

Considered the incidence of a wave on the surface S of an obstacle as depicted in Fig. 1, the basic principle of the WCIP is the concept of waves, the incident one A, and the reflected one B, defined from the tangent electric field E, and the incident magnetic tangent field H.

For simplicity reasons, instead of H, the "current density" vector J is preferred. The following relation defines the J vector:

$$\vec{J} = \vec{H} \times \hat{n} \quad (1)$$

where $\hat{n}$ is the normal versor to the scattering surface, defined in each point.

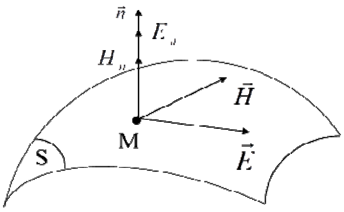

Fig.1 Scattering surface

The waves A and B are defined by (2):

$$\begin{cases} \vec{A} = \dfrac{1}{2\sqrt{Z_0}}(\vec{E} + Z_0\vec{J}) \\ \vec{B} = \dfrac{1}{2\sqrt{Z_0}}(\vec{E} - Z_0\vec{J}) \end{cases} \quad (2)$$

where Z_0 is an arbitrary parameter.

The analytic expression of the iterative process is issued from the continuity and border conditions of the electromagnetic field. In the case of free space scattering, the iterative process is described by (3):

$$\begin{cases} \vec{B}_n + \vec{B}_0 = \hat{S}(\vec{A}_{n-1} + \vec{A}_0) \\ \vec{A}_n = \hat{\Gamma}\vec{B}_n \end{cases} \quad (3)$$

where n is the number of the iteration, $\hat{S}$ is the real domain scattering operator, and $\hat{\Gamma}$ is the modal domain scattering operator.

The two scattering operators are given by (4) and (5):

$$\hat{S} = I_S - I_M \quad (4)$$

where I_S and I_M are the dielectric and respectively metallic domain definition functions.

$$\hat{\Gamma} = \sum_m |f_m\rangle \frac{1 - Z_0 Y_m}{1 + Z_0 Y_m} \langle f_m| \quad (5)$$

where $\{f_m\}$ is a complete orthonormal modal base and Y_m is the modal admittance of the m^{th} mode.

A schematic of the current density calculation on the scatterer's surface using the WIP is presented in Fig. 2.

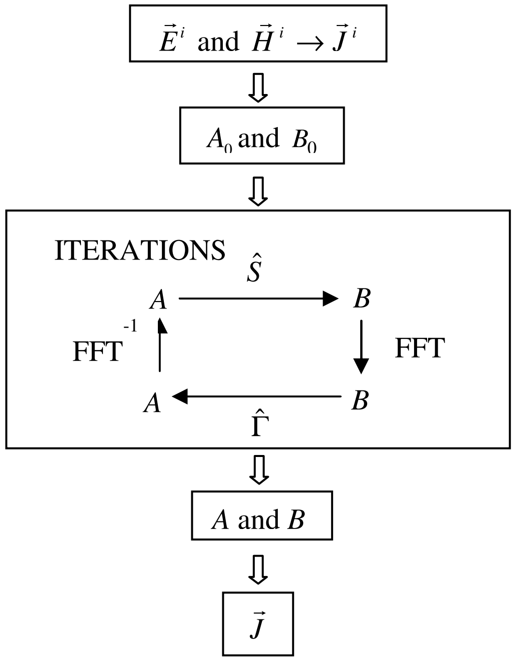

Fig.2 Iterative process schematic

A_0 and B_0 are the waves corresponding to the incident electromagnetic field on the studied structure, defined as:

$$\begin{cases} \vec{A}_0 = \dfrac{1}{2\sqrt{Z_0}}(\vec{E}^i + Z_0\vec{J}^i) \\ \vec{B}_0 = \dfrac{1}{2\sqrt{Z_0}}(\vec{E}^i - Z_0\vec{J}^i) \end{cases} \quad (6)$$

The first equation of (3) is written in the real domain, the second one being written in the spectral domain, and therefore a direct and inverse fast modal (*Fourier*) transformations (FFT and FFT^{-1}) are used to pass from one to the other.

Scattering by a Metallic Sphere:

Let us consider the incidence of a z-polarized plane wave on the surface of a metallic spherical obstacle, as depicted in Fig. 3. The radius of the sphere is a.

The incident electromagnetic field is given by:

$$\begin{aligned} \vec{E}^i &= E_0 e^{-jkz}\vec{x} \\ \vec{H}^i &= \frac{E_0}{\zeta} e^{-jkz}\vec{y} \end{aligned} \quad (7)$$

where $\zeta \cong 120\pi$ [Ω] is the intrinsic space impedance.

Taken into account the geometry of the problem, it is obvious a change of coordinate system type is necessary, spherical coordinates being used.

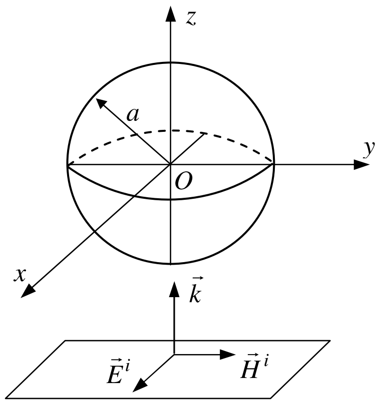

Fig.3 Plane wave incidence upon the surface of a sphere

The following values of the initial current density are obtained:

$$J_\rho^i = 0$$

$$J_\theta^i = \frac{E_0}{\zeta}\cos\phi\; e^{-jk\rho\cos\theta} \qquad (10)$$

$$J_\phi^i = -\frac{E_0}{\zeta}\sin\phi\cos\theta\; e^{-jk\rho\cos\theta}$$

In the following, the amplitude of the electrical field will be supposed as unitary $E_0 = 1$. If the arbitrary parameter's value is set as $Z_0 = \zeta$, the components of the initial waves A_0 and B_0 are calculated accordingly to (6) and have the following expressions:

$$A_{0\rho} = \frac{1}{2\sqrt{Z_0}}\sin\theta\sin\phi\, e^{-jk\rho\cos\theta}$$

$$A_{0\theta} = \frac{1}{2\sqrt{Z_0}}(1+\cos\theta)\cos\phi\, e^{-jk\rho\cos\theta} \qquad (11)$$

$$A_{0\phi} = \frac{1}{2\sqrt{Z_0}}(1-\cos\theta)\cos\phi\, e^{-jk\rho\cos\theta}$$

and, respectively:

$$B_{0\rho} = \frac{1}{2\sqrt{Z_0}}\sin\theta\sin\phi\, e^{-jk\rho\cos\theta}$$

$$B_{0\theta} = \frac{1}{2\sqrt{Z_0}}(\cos\theta-1)\cos\phi\, e^{-jk\rho\cos\theta} \qquad (12)$$

$$B_{0\phi} = \frac{1}{2\sqrt{Z_0}}(\cos\theta+1)\cos\phi\, e^{-jk\rho\cos\theta}$$

The spherical structure of the studied scatterer imposes the choice of a spherical modal base. The TE and TM components of the modal base are given by the tesseral harmonics [15]:

$$f_m^{TE} = \alpha_m P_m^1(\cos\theta)\cos m\phi$$
$$f_{mn}^{TM} = \alpha_m P_m^1(\cos\theta)\sin m\phi \qquad (13)$$

where P_m^1 are the first order associated *Legendre* polynomials.

The α_m coefficient issues from the ortonormation condition of the modal base and is given by:

$$\alpha_m = \frac{2\pi\tau_m}{2n+1}\cdot\frac{(m+1)!}{(m-1)!} \qquad (14)$$

where

$$\tau_m = \begin{cases} 2 & \text{for } m = 0 \\ 1 & \text{for } m \neq 0 \end{cases} \qquad (15)$$

The modal admittances for the TE and TM modes are respectively given by [15]:

$$Y_m^{TE} = -\frac{j}{Z_0}\cdot\frac{H_m^{(2)}(ka)}{H_m^{(2)\prime}(ka)} \qquad (16)$$

$$Y_m^{TM} = \frac{j}{Z_0}\cdot\frac{H_m^{(2)\prime}(ka)}{H_m^{(2)}(ka)} \qquad (17)$$

where $H_m^{(2)}$ is the second kind *Hankel* function.

The existence of the two mode types determines (5) to be rewritten as:

$$\hat{\Gamma} = \sum_m \left| f_m^{TE}\right\rangle \frac{1-Z_0Y_m^{TE}}{1+Z_0Y_m^{TE}}\left\langle f_m^{TE}\right| + \sum_m \left| f_m^{TM}\right\rangle \frac{1-Z_0Y_m^{TM}}{1+Z_0Y_m^{TM}}\left\langle f_m^{TM}\right| \qquad (18)$$

All the necessary elements needed to run the iterative process are now defined.

The case of the scattering of a plane wave by a metallic sphere is among the very few that have an exact analytical solution given by *Harrington* [15]. The values of the two components of the current density on the obstacle's surface have the following expressions:

$$J_\theta = \frac{j}{\zeta}\cdot\frac{\cos\phi}{ka}\sum_n \frac{j^{-n}}{\alpha_n}\left[\frac{\sin\theta\, P_n^{1\prime}(\cos\theta)}{H_n^{(2)\prime}(ka)} + \frac{P_n^1(\cos\theta)}{\sin\theta\, H_n^{(2)}(ka)}\right] \qquad (19)$$

$$J_\phi = \frac{j}{\zeta}\cdot\frac{\sin\phi}{ka}\sum_n \frac{j^{-n}}{\alpha_n}\left[\frac{j\sin\theta\, P_n^{1\prime}(\cos\theta)}{H_n^{(2)}(ka)} + \frac{P_n^1(\cos\theta)}{\sin\theta\, H_n^{(2)\prime}(ka)}\right] \qquad (20)$$

We will present in Paragraph 3 the comparison between the current density given by the use of the Wave Concept Iterative Process with the one issued by applying (19) and (20). This will allow us to validate the iterative method for this type of scattering obstacle.

3. NUMERICAL RESULTS

Let us consider the incidence of a 10 GHz plane wave upon the surface of a metallic sphere. The first investigated issue is the iterative convergence of the WIP. Fig. 4 presents the results obtained for the point $\theta = \pi/2$. As expected, the smaller the geometrical dimension of the problem, the larger the number of iterations needed to achieve the convergence.

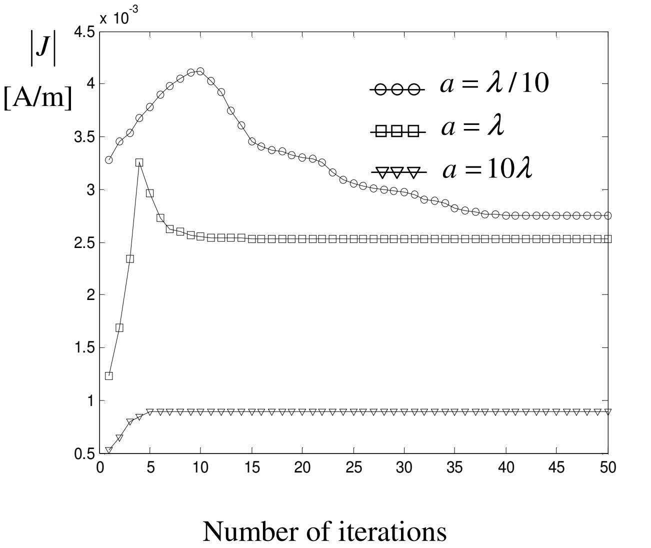

Fig.4 Iterative convergence of WCIP for different radius values

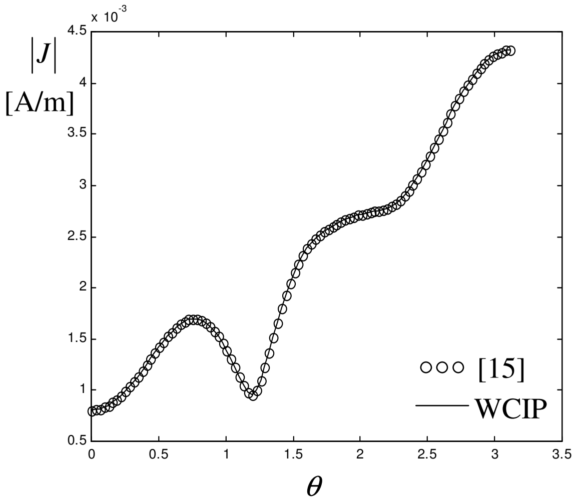

Fig.5 Current density on a longitude circle for a radius $a = \lambda$

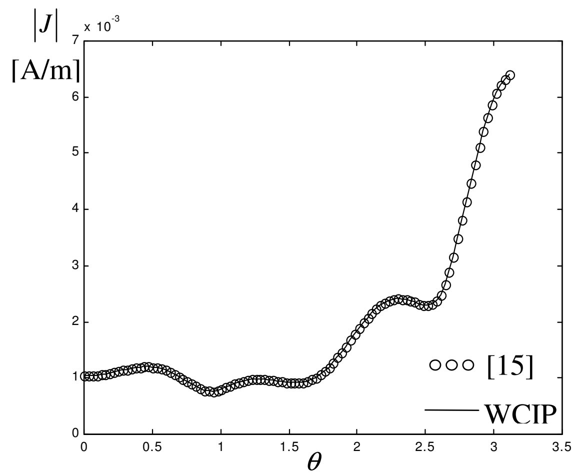

Fig.6 Current density on a longitude circle for a radius $a = 10\lambda$

The comparison between the WCIP and the exact analytic solution [15] is depicted in Fig. 5 for a radius $a = \lambda$, in Fig. 6 for $a = 10\lambda$, and, respectively, in Fig. 7 for $a = \lambda / 10$.

Due to the symmetry of the problem, the current density is evaluated for $\theta \in [0, \pi]$.

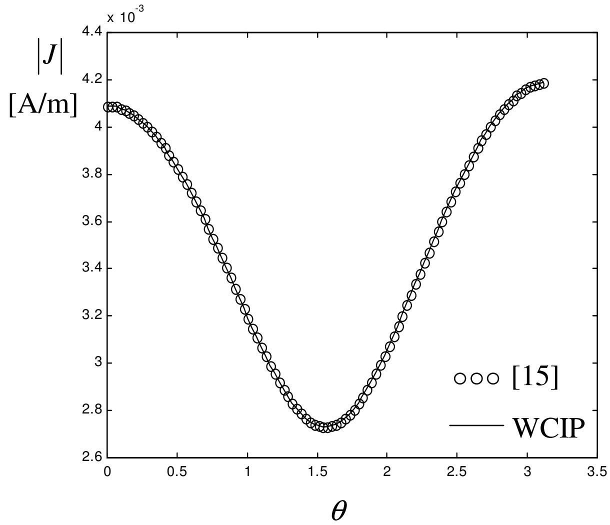

Fig.7 Current density on a longitude circle for a radius $a = \lambda/10$

One can notice the excellent correspondence between the WCIP results and the exact solution [15], allowing us to conclude that WCIP is validated for this case.

4. CONCLUSION

An application of the Wave Concept Iterative Process was made, for a spherical scattering metallic obstacle. The WCIP was proved convergent for small and for large geometrical dimensions of the scatterers, in terms of wavelength.

The results given by WCIP for the scattering of a plane wave by a sphere are practically coincident with the exact analytic solution available for this particular structure. We can conclude that WCIP was successfully validated for the 3D spherical diffraction structure, opening the path of further free space scattering obstacles study.

5. AKNOLEDGEMENT

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