

# STUDY ABOUT THE DIAGRAM OF A TRANSFORMATIONS FOR A CLOCK-HOUR FIGURE OF THE VECTOR GROUP AT AN THREE-PHASE TRANSFORMER

Eng. Cristina PRODAN PhD, Prof. Eng. Adrian GRAUR PhD

University „Ștefan cel Mare” from Suceava, Faculty of Electrical Engineering and Computer Science, Department of Electrotechnics, Suceava, România

**REZUMAT.** Articolul evidențiază că trecerea de la un indice orar la altul se realizează prin *parcurea diagramei transformărilor* fie în sens trigonometric, fie în sens invers trigonometric. *Parcurea diagramei transformărilor* se poate face prin permutarea circulară a liniilor sau coloanelor în sens direct sau invers, ceea ce echivalează cu modificarea legăturilor din N în Z sau din Z în N, la bornele înfășurării primare sau secundare, modificări care se efectuează de câte ori este nevoie, în funcție de indicele orar la care se ajunge.

**Cuvinte cheie:** transformatoare, indice orar, diagrama transformărilor

**ABSTRACT.** Article shows that the transition from one clock hour figure of the vector group to another, is achieved by following the transformations diagram, be in the clockwise, or counterclockwise. Crossing through the transformations diagram, can be made by the circular permutation of the lines or of the columns, in direct sense or in inverse sense, which is equivalent to changing linkages from N in Z or from Z in N, from the primary winding terminals or to the secondary winding terminals, changes which are made whenever needed, depending on the clock hour figure which is reached.

**Keywords:** transformers, clock hour figure of the vector group, the transformations diagram

## 1. INTRODUCTION

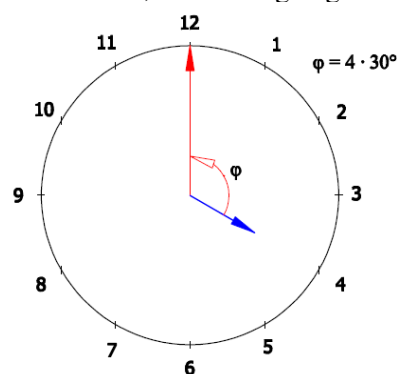
The parallel operation, of the electrical transformers, operation with the unbalanced loads [7, 11, 12] and the deforming regime at the magnetization to the transformers [1, 7, 8, 13], are issues directly related to wiring diagrams, the groups of connections and the clock hour figure of the vector groups, at the three-phase transformers.

The phase displacement angle, on that makes it the phasor of the voltage measured between two terminals of the low voltage winding, with the phasor of the voltage, measured between the corresponding terminals of the high voltage winding, is in all cases a multiple of 300, is indicated conventionally, by a number.  $\phi$  angle, is practically the same of the phase displacement, as well as between the clock needles, that shows one of the hours, fixed between 1 and 12 (Fig. 1) [6, 7]. The  $i$  number, is named the sequence number [7] or clock hour figure of the vector group [3, 6] and the  $\phi$  phase displacement between the corresponding voltage, from the primary, and, from the secondary, is obtained by multiplying the  $i$  number, with 300.

As shown, the  $i$  multiple, was assigned as the clock hour figure at the vector group [6], because it was found

that, as between the clock needles, that shows one of the hours, marked on the dial and equal with  $i$ , is the same the  $\phi$  phase displacement, measured between the corresponding voltage, from the primary, and, from the secondary, considered between the corresponding terminals, at the checked transformer [2, 6].

At the three-phase transformers, the phase windings can be connected in star, delta or zig-zag.



**Fig. 1** Explanatory for defining the clock hour figure of the vector group.

In the case of the delta connections, the series connection of the phase windings, can be achieved in

two ways [4, 5, 8, 14], as shown in Fig. 2. The series connection can be achieved to "left" or to the "right" [8].

Be adopted [5], term of linkages, in "N" ("left") or in "Z" ("right"), because the inclination the linkages to the schema configuration, suggests the letter "N" or "Z" (see Figure 2).

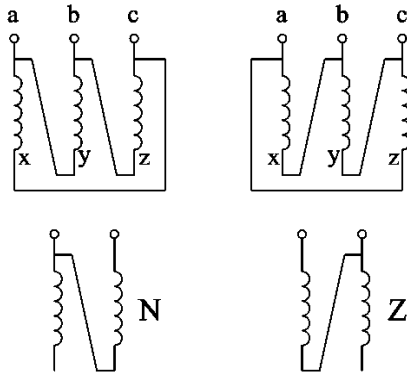


Fig. 2 Delta connection diagram, with linkages in "N" and with linkages in "Z".

To identify the clock hour figure of the vector group, at a transformer that has suffered changes to terminal connections, the authors propose using the mathematical model of the clock hour figure of the vector group and his connection to the connection diagram, after the changes suffered on the transformer.

In [5, 14] is presented the mathematical model of clock hour figure of the vector group, expressed as a matrix, with form:

$$G = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix} \quad (1)$$

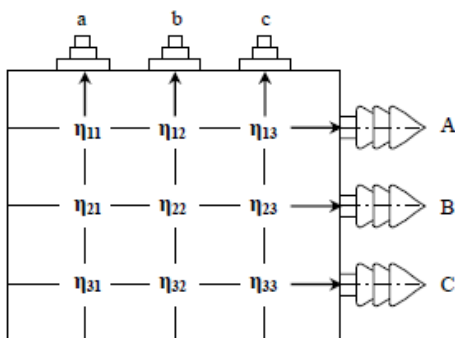


Fig. 3 The link between the code matrix configuration and the connections configuration to the terminals of a transformer.

The studies made, highlight the interdependence that exists between the configuration of the matrix code (the position of the lines and of the columns) and the configuration of the linkages to terminals fixed on the transformer cover.

Position of the lines reflects changes across the terminals at the high voltage winding, while position of the columns reflects changes across the terminals at the low voltage winding (Fig.3).

## 2. NOTATIONS AND DEFINITIONS USED [4, 5, 9, 10, 14, 15]

Considering a 3x3 square matrix  $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ , we adopt the notations and

definitions listed below:

$A \downarrow$  - for the matrix obtained by permutation the lines in direction of arrow (in direct sense):

$$A \downarrow = \begin{pmatrix} a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = A \downarrow$$

$A \uparrow$  - for the matrix obtained by permutation the lines in direction of arrow (in reverse sense):

$$A \uparrow = \begin{pmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \end{pmatrix} = A \uparrow$$

$\vec{A}$  - for the matrix obtained by permutation the columns in direction of arrow (in direct sense):

$$\vec{A} = \begin{pmatrix} a_{13} & a_{11} & a_{12} \\ a_{23} & a_{21} & a_{22} \\ a_{33} & a_{31} & a_{32} \end{pmatrix} = \vec{A}$$

$\overleftarrow{A}$  - for the matrix obtained by permutation the columns in direction of arrow (in reverse sense):

$$\overleftarrow{A} = \begin{pmatrix} a_{12} & a_{13} & a_{11} \\ a_{22} & a_{23} & a_{21} \\ a_{32} & a_{33} & a_{31} \end{pmatrix} = \overleftarrow{A}$$

### 3. DIAGRAM OF TRANSFORMATION FOR THE CLOCK HOUR FIGURE OF THE VECTOR GROUP

#### 3.1. DEFINING THE POLAR AXIS

In the explanatory figure for defining the clock hour figure of the vector group (Fig.1), one can identify 6 axes, shown in Fig.4; the authors have adopted for this explanatory, the diagram of transformations, and for the six circle diameters, on which are willing the twelve clock hour figure of the vector groups, were adopted the names as follows:

- axis (12-6) – the main polar axis 0;
- axis (11-5) – the secondary polar axis 1;
- axis (10-4) – the secondary polar axis 2;
- axis (9-3) – the secondary polar axis 3;
- axis (8-2) – the secondary polar axis 4;
- axis (7-1) – the secondary polar axis 5;

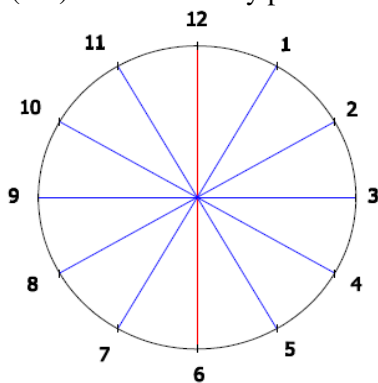


Fig. 4 Explanatory for defining the polar axis.

#### 3.2. PASSING FROM ONE CLOCK HOUR FIGURE OF THE VECTOR GROUP TO ANOTHER, USING THE POLAR AXES

The transition from one clock hour figure of the vector group to another, is achieved by following the transformations diagram, be in the clockwise, or counterclockwise. Crossing through the transformations diagram, can be made by the circular permutation of the lines or of the columns, in direct sense or in inverse sense, which is equivalent to changing linkages from N in Z or from Z in N, from the primary winding terminals or to the secondary winding terminals, changes which are made whenever needed, depending on the clock hour figure of the vector group which is reached.

In this regard we can mention that:

- modification of the linkages from N in Z to the primary winding, is equivalent with the circular permutation of the columns in reverse sense, followed by change the sign to the matrix elements;
- modification of the linkages from Z in N to the primary winding, is equivalent with the circular permutation of the columns in direct sense, followed by change the sign to the matrix elements;
- modification of the linkages from N in Z to the secondary winding, is equivalent with the circular permutation of the lines in reverse sense, followed by change the sign to the matrix elements;
- modification of the linkages from Z in N to the secondary winding, is equivalent with the circular permutation of the lines in direct sense, followed by change the sign to the matrix elements.

Consider the polar secondary axis (11-5), which has as axes perpendicular, the axes: (10-12), (9-1), (2-8), (3-7) and (4-6) (fig. 5). In Fig.6, it can be seen the crossing per the transformations diagram, to changing from the clock hour figure of the vector group 10 to the clock hour figure of the vector group 12, which can be done as follows:

$$G_{10} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \quad G_{12} = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

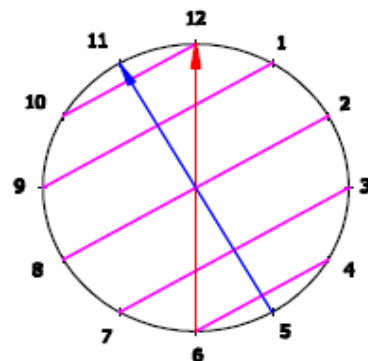


Fig. 5 The perpendicular axes to the secondary polar axis 11 - 5.

- the passing on the small outline - is done by changing the linkages in the secondary winding, from N to Z, or in the primary winding from Z to N:

$$(-1) \cdot G_{10} \uparrow = (-1) \cdot \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} = G_{12}$$

$$(-1) \cdot G_{10} \rightarrow = (-1) \cdot \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} = G_{12}$$

- the passing on the large outline - is done by six possibilities:

- modification of the linkages from Z to N, to the secondary winding, by five times;
  - modification of the linkages from Z to N, to the secondary winding, by four times, and from N to Z, to the primary winding;
  - modification of the linkages from Z to N, to the secondary winding, by three times, and from N to Z, to the primary winding, by twice;
  - modification of the linkages from Z to N, to the secondary winding, by twice, and from N to Z, to the primary winding, by three times;
  - modification of the linkages from Z to N, to the secondary winding, and from N to Z, to the primary winding, by four times;
- modification of the linkages from N to Z, to the primary winding, by five times.

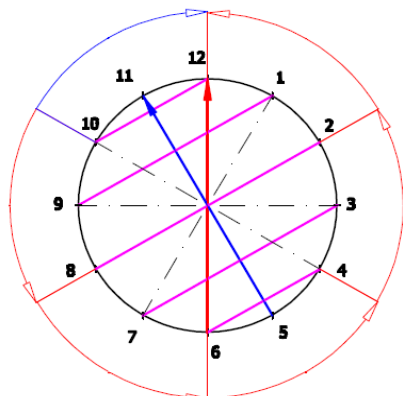


Fig. 6 Possibilities to change from the clock hour figure of the vector group 10 to 12.

#### 4. CONCLUSIONS

Were identified relations for crossing from an clock hour figure of the vector group, to another, for all polar axes:

- ✓ For the axes (1-3), (2-4), (3-5), (4-6), (6-8), (7-9), (8-10), (9-11), (10-12) and (12-2), who are perpendicular to the polar secondary axis, moving from the clock hour figure of the vector groups 1, 2, 3, 4, 6, 7, 8, 9, 10 and 12, to the clock hour figure of the vector groups 3, 4, 5, 6, 8, 9, 10, 11, 12 and 2, it do with relations:

$$\begin{aligned}
 (-1) \cdot G_i \uparrow &= G_{i+2} \\
 (-1) \cdot \vec{G}_i &= G_{i+2} \\
 5 X [(-1) \cdot G_i \downarrow] &= G_{i+2} \\
 4 X [(-1) \cdot G_i \downarrow] + [(-1) \cdot \vec{G}_i] &= G_{i+2} \\
 3 X [(-1) \cdot G_i \downarrow] + 2 X [(-1) \cdot \vec{G}_i] &= G_{i+2} \\
 2 X [(-1) \cdot G_i \downarrow] + 3 X [(-1) \cdot \vec{G}_i] &= G_{i+2} \\
 [(-1) \cdot G_i \downarrow] + 4 X [(-1) \cdot \vec{G}_i] &= G_{i+2} \\
 5 X [(-1) \cdot \vec{G}_i] &= G_{i+2}
 \end{aligned}$$

for  $i = 1 \div 4, 6 \div 10, 12$ .

This means one of the following operations made at the terminals transformer:

- modification of the linkages from N to Z, to the secondary winding;
- modification of the linkages from Z to N, to the primary winding;
- modification of the linkages from Z to N, to the secondary winding, by five times;
- modification of the linkages from Z to N, to the secondary winding, by four times, and from N to Z, to the primary winding;
- modification of the linkages from Z to N, to the secondary winding, by three times, and from N to Z, to the primary winding, by twice;
- modification of the linkages from Z to N, to the secondary winding, by twice, and from N to Z, to the primary winding, by three times;
- modification of the linkages from Z to N, to the secondary winding, and from N to Z, to the primary winding, by four times;
- modification of the linkages from N to Z, to the primary winding, by five times.

✓ For the axes (1 - 5), (2 - 6), (3 - 7), (5 - 9), (6 - 10), (7 - 11), (8 - 12), (9 - 1), (11 - 3) and (12 - 4), who are perpendicular to the polar secondary axis, moving from the clock hour figure of the vector groups 1, 2, 3, 5, 6, 7, 8, 9, 11, and 12, to the clock hour figure of the vector groups 5, 6, 7, 9, 10, 11, 12, 1, 3, and 4, it do with relations:

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$$\begin{aligned}
 G_i \downarrow &= G_{i+4} \\
 \overleftarrow{G}_i &= G_{i+4} \\
 2 X [(-1) \cdot G_i \uparrow] &= G_{i+4} \\
 [(-1) \cdot G_i \uparrow] + [(-1) \cdot \overrightarrow{G}_i] &= G_{i+4} \\
 2 X [(-1) \cdot \overrightarrow{G}_i] &= G_{i+4} \\
 4 X [(-1) \cdot G_i \downarrow] &= G_{i+4} \\
 3 X [(-1) \cdot G_i \downarrow] + [(-1) \cdot \overleftarrow{G}_i] &= G_{i+4} \\
 2 X [(-1) \cdot G_i \downarrow] + 2 X [(-1) \cdot \overleftarrow{G}_i] &= G_{i+4} \\
 [(-1) \cdot G_i \downarrow] + 3 X [(-1) \cdot \overleftarrow{G}_i] &= G_{i+4} \\
 4 X [(-1) \cdot \overleftarrow{G}_i] &= G_{i+4}
 \end{aligned}$$

for  $i = 1 \div 3, 5 \div 9, 11, 12$ .

This means one of the following operations made at the terminals transformer:

- the circular permutation of the lines to the matrix of code, in direct sense;
- the circular permutation of the columns to the matrix of code, in reverse sense;
- modification of the linkages from N to Z, to the secondary winding, by twice;
- modification of the linkages from N to Z, to the secondary winding, and from Z to N, to the primary winding;
- modification of the linkages from Z to N, to the primary winding, by twice;
- modification of the linkages from Z to N, to the secondary winding, by four times;
- modification of the linkages from Z to N, to the secondary winding, by three times, and from N to Z, to the primary winding;
- modification of the linkages from Z to N, to the secondary winding, by twice, and from N to Z, to the primary winding, by twice;
- modification of the linkages from Z to N, to the secondary winding, and from N to Z, to the primary winding, by three times;
- modification of the linkages from N to Z, to the primary winding, by four times.

✓ For the axes (1 - 7), (2 - 8), (4 - 10), (5 - 11) and (6 - 12), who are perpendicular to the polar secondary axis, moving from the clock hour figure of the vector groups 1, 2, 4, 5, and 6, la indicii orari 7, 8, 10, 11, respectiv 12, se face cu relațiile:

$$\begin{aligned}
 (-1) \cdot G_i &= G_{i+6} \\
 3 X [(-1) \cdot \overleftarrow{G}_i] &= G_{i+6} \\
 2 X [(-1) \cdot \overleftarrow{G}_i] + [(-1) \cdot G_i \downarrow] &= G_{i+6} \\
 [(-1) \cdot \overleftarrow{G}_i] + 2 X [(-1) \cdot G_i \downarrow] &= G_{i+6} \\
 3 X [(-1) \cdot G_i \downarrow] &= G_{i+6} \\
 3 X [(-1) \cdot \overrightarrow{G}_i] &= G_{i+6} \\
 2 X [(-1) \cdot \overrightarrow{G}_i] + [(-1) \cdot G_i \uparrow] &= G_{i+6} \\
 [(-1) \cdot \overrightarrow{G}_i] + 2 X [(-1) \cdot G_i \uparrow] &= G_{i+6} \\
 3 X [(-1) \cdot G_i \uparrow] &= G_{i+6}
 \end{aligned}$$

for  $i = 1, 2, 4, 5, 6$ .

This means one of the following operations made at the terminals transformer:

- the change of the sign, of the code matrix elements;
- modification of the linkages from N to Z, to the primary winding, by three times;
- modification of the linkages from N to Z, to the primary winding, by twice, and from Z to N, to the secondary winding;
- modification of the linkages from N to Z, to the primary winding, and from Z to N, to the secondary winding, by twice;
- modification of the linkages from Z to N, to the secondary winding, by three times;
- modification of the linkages from Z to N, to the primary winding, by three times;
- modification of the linkages from Z to N, to the primary winding, by twice, and from N to Z, to the secondary winding;
- modification of the linkages from Z to N, to the primary winding, and from N to Z, to the secondary winding, by twice;
- modification of the linkages from N to Z, to the secondary winding, by three times.

**ACKNOWLEDGMENT:** This paper was supported by the project "Progress and development through post-doctoral research and innovation in engineering and applied sciences– PRiDE - Contract no. POSDRU/89/1.5/S/57083 ", project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

## BIBLIOGRAPHY

- [1] **BĂLĂ, C.** *Mașini electrice – teorie și încercări*. București: Editura Didactică și Pedagogică, 1982, p.120-129.
- [2] **BULGAKOV, N. I.** *Grupele de conexiuni ale transformatoarelor. Traducere din limba rusă*. București: Editura Tehnică, 1957, p.31-40.
- [3] **CERNOMAZU, D.; MANDICI, L.**; et al. *Studiul privind modernizarea tehnologiei de diagnosticare a transformatoarelor de mare putere sosite pentru reparat*. Suceava: Universitatea "Ștefan cel Mare", Facultatea de Inginerie Electrică, Contract de Cercetare Nr.1/1990, Beneficiar: Uzina de Reparat Transformatoare și Aparataj Electric Roman (U.R.T.A.E. Roman), 1990.
- [4] **CERNOMAZU, D.** *Étude du modèle mathématique de l'indice horaire d'un transformateur triphasé – ouvrage de stage à l'Université des Sciences et Technologies de Lille*. France: Coordonnateur: le professeur Philippe Delarue, 1995.
- [5] **CERNOMAZU, D.** *Études sur le modèle mathématique de l'indice horaire d'un transformateur triphasé*. Suceava: Éditions de l'Université Suceava, 1997.
- [6] **CERNOMAZU, D.; RAȚĂ, M.; RAȚĂ, G.** *Transformatorul electric. Proiectare. Construcție. Exploatare. Colecție de standarde, norme, regulamente și instrucțiuni tehnice*. Suceava: Universitatea "Ștefan cel Mare", Facultatea de Inginerie Electrică, 1999.
- [7] **CIOC, V.; CALOTĂ, C.**; et al. *Transformatorul electric. Construcție. Teorie. Exploatare. Fabricare. Exploatare*. Craiova: Editura Scrisul Românesc, 1989, p.71-78, p.111-116.
- [8] **COJAN, M.; SIMION, Al.; LIVADARU, L.**; et al. *Mașini electrice. Aplicații practice*. Iași: Editura Shakti, 1998, p.41-46.
- [9] **COLIN, M.; LAVOIE, P.** *Mathématiques pour techniques de l'industrie*. Canada, Québec: Éditions Gaëtan Marin, 1986, p.173-193.
- [10] **EDELMANN, H.** *Calculul electric al rețelelor interconectate*. București: Editura Tehnică, 1966, p.266-289.
- [11] **GHEORGHIU, I. S.; FRANSUA, Al.** *Tratat de mașini electrice - Transformatoare. Volumul II*. București: Editura Academiei Române, 1970, p.56-61, p.81-102.
- [12] **LAZU, C.** *Mașini electrice*. București: Editura Didactică și Pedagogică, 1966, p.247-260.
- [13] **POPOVICI, A.; SIMION, Al.**; et al. *Îndrumar pentru lucrările de laborator de mașini electrice*. Iași: Institutul Politehnic Iași, Facultatea de Electrotehnică, 1973, p.188-201.
- [14] **PRODAN, Cr.** *Contribuții teoretice și experimentale privind conexiunile și grupele de conexiuni la transformatoarele electrice de forță – Teza de doctorat*. Suceava: Universitatea "Ștefan cel Mare", Facultatea de Inginerie Electrică și Știința Calculatoarelor, 2008.
- [15] **THUILLER, P.** *Cours de mathématiques supérieures – Enseignement technique*. Paris: Éditions Masson et Cie, 1973, p.155-179.

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### About the authors

Eng. **Cristina PRODAN**, PhD.  
University „Ștefan cel Mare” from Suceava.  
email:cristinap@eed.usv.ro

Graduated at the University „Ștefan cel Mare” Suceava. In 2008 she obtained PhD degree in electrical engineering field. Presently, Postdoctoral Researcher in the School of the University "Stefan cel Mare" Suceava, Electrical Engineering Department.