VECTORIAL PROBABILISTIC MODEL FOR THE UNBALANCE FACTOR OF A THREE-PHASE ELECTRICAL NETWORK

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ABSTRACT. In a three-phase power electrical network, the voltage asymmetry can be split into two different components. The first component results from the network structure and possesses almost constant value caused by asymmetry of circuit elements (e.g., lines, transformers, etc.). The second component is a quasi-stationary fluctuation process because it comes from temporary asymmetry of electric loads, random switching on/off of single phase loads, transient response of three phase loads. For the previous reasons, a probabilistic vectorial model is proposed to take into account the random properties of voltage asymmetry. The probability distribution of its magnitude is determined by the two previous components, each one being characterized by its specific parameter. Finally, a new simple criterion characterizing asymmetry is proposed, useful for electrical power quality study.

Keywords: electrical power network, 3-phase unbalance, structural asymmetry, functional asymmetry, fluctuation process, unbalance factor, exceeding probability

1. INTRODUCTION

In industrialized countries, the reliability and productivity of a technical-economic system depends heavily upon the quality of the electric power delivered to customers. Increasing sophistication of industrial processes, especially their automation, requires diverse apparatus that are sources (and sometimes, victims, as well) of electrical disturbances. Electrical power system is a very complex system and its operation is subjected to several imperfections that are more or less inevitable. These ones become perceptible when customers' electrical appliances are affected.

Consequently, it is necessary to mitigate these imperfections but an improvement of power supply quality is usually limited by the knowledge of physical phenomena. Thus, more profound knowledge of a particular imperfection and the modeling of its mechanisms are the basis of this kind of study.

Among the imperfection of three-phase supply systems, the voltage asymmetry is often encountered [1]. The voltage asymmetry may affect the normal operation of certain power system apparatus: for example, harmonic spectrum generated by static power converters [2], supplementary losses and heating of cables, transformers and rotating machines [3, 4].

The purpose of this paper is to describe the phenomena generated by asymmetry and built up an explicative and usable model for the quantitative prediction of its effects and of the limits of possible improvements.

2. MODEL OF ASYMMETRY

2.1. Deterministic model

A three-phase balanced source is represented as a system of three equal magnitude vectors spaced at 120 degrees. Real systems do not conform to this ideal case and the theory of symmetrical components [5] has been introduced to describe the asymmetry that is practically always present.

A real system of three vectors $\underline{V}_1$, $\underline{V}_2$, $\underline{V}_3$ can be decomposed into a set of balanced positive and negative sequences given respectively by the equations:

$$\underline{V}_p = \frac{1}{3}\left(\underline{V}_1 + a\underline{V}_2 + a^2\underline{V}_3\right) \quad (1)$$

$$\underline{V}_n = \frac{1}{3}\left(\underline{V}_1 + a^2\underline{V}_2 + a\underline{V}_3\right) \quad (2)$$

Where $a = \exp(j2\pi/3)$ is an operator that defines a rotation of 120 degrees in the complex plane ;

Here, the zero phase sequence is ignored since it does not affect the energy conversion processes occurring in power electronic and electromechanical converters.

By definition, the sequences $\underline{V}_p$ and $\underline{V}_n$ are intrinsically balanced but the system asymmetry is given by a vectorial ratio usually named *unbalance factor*:

$$\underline{\tau} = \frac{\underline{V}_n}{\underline{V}_p} = \tau \exp(j\theta) \qquad (3)$$

This vectorial unbalance factor τ has two dimensions and is defined above in a polar coordinate system. Its two parameters (τ, θ) may be calculated from the actual system parameters which are easily measured [6].

Two types of asymmetry components are important in three phase networks: one of them is structural and the other one is functional.

The structural asymmetry is caused by the unequal phase impedances of the power network (lines and transformers) and of the consumer loads. That is why asymmetry low voltage drops would be present even if the generator voltages were perfectly balanced. Due to the slowly changing of the network structure, such an asymmetry is practically constant and can be considered as a quasi-deterministic value.

The functional asymmetry is due to consumer loads which may have variable cycles of demand, so that the phase currents are fluctuating and transitory unbalanced. Contrary to the quasi-deterministic nature of the structural asymmetry, the functional one due to consumer loads is frequently random and must be characterized in a probabilistic meaning.

2.2. Probabilistic model

Although the expression (3) describes mathematically the asymmetry, it is not always well employed and does not completely represent observations on a real network [6].

The model developed here covers both the vectorial nature of asymmetry and its random character resulting from its structural and functional components [7].

Let's consider a system of three vectors $\underline{V}_1$, $\underline{V}_2$, $\underline{V}_3$, 120 degrees phase displaced but having different amplitudes of mutually correlated *Gaussian* random variables [8] with means (μ_1, μ_2, μ_3) and a variance-covariance matrix (σ^2, ρ).

The structural component characterized by the differences in the mean values (μ_1, μ_2, μ_3) corresponds to the classic formulation by symmetrical components such as modulus (τ) in (3).

The functional component, due to random variation of consumer loads has stationary properties and its *Gaussian* nature corresponds to the limit of a flux of events defined by a *Poisson* process. When combined, the observable consumer loads may be considered as two "sub-ensembles" which, according to their unbalance level, determine the mutual correlation coefficient between the three vectors. It should be noted that some loads can be mutually independent (i.e. single phase domestic loads), while others are three-phase but mutually dependent (i.e. three-phase induction motors which the stator coils are electrically coupled in the slots).

These assumptions are realistic and compatible with a simplest model of fluctuating power loads [9] and with the observed experimental data on real networks [10].

In general, simplifications may be made noting that the positive phase sequence does not fluctuate to a high degree and remains close to the rated value of the voltage network. In a well balanced network the amplitude of the positive phase sequence voltage is practically equal to the nominal value but its relative fluctuation is comparable with the low level of negative phase sequence in per unit values. Practically, this one justifies the current definition for the positive phase sequence to be equal to the arithmetic mean of the amplitudes of the three phase voltages [6].

Because the unbalance factor (τ) has usually a low value and it is independent of the positive phase sequence (1), it can be assimilated only to the negative phase sequence value (2).

Taking into account the preceding hypothesis, the probability density function (pdf) of τ is given by [11]:

$$f(\tau) = \left(\frac{\tau}{s^2}\right) \exp\left[-\frac{\left(\tau^2 + m^2\right)}{2s^2}\right] I_0\left(\frac{m\tau}{s^2}\right) \qquad (4)$$

$$m = \frac{1}{3}\left[\sum_{i=1}^{3}\mu_i^2 - \sum_{j\neq k}^{3}\mu_j\mu_k\right]^{1/2} \qquad (5)$$

$$s = \left[(1-\rho)/6\right]^{1/2}\sigma \qquad (6)$$

$I_0(x)$: modified *Bessel* function of order zero, for the variable *x* [12].

In order to interpret the model and to characterize the asymmetry, it is sufficient to know only the two parameters (m, s).

The parameter (m) corresponds to the modulus and is fixed by the structural component of asymmetry when the system is perfectly balanced ($m = 0$).

The parameter (s) is the standard deviation of the functional component of asymmetry and is proportional to the standard deviation (σ) of the fluctuation induced by consumer loads. It is theoretically zero only when the random loads are totally correlated and perfectly balanced ($\rho = 1$) and/or when the loads are constant ($\sigma = 0$).

Relative parametric sensitivity of the probabilistic model may be investigated by considering asymptotic forms of the pdf (4) [7].

If, for example, the structural asymmetry is high but fluctuation of the load currents is low, then ($s/m << 1$) and the distribution (4) tends to a *Gaussian* distribution having the pdf:

$$f(\tau) \approx \left(1/s\sqrt{2\pi}\right)\exp\left[-(\tau-m)^2/2s^2\right] \quad (7)$$

Its mean is (m) and its standard deviation is (s).

When ($s \to 0$), the *Gaussian* pdf degenerates to a *Dirac* with fixed values (τ) and (θ) resulting from the deterministic model (3).

If the structural asymmetry tends to zero, the distribution can be assimilated to a *Rayleigh* stochastic process where the phase has a uniform pdf between 0 and 2π and the amplitude has the *Rayleigh* pdf:

$$f(\tau) \approx \left(\frac{\tau}{s^2}\right)\exp\left[-\frac{\tau^2}{2s^2}\right] \quad (8)$$

2.3. New quality criterion

The probabilistic model (4) can be used to define a new measure of voltage asymmetry.

A structurally balanced three-phase network usually contains a functional asymmetry caused by fluctuating single phase and partially three-phase correlated loads. A fundamental property results from (8) above: even if the three-phase network is perfectly structurally balanced (i.e. $m \equiv 0$), the natural load fluctuation (i.e. $s \neq 0$) induces inevitably a finite mean value for ($\tau_.$). So, from a probabilistic point of view, a real three-phase network is always slightly unbalanced.

We may therefore define a new quality scalar measure form assimilated to the inverse of a coefficient of variation:

$$Q = m/s \quad (9)$$

Note that (m) is due to the network structure under the responsibility of the utility and that (s) is caused by the consumer loads. From the consumer point of view, a perfectly balanced network is the ideal case where Q=0 and from the utility point of view, a constant and steady load corresponds to an ideal case where $Q >> 1$.

Obviously the reality is a compromise between these two situations.

3. APPLICATION CASE

3.1. Network characterization

The interest of the model lies in its ability to represent phenomenological effects and on its simple interpretation. The principal application concerns the utilization of criterion (9) which permits both the validation of the model and a quantitative characterization of the network supply quality.

In order to identify the model parameters, it is necessary to measure the asymmetry for obtaining a usable statistical distribution.

In practice, it is important to choose a sufficiently long duration for measurements where the random properties can be assumed stationary.

So, a sample statistic of $\tau_.$ can be characterized by its two first moments, that is a mean ($m_1^{'}$) and a standard deviation ($\sqrt{m_2}$).

Direct identification of expression (4) is intrinsically difficult, therefore it is preferable to choose an equivalent distribution such that:

$$f(\tau, r, s) \approx f(\tau, m, s) \quad (10)$$

The identification of (r, s) from empirical moments ($m_1^{'}, \sqrt{m_2}$) permits an explicit identification of (m, s) as a function of ($m_1^{'}, \sqrt{m_2}, r$). The non dimensioned parameter (r) is an auxiliary variable more easily identified statistically than (m) [13].

Table 1 presents the characteristics of two three-phase low voltage networks, a North American system (120/258V) and a European system (230/400V) denoted A and E respectively. In this table, except (s, Q) the other parameters are in percent.

Table 1

Measured parameters of asymmetry

Case	$m_1^{'}$	$\sqrt{m_2}$	r	m	s	Q
A	1.08	0.632	1.93	1.204	0.565	2.131
E	0.205	0.107	1.00	0	0.163	0

The choice of the two systems was made to highlight the difference between the two sets of data which are not necessarily typical for the countries in question [7].

Network A has marked structural and functional asymmetries, ($m = 1.204\%$) and ($s = 0.565\%$) respectively, so ($Q > 2$). The distribution tends to be *Gaussian* since the structural asymmetry predominates.

Network E is structurally well balanced ($Q \approx 0$) and feeds loads of low fluctuation ($m = 0, s = 0.163\%$). Its asymmetry is best described by the *Rayleigh* distribution (8).

In terms of the supply quality criterion (9), network E approaches the perfection, while network A is less perfect. These results can be explained by the study of other parameters such as the up-side short circuit level of the power network and the spatial distribution of connected single and three phase loads to the network structure.

3.2. Risk level of asymmetry

Knowledge of the parameters (m, s, r) may be used to calculate the asymmetry factor (τ_p) associated with a probability (p) of exceeding a predetermined level.

An approximate Gaussian form is explicitly derived in [14] as:

$$\tau_p \approx \sqrt{m^2 + s^2}\left[1 - (1/9r) + x_p\sqrt{1/9r}\right]^{3/2} \qquad (11)$$

For example, an exceeding probability of 5% corresponds to ($p = 95\%$) and to ($x_p = 1.645$).

Taking into account the standardized compatibility asymmetry level of 2% [1], we can notice that in case of network A with an exceeding probability of 5%, we obtain a value ($\tau_p \approx 2.06\%$). Such a network would not respect rigorously the acceptable standardized value.

5. CONCLUSIONS

Electromagnetic compatibility considers different forms of disturbances from the point of view of their specific effects and their interactions. Considering these aspects, three-phase asymmetry may introduce additional problems by interaction with power electronic converters and electrical machines. The asymmetry level must be limited to avoid the generation of excessive non characteristic harmonic currents and the overheating of induction motors.

The probabilistic approach developed here provides an analytical model characterized by only two fundamental parameters linked to the components of network voltage asymmetry. A criterion of supply quality has been proposed to separate the structural asymmetry due to network from the functional one caused by individual customer loads.

This model, simple and representative, is easily interpreted and its parameters may be identified experimentally. It may be used to demonstrate that in a real three-phase network system, even perfectly balanced in a structural sense, the random current fluctuations lead inevitably to a functional asymmetry. The last one can not be attenuated by means of passive compensation but only by active attenuation having the capability to compensate the process fluctuations dynamically.

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