COMPUTER ASSISTED SIMULATION OF MARKET SHARES FOR COMPETING PRODUCTS USING MARKOV CHAINS

Radu BUCEA-MANEA-ȚONIȘ[1], Rocsana BUCEA-MANEA-ȚONIȘ[1]

[1] Spiru Haret University

ABSTRACT. Article implements Markov chains in prediction of market shares for competing products, in a given time horizon. Practical approach is implemented in C code, recursively generates transition probabilities to determine the limit vector π. The case study is performed on the data of a company producing computer motherboards. Ms.Excel is used to calculate the vector of initial market shares, the matrix of transition probabilities, steady state and recurrence time. Based on the results is the economic interpretation in terms of marketing manager.

Keywords: Markov chains, probabilities, forecast, market share, marketing simulation.

1. INTRODUCTION

In simulations marketing Markov chains are often used for forecasting market shares of competing products in a given time horizon and for determining the state of equilibrium, at which customers do not switch from one brand to another, staying true to a one brand. Forecast of market share will be based on preferences and customer satisfaction for a product generally expressed in a survey. Customer satisfaction is measured by:

• degree of precision - the percentage of customers who are given is to buy the same brand;

• the attraction - gave the percentage of customers who have switched from one brand to another - is called the transition probability pij.

These transition probabilities are obtained from a panel of clients, participating in many surveys over time to follow their preferences change

They say that the random variables $(X_n)_{n\in N}$ form a Markov chain if any $n \in N$, the probability of reaching the state x_{n-1} in one of the states of a set $A_n \in S$ does not depend on the trajectory to x_{n-1}, that the following relation according to [Popovici, 2013]:

$$P(X_n = x_n|X_{n-1} = x_{n-1}, \ldots, X_1 = x_1) = P(X_n = x_n|X_{n-1} = x_{n-1}) = p(x_n|x_{n-1})$$

where $p(x_n|x_{n-1})$ is called the transition probability at step n, and $p_0(x) = P(X_0 = x)$ is called the initial distribution.

Analysis of transition probabilities is especially important at the stage of launching a new brand; it allows both after the first purchase behavior analysis and forecast of the change this behavior.

In the literature [Ratiu, 2013] Statistical estimation of transition probabilities can be calculated using the formula $q_{i\,j}(t) = \frac{N_{ij}(t)}{N_i(t-1)}$, where:

- $t = 1,2, \ldots, q$. is more successive periods,
- $N_{i\ (t-1)}$ is the number of consumers who bought the sample selected brand *i* in period *t-1*,
- $N_{ij\ (t-1)}$ is the number of those consumers who have purchased brand sample *i* in the period *t-1* and brand *j* in period *t*.

This ratio represents the proportion of consumers who have switched from buying brand *i* in period *t-1* to purchase brand *j* in period *t* interpreting the ratio $q_{ij}(t)$ as a random variable, we can prove that its average value is $\overline{q_{ij}(t)} = p_{ij}(t)$.

It is said that $(X_n)_{n\in N}$ is a homogeneous Markov chain where the transition probabilities $p_n(A|x)$ do not depend explicitly to n, according to:

$$p_n(x_j, x_i) = p_{ij}, \forall i,j \in I, I = \{i|x_i \in S\}$$

Markov chains base on the assumption that the client will tend to buy today, same brand purchased in a prior period.

Whether probability transition matrix $P = (p_{ij})_{i,j\in I}$ and the initial probability vector states $p = (p_i)_{i\in I}$, we call π states limit probability vector and $\lim_{n\to\infty} p^{(n)} = \pi$ true relationship [Popovici, 2013].

Follows from the above conditions for the application of a Markov model:

- all consumers is homogeneous in terms of choosing brands considered
- transition probabilities do not change over time;
- quantity acquired within a period to be, at least, on the average, the same for all consumers;
- the total number of consumers to be stable over time.

2. IMPLEMENTATION

It is implemented in C code the recursive generation program of transition probabilities to determine the limit vector π. It is a heuristic algorithm, not knowing the previous number of steps after which stabilizes the state vector probability matrix of transition probabilities.

Step 1. Declare variables that will change the value for each recursive call (prod, h, p) and initialize vector for the initial transition probabilities matrix and initial distribution of the states:

```
float matrix[][3]={{ 0.8, 0.1, 0.1},{0.2, 0.6, 0.2},{0.1, 0.2, 0.7}};
// probability matrix of the initial transition
float prod[3][3];    // transition probability matrix
float h[3][3];       // average share of input matrix
float p[3];          // state probability vector
float p2[3]={0.52, 0.3, 0.37}; // previous state probability vector
int i,j,k;
```

Step 2. It implements the method for calculating state probabilities vector - p after each transition:

```
int pi(float temp[][3])
{
        for(i=0; i<3; i++)
        {
                p[i]=0;
                for(j=0; j<3; j++)
                        p[i]+=temp[j][i]*p2[j];
                for(int l=0; l<3; l++)
                        p2[l]=p[l];
        }
        return 0;
}
```

Step 3. ***Hmed*** method generates average share of input-H matrix using the matrix obtained in the previous step:

```
int hmed()
{
    for(i=0; i<3; i++)
        for(j=0; j<3; j++)
        {
            h[i][j]=(prod[i][j]+matrix[i][j])/2;
        }

    for(i=0; i<3; i++)
        for(j=0; j<3; j++)
        {
            matrix[i][j]=h[i][j]; // initial matrix is updated with
                                  //each transition
        }
    return 0;
}
```

Step 4. ***MxM*** method recursively calculates the product P x H for each transition, accepting as input parameter vector of initial probabilities of transition:

```
int MxM(float temp[][3])  // calculated recursively for each
                          // transition the product PxH
{
    for(i=0; i<3; i++)
        for(j=0; j<3; j++)
        {
            prod[i][j] = 0;
            for(k=0; k<3; k++)
                prod[i][j] += temp[i][k] * temp[k][j];
        }
    pi(prod);
    hmed();
    MxM(h);
    return 0;
}
```

3. CASE STUDY: FORECAST MARKET SHARE OF COMPETITIVE PRODUCTS

A company producing motherboards for computers is interested in the study of the market for one of its brands. The information sought estimates the market share in comparison with those of competing brands (their names are B and C). For this product, in any of the three brands, the interval between successive purchases is about one month, and the market study revealed the following information relating to the behaviour of the 10,000 people surveyed (Table 1).

Table 1

General information – description of the behavior of clients

Produced by brand	Number of clients	Change purchase option			
		From A	From B	From C	Total “from”
A	5200	-	570	380	950

B	3000	700	-	615	1315
C	1800	200	300	-	500
Total	10000	900	870	995	-

The following data are analyzed:

In our case study the vector of ***initial market shares*** is: S0 = (0.52 0.3 0.37). Determination of initial probabilities are calculated as in the next example $\frac{5200}{10000}=0.52$. This is the initial probability of the market share for brand A at the moment 0, etc.

Matrix of transition probabilities, calculated as described below, is: $P=\begin{pmatrix}0.8 & 0.1 & 0.1\\ 0.2 & 0.6 & 0.2\\ 0.1 & 0.2 & 0.7\end{pmatrix}$.

Transition probabilities for the brand A are: $p_{11}=\frac{5200-570-380}{5200}=0.8$ (with interpretation of the reliability coefficient for the product A), respectively $p_{12}=\frac{570}{5200}=0.1$ etc.

In the Excel worksheet we write the formulas: (D5-F5-G5)/D5 and F5/D5 etc. (Fig. 1)

Produs de marca	Numărul clienților	Schimbarea opțiunii de cumpărare: De la A	De la B	De la C	Total "de la"
A	5200	-	570	380	950
B	3000	700	-	615	1315
C	1800	200	300	-	500
total	10000	900	870	995	-

Matricea stohastica (P)			S_0
0,8	0,1	0,1	0,52
0,2	0,6	0,2	0,3
0,1	0,2	0,7	0,18
1,161752137	0,837948718	1,00029915	

Matricea stohastica			S_0
=(D5-F5-G5)/D5	=F5/D5	=G5/D5	=D5/$D$8
=E6/D6	=(D6-E6-G6)/D6	=G6/D6	=D6/$D$8
=E7/D7	=F7/D7	=(D7-F7-E7)/D7	=D7/$D$8

Fig. 1. The input data and the probability transition matrix (with related formulas)

In the matrix P, the first column describes the ***"attractiveness" of the product:***

- 80% (in absolute values: 4160 people = 5200 * 0.5) of customers who purchase in present the product A, will continue to buy it and next period;
- a percentage of 20%, *i*.e. 600 people (= 3000 * 0.2), of the current customers of product B will refocus on product A in the future;
- 10% (180 people = 1800 * 0.1) of product C customers will buy in the future (the next purchase) product A.

In Fig. 2 are past formulas that are written in the Excel worksheet for the calculation of market shares at different periods. Keeping up the algorithm until it reaches a steady state in which market shares tend to remain unchanged.

The components of the **steady state vector (SE)** represents the distribution of the corresponding market shares of competitive products analyzed in the hypothesis that the probability transition matrix will not change over the long term (so between different moments of time customers will not switch from one brand to another).

Steady state vector is determined by solving the equation: $S_E = S_E \cdot P$. This vector of limit probability is called the state of equilibrium or stability for the new brands (because over the long term, the likelihood that a consumer to buy a brand *i* tend to stabilize, approaching increasingly more of probability p_i).

Market shares of products A, B and C for the steady state are {42.27%, 26.25%, 31.47%}(Fig. 3)

Table 2

The evolution of market shares for 11 periods and the steady state

	A	B	C	D	E	F	G	H	I
11	**Step**	**Product**	**Matrix of transition probabilities**			**State probabilities vector (SE)**	**Average share of input matrix**		
12	1	A	0,8	0,1	0,1	0,52	0,8	0,1	0,1
13		B	0,2	0,6	0,2	0,3	0,2	0,6	0,2
14		C	0,1	0,2	0,7	0,18	0,1	0,2	0,7
15	2	A	0,70169	0,16334	0,13498	0,50603	0,75950	0,13648	0,10403
16		B	0,34454	0,37521	0,28025	0,23821	0,28894	0,46844	0,24262
17		C	0,20995	0,22616	0,56389	0,25576	0,16053	0,19641	0,64306
18	3	A	0,62660	0,19115	0,18224	0,49758	0,66415	0,17724	0,15861
19		B	0,40028	0,29522	0,30450	0,23189	0,37241	0,33522	0,29237
20		C	0,28702	0,24402	0,46896	0,27053	0,24848	0,23509	0,51643
21	4	A	0,57698	0,20642	0,21660	0,49084	0,60179	0,19879	0,19942
22		B	0,42987	0,26044	0,30969	0,22988	0,41508	0,27783	0,30709
23		C	0,34363	0,24668	0,40969	0,27928	0,31532	0,24535	0,43933
24	5	A	0,54380	0,21529	0,24091	0,48584	0,56039	0,21085	0,22875
25		B	0,44652	0,24502	0,30847	0,22946	0,43819	0,25273	0,30908
26		C	0,38393	0,24450	0,37157	0,28470	0,36378	0,24559	0,39063
27	6	A	0,52145	0,22068	0,25786	0,48225	0,53263	0,21798	0,24939
28		B	0,45639	0,23797	0,30564	0,22959	0,45145	0,24150	0,30705
29		C	0,41212	0,24134	0,34654	0,28816	0,39803	0,24292	0,35905
30	7	A	0,50633	0,22409	0,26958	0,47974	0,51389	0,22238	0,26372
31		B	0,46250	0,23463	0,30288	0,22984	0,45944	0,23630	0,30426
32		C	0,43165	0,23848	0,32987	0,29042	0,42189	0,23991	0,33820
33	8	A	0,49607	0,22629	0,27764	0,47799	0,50120	0,22519	0,27361
34		B	0,46640	0,23296	0,30064	0,23008	0,46445	0,23379	0,30176
35		C	0,44509	0,23624	0,31867	0,29192	0,43837	0,23736	0,32427
36	9	A	0,48909	0,22775	0,28316	0,47679	0,49258	0,22702	0,28040
37		B	0,46895	0,23208	0,29897	0,23028	0,46768	0,23252	0,29980
38		C	0,45430	0,23459	0,31111	0,29293	0,44970	0,23542	0,31489
39	10	A	0,48434	0,22873	0,28693	0,47596	0,48672	0,22824	0,28504
40		B	0,47065	0,23158	0,29777	0,23043	0,46980	0,23183	0,29837
41		C	0,46061	0,23341	0,30598	0,29361	0,45746	0,23400	0,30854
42	11	A	0,48111	0,22938	0,28951	0,47540	0,48272	0,22905	0,28822
43		B	0,47179	0,23129	0,29692	0,23053	0,47122	0,23144	0,29734
44		C	0,46492	0,23259	0,30249	0,29407	0,46277	0,23300	0,30424

	A	B	C	D	E	F	G	H	I
11	Pa	Pr	Matricea probabilitatilor de tranzitie			V.prob. Stare SE	Matricea med. prop. intarilor		
12	1	A	=J5	=K5	=L5	=0,52	=J5	=K5	=L5
13		B	=J6	=K6	=L6	=0,3	=J6	=K6	=L6
14		C	=J7	=K7	=L7	=0,18	=J7	=K7	=L7
15	2	A	=C12*G12+D12*G13+E12*G14	=C12*H12+D12*H13+E12*H14	=C12*I12+D12*I13+E12*I14	=C15*F12+C16*F13+C17*F14	=(C12+C15)/2	=(D12+D15)/2	=(E12+E15)/2
16		B	=C13*G12+D13*G13+E13*G14	=C13*H12+D13*H13+E13*H14	=C13*I12+D13*I13+E13*I14	=D15*F12+D16*F13+D17*F14	=(C13+C16)/2	=(D13+D16)/2	=(E13+E16)/2
17		C	=C14*G12+D14*G13+E14*G14	=C14*H12+D14*H13+E14*H14	=C14*I12+D14*I13+E14*I14	=E15*F12+E16*F13+E17*F14	=(C14+C17)/2	=(D14+D17)/2	=(E14+E17)/2
18	3	A	=C15*G12+D15*G13+E15*G14	=C15*H12+D15*H13+E15*H14	=C15*I12+D15*I13+E15*I14	=C18*$F$12+C19*$F$13+C20*$F$14	=(C15+C18)/2	=(D15+D18)/2	=(E15+E18)/2
19		B	=C16*G12+D16*G13+E16*G14	=C16*H12+D16*H13+E16*H14	=C16*I12+D16*I13+E16*I14	=D18*$F$12+D19*$F$13+D20*$F$14	=(C16+C19)/2	=(D16+D19)/2	=(E16+E19)/2
20		C	=C17*G12+D17*G13+E17*G14	=C17*H12+D17*H13+E17*H14	=C17*I12+D17*I13+E17*I14	=E18*$F$12+E19*$F$13+E20*$F$14	=(C17+C20)/2	=(D17+D20)/2	=(E17+E20)/2

Fig 2. Formulas for estimating market share at different moments

Recurrence time (TR) represents the time interval between two successive purchases of the same product. The numeric value for the recurrence time of *i* product is obtained as $TR_i = \dfrac{1}{p_i(S_E)}$.

For the product with the brand A, the recurrence time is 2.1503 and it represents the number of purchase processes taking place between successive purchases of the product (e.g. a person who buys the product at this time, is expected to buy the same product over about 2 months). For the product of the brand B, recurrence time is 3.4631 and for the product of the brand C recurrence time is 2.88. A person who buys the product B at this time, is expected to buy the same product over about 3 months and a half. A person who buys the product C at this time, is expected to buy the same product over about 3 months. In Fig. 3 are presented these values and the formulas that are written in the Excel worksheet for their calculation.

	A	B	C	D	E	F	G	H	I	J	K
42	11	1	0,42742	0,26072	0,31185	0,42277	0,42905	0,26010	0,31085	2,1503	=1/(1,1*F42)
43		2	0,41964	0,26376	0,31661	0,26251	0,41926	0,26394	0,31680	3,4631	=1/(1,1*F43)
44		3	0,41453	0,26559	0,31987	0,31472	0,41288	0,26619	0,32093	2,8885	=1/(1,1*F44)

Fig. 3. Steady state and time of recurrence

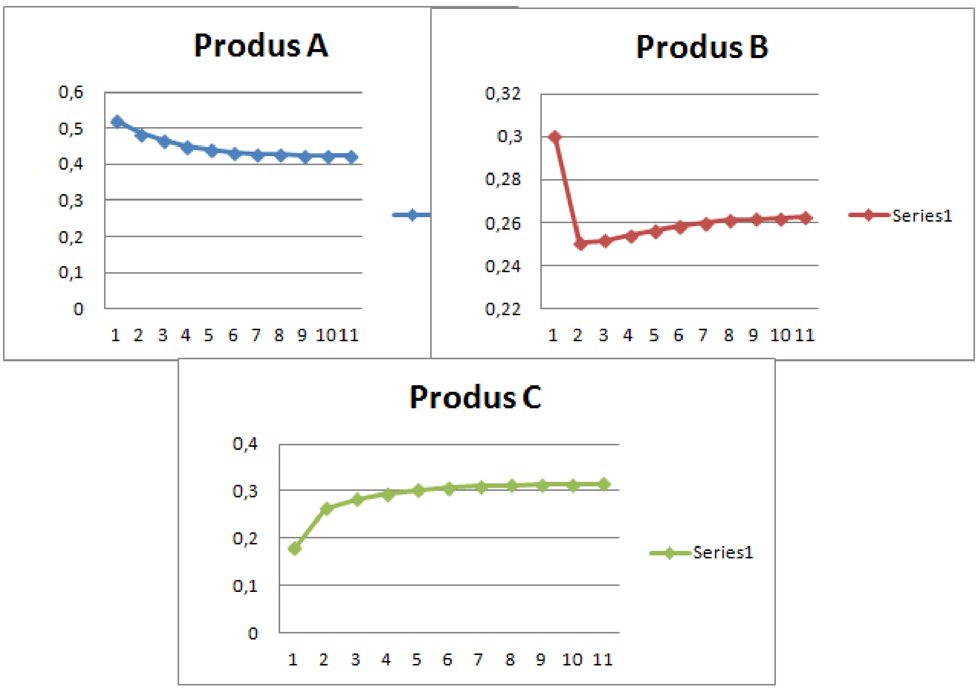

Fig.4. Representation of the evolution of the market share (forecast for 11 periods) for products A, B and C

Fig. 4 represents the evolution of the market share (forecast for 14 periods) for products A, B and c. Product A and B appear to be in decline phase of the life cycle, when it is recommended to restrict their share in the company's portfolio, or even the avoidance of maintaining them in the manufacturing process and restriction of investments that would mean wasting unnecessary because these products will not recover anyway. These products have a low market share.

Product C appears to be in a phase of transition from growth to maturity. It is a product with a slow-growing but having a high market share. If the competitive advantage has been achieved, these products bring big profit and generate a significant cash flow. Due to the slow growth does not require extra investment for promotion and distribution, however, are required investments to support the sale infrastructure. This product can lead to increased efficiency in the company and can bring extra cash flow. In general marketing managers want to have in their portfolio this kind of product, which corresponds to the category of milking cows in the Boston Consulting Group matrix

4. CONCLUSIONS

Marketing simulations with Markov chains are highly useful to forecast the market shares of some competing products on a specific time-frame, as well as for determining the state of equilibrium, at which time customers do not pass from one brand to another, remaining loyal to a single brand. So marketing managers may determine marketing strategies, setting in which product to invest, which product is recommended to be removed from the market, depending on the stage of the life cycle of the product.

BIBLIOGRAPHY

[1] **Popovici, A.**, *Probabilități, Statistică şi Econometrie asistate de programul Excel*, Ed. Niculescu, 2013, ISBN: 973-748-722-3

[2] Camelia RAȚIU-SUCIU, Florica LUBAN, Daniela HINCU, Nadia ENE, *Modelarea şi simularea proceselor economice*, http://www.biblioteca-digitala.ase.ro