FIXED END POINT OPTIMAL CONTROL OF A VOLTAGE CONTROLLED ELECTRICAL DRIVE SYSTEM WITH VARIABLE TORQUE

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REZUMAT. Se prezintă o problemă de control optimal din punct de vedere energetic a unui sistem de actionare electrică. Se are în vedere o variație treaptă a cuplului rezistent în intervalul de optimizare. Criteriul de performanță ține cont doar pierderile Joule, deoarece acestea depăşesc în mod semnificativ alte pierderi în perioada tranzitorie. Lucrarea se referă la o structură cu control în tensiune a sistemului de acționare. Studiul se prezintă în domeniu discret.

Cuvinte cheie: actionare electrică, cuplu rezistent variabil, control optimal linear patratic discret, stare finala fixată.

ABSTRACT. The optimal control from the energetic point of view of the transient state of electrical drive systems is presented. A step variation of the load torque in the optimization interval is considered. The performance criteria consider only Joule losses, since they significantly overcome other ones in the transient state. The paper refers to a structure with voltage control of the drive. The study is performed in discrete time domain.

Keywords: electrical drive, variable load torque, discrete linear quadratic optimal control, fixed end point.

1. INTRODUCTION

The paper deals with an optimal control [1], [2] from energetic point of view of an electrical drive system This problem is important because can ensure a significant way for energy saving. There are numerous studies dedicated to this problem, for different types of motors, control strategies, criteria, or used methods (for instance, we mention [3], [4], [5], [6], [7], but many other papers can be indicated). The optimization is appreciated as a main direction of the developing of the electrical drive systems in the future [8].

It should be also noticed that the optimal control is useful not only for energy saving, but in many cases offers the possibility to reduce the motor rated power and therefore the weight and volume. Indeed, the motor rated power is chosen from heat consideration and the optimal control leads just to a diminished Joule losses. Moreover, the decrease of the weight of some sub-ensemble leads in certain applications to the decrease of the energy consumption of whole plant.

The control of an electrical drive must be chosen so that to obtain a small energy losses and an acceptable behaviour of the system. The demands and conditions for different applications are not the same and therefore, one can formulate different optimal control problems for an electrical drive system. Some considerations for various applications are indicated below.

- It is possible to adopt one or two control variables. The last variant is useful for optimal control if the drive operates in many situations with a reduced load. However, the most used case refers to a system with one control variable.
- In order to ensure small energy losses and an acceptable behaviour, the optimization criterion contains different components. Also, the formulated optimal problem can be with free or fixed final time, or with free or fixed final states variables.
- The structure of the optimal drive system may be with current or with voltage control, depending on the type of the power electronic converter. The first one can be easier implemented, but different other considerations can influence the choice of the structure.
- The optimization can refer to the steady state (more frequently), or to the dynamic optimization of the transient state. It should be noted that the criterion express the power in the first case and the energy in the dynamic optimization problems. In the last case, the criterion refers only to the cooper losses, since they significantly overcame all other losses, because of very great values of currents.
- The problems can present differences depending on the used motor type, but a general approach can be adopted.

The authors have presented in some previous paper different variants for optimal control problems for transient period of the electrical drives. For instance, the voltage control for the drives with constant load torque was studied in [6] and [9]. The current control variant was presented in [10] for constant load torque and in [11] for a variable one. The fixed end point optimal control for constant torque is discussed in [12]. A control problem for a voltage controlled drive system with variable torque, for a free end point problem is presented in [13].

The paper deals with optimal control of a voltage controlled drive system with variable load torque. Of course, a general algorithm for a variable torque can be established, but the implementation is significantly simpler for a constant load torque. Therefore, a suboptimal solution can be obtained if the variation of the load torque in the transient period is approximated with a step function. Such situations are frequently met in the electrical drive systems, when the no-load or small load torque is succeeded by a great one. Examples for such operation are the rolling mills, or cutting processes.

The studied problem refers to one with fixed end point and with fixed final time and it is performed in discrete time domain.

Only the case of one control variable is considered. The results are valid for different motor types, because the mathematical model is the same with adequate assumptions.

2. PROBLEM FORMULATION

A linear electrical drive system is described by the state equation

$$\dot{\tilde{x}}(t) = A_c\tilde{x}(t) + B_c u(t) + \tilde{w}(t), \quad \tilde{x}(0) = \tilde{x}^0, \tag{1}$$

where $\tilde{x}(t) = [\omega(t)\ i(t)]^T$ is the state vector (T denotes the transposition), $\tilde{w}(t) = W_c m(t)$ is the disturbance variable (with $m(t)$ the load torque) and $\omega(t)$ is the rotor speed. The variables i and u are the rotor current and voltage in the case of a brushed d.c. motor (the flux is constant). For synchronous and asynchronous motors, i and u correspond to the q current and voltage components and similar equations may be adopted with adequate assumptions (mainly, the i_d component is constant or very small). The matrices A_c, B_c and the vector $\tilde{w}(t)$ are in the form

$$A_c = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \ B_c = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}, \ \tilde{w}(t) = W_c m(t) = \begin{bmatrix} w_1 \\ 0 \end{bmatrix} m(t), \tag{2}$$

depending on the constant parameters of the drive system.

In the case of the electrical drive system, the aim of the optimal control is to obtain small energy consumption and steady state and transient errors as smaller as possible for the state variables.

Since the optimal controller will be implemented in a discrete manner, it is useful to approach the problem in discrete time. With the notations

$$A = e^{A_c\tau}, \ B = \int_0^\tau e^{A_c\tau} dt B_c, \ W = \int_0^\tau e^{A_c\tau} dt W_c, \tag{3}$$

where τ is the sampling period, the discrete model of the drive system is

$$\tilde{x}(k+1) = A\tilde{x}(k) + Bu(k) + w(k), \tag{4}$$

with $w(k) = Wm(k)$.

The discrete state was denoted $\tilde{x}(k) = \tilde{x}(k\tau)$, $k \in Z$ and similar notations were used for all other variables.

In order to achieve the mentioned goals and tacking into account that the problem is with fixed end point (we have to obtain exactly the desired speed ω_d at the final moment $t_f = k_f\tau$) it is adopted the performance index

$$J = \frac{\tau}{2}\sum_{k=0}^{k_f-1} q_2[\omega(k) - \omega_d]^2 + q_1 i^2(k) + pu^2(k) + \ell u(t)i(t), \tag{5}$$

which penalizes, by means of first and third terms, the transient error of the speed and the great values of the control variable. Also, the cooper energy losses are penalized by the second term and the last term refers to the global energy consumption. The presence of the last term in the criterion is not mandatory because the cooper losses significantly overcame all other losses in the transient period.

The performance index (5) can be written in the general form

$$J = \frac{1}{2}\sum_{k=0}^{k_f-1}\{\tilde{x}^T(k)Q_1\tilde{x}(k) + [\tilde{x}(k) - x_d]^T Q_2[\tilde{x}(k) - x_d] + u^T(k)Pu(k) + [\tilde{x}(k) - x_d]^T Lu(k) + u^T(k)L^T[\tilde{x}(k) - x_d]\} \tag{6}$$

with

$$Q_1 = \tau \cdot diag(0, q_1) \geq 0, \quad Q_2 = \tau \cdot diag(q_2, 0) \geq 0, \quad P = \tau \cdot p > 0, \quad L = \tau \cdot [0 \quad \ell/2]. \tag{7}$$

and the desired state vector $x_d = [\omega_d\ 0]^T$.

We introduce the deviation of the state vector $x(k) = \tilde{x}(k) - x_d$ and the system equation becomes

$$x(k+1) = Ax(k) + Bu(k) + e(k) \tag{8}$$

with $$e(k) = (A - I_n)x_d + Wm(k) \tag{9}$$

the vector of exogenous variables $\tilde{w}(t)$ and x_d (I_n is the n-order identity matrix).

With the previous translation the performance criterion becomes

$$J=\frac{1}{2}\sum_{k=k_0}^{k_f-1}\{[x(k)+x_d]^T Q_1[x(k)+x_d]+x^T(k)Q_2x(k)+ +u^T(k)Pu(k)+x(k)^T Lu(k)+u^T(k)L^T x(k)\}. \quad (10)$$

The optimal control problem is to find the closed loop control $u(x(k))$ which transfers the system (8) from the initial state $x(0)$ in the final state $x(k_f)=0$ and minimizes the criterion (10).

3. OPTIMAL CONTROLLER DESIGN

One obtain from the necessary conditions for optimality [2]

$$u(k)=-P^{-1}[B^T\lambda(k+1)+L^T x(k)], \quad (11)$$

where $\lambda(k)\in \mathbf{R}^n$ is the co-state vector and

$$x(k+1)=Ax(k)-BP^{-1}B^T\lambda(k+1)-BP^{-1}L^T x(k)+e(k) \quad (12)$$

$$\lambda(k)=(Q-LP^{-1}L^T)x(k)+(A-BP^{-1}L^T)^T\lambda(k+1)+Q_1x_d \quad (13)$$

with

$$N=BP^{-1}B^T, \tilde{A}=A-BP^{-1}L^T, \tilde{Q}=Q-LP^{-1}L^T. \quad (14)$$

The system (12), (13) can be written in the form

$$\gamma(k+1)=G\gamma(k)+r(k) \quad (15)$$

where

$$\gamma(k)=\begin{bmatrix}x(k)\\ \lambda(k)\end{bmatrix}\in R^{2n}, G=\begin{bmatrix}\tilde{A}+N\tilde{A}^{-T}\tilde{Q} & -N\tilde{A}^{-T}\\ -\tilde{A}^{-T}\tilde{Q} & \tilde{A}^{-T}\end{bmatrix}\in \mathbf{R}^{2n\times 2n}, \quad r(k)=\begin{bmatrix}r_1(k)\\ r_2(k)\end{bmatrix}=\begin{bmatrix}N\tilde{A}^{-T}Q_1x_d+e(k)\\ -\tilde{A}^{-T}Q_1x_d\end{bmatrix}\in \mathbf{R}^{2n}. \quad (16)$$

where $A^{-T}=\left(A^T\right)^{-1}$ (the matrix A is non-singular in the discrete case).

The solution to the linear non-homogenous equation (16) is

$$\gamma(k)=\Gamma(k)\gamma(0)+\sum_{i=0}^{k-1}\Gamma(i)r(k-i-1) \quad (17)$$

where Γ is the transition matrix for G:

$$\Gamma(k)=G^k=\begin{bmatrix}\Gamma_{11}(k) & \Gamma_{12}(k)\\ \Gamma_{21}(k) & \Gamma_{22}(k)\end{bmatrix}\in \mathbf{R}^{2n\text{x}2n},\ \Gamma_{ij}\in \mathbf{R}^{n\text{x}n},\ i,j=1,2. \quad (18)$$

Tacking into account that $x(k_f)=0$ one can finally obtain from (17)

$$\lambda(0)=-\Gamma_{12f}^{-1}[\Gamma_{11f}x(0)+e_\Sigma], \quad e_\Sigma=\sum_{i=0}^{k_f-1}[\Gamma_{11i}r_1(k_f-i-1)+\Gamma_{12i}r_2(k_f-i-1)] \quad (19)$$

with $\Gamma_{11f}=\Gamma_{11}(k_f)$, $\Gamma_{12f}=\Gamma_{12}(k_f)$ and $\Gamma_{12i}=\Gamma_{12}(i)$.

The matrix Γ_{12f} is non-singular if the system is completely controllable [13].

At this moment the vector $\gamma(0)$ is known and the solution (17) can be computed. Thus, the optimal control variable results from (11), (12) and (13)

$$u(k)=P^{-1}B^T\tilde{A}^{-T}Qx(k)-P^{-1}B^T\tilde{A}^{-T}\lambda(k)+P^{-1}B^T\tilde{A}^{-T}Q_1x_d, \quad (20)$$

where $x(k)$ and $\lambda(k)$ are replaced from (17). Finally, $u(k)$ is expressed in terms of $x(0)$ and $w(k)$ and this is an open loop control. In order to obtain a closed loop optimal control, $x(0)$ is expressed in terms of $x(k)$ from (17) and then it is introduced in (20). However, the real time computing of $u(k)$ is difficult since the formula is complicated and implies to compute in real time the inverse of a time variant matrix.

In order to avoid the mentioned drawbacks the paper proposes another method. This approach uses a change of variables:

$$\gamma(k)=U\rho(k),\quad \rho(k)=\begin{bmatrix}x(k)\\ v(k)\end{bmatrix}\in \mathbf{R}^{2n}, \quad U=\begin{bmatrix}I_n & 0\\ R & I_n\end{bmatrix}\in \mathbf{R}^{2n\times 2n},\quad U^{-1}=\begin{bmatrix}I_n & 0\\ -R & I_n\end{bmatrix}, \quad (21)$$

where R is a constant matrix (and not variable, as in classical procedures) and $v(k)$ is a supplementary vector. This vector contains terms depending on the exogenous variables x_d and m and a corrective component, which compensate the fact that R is a constant and not a time-variant matrix. The new system is

$$\rho(k+1)=H\rho(k)+U^{-1}r(k) \quad (22)$$

with

$$H=U^{-1}GU=\begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}\in \mathbf{R}^{2n\times 2n},\quad H_{ij}\in \mathbf{R}^{n\times n},\quad i,j=1,2 \quad (23)$$

Using (16), (21) and (23), yields

$$H_{21}=(I+RN)\tilde{A}^{-T}[R-\tilde{Q}-\tilde{A}^T(I+RN)^{-1}R\tilde{A}] \quad (24)$$

If we choose $\quad R-\tilde{Q}-\tilde{A}^T(I+RN)^{-1}R\tilde{A}=0 \quad (25)$

one obtain $H_{21}=0$ and

$$H_{11}=(I+NR)^{-1}\tilde{A},\ H_{12}=-N\tilde{A}^{-T},\ H_{22}=H_{11}^{-T}=(I+RN)\tilde{A}^{-T}. \quad (26)$$

Note that (25) is the Riccati algebraic equation that appears in the discrete linear quadratic problem with infinite final time.

The transition matrix $\Omega(k)$ of H can be partitioned in $n \times n$ blocks

$$\Omega(k) = \begin{bmatrix} \Omega_{11}(k) & \Omega_{12}(k) \\ 0 & \Omega_{22}(k) \end{bmatrix} \quad (27)$$

where

$$\Omega_{11}(k) = H_{11}^k,\ \Omega_{22}(k) = H_{22}^k,\ \Omega_{12}(k) = H_{12k} = \sum_{i=0}^{k-1} H_{11}^i H_{12} H_{22}^{k-i-1} \quad (28)$$

One can easily prove, starting from (16), (21) and (27), that

$$\begin{aligned} &\Gamma_{11} = \Omega_{11}(k) - \Omega_{12}(k)R,\ \Gamma_{12} = \Omega_{12}(k) \\ &\Gamma_{21} = R\Omega_{11}(k) - R\Omega_{12}(k)R - \Omega_{22}(k)R, \Gamma_{22} = R\Omega_{12}(k) + \Omega_{22}(k) \end{aligned} \quad (29)$$

The solution to the system (22) is

$$\rho(k) = \Omega(k)\rho(0) + \sum_{i=0}^{k-1} \Omega(k-i-1)U^{-1} r(i) \quad (30)$$

and we can explicit the components of the vector $p(k)$:

$$\begin{aligned} &x(k) = \Omega_{11}(k)x(0) + \Omega_{12}(k)v(0) + \\ &+\sum_{i=0}^{k-1} \{\Omega_{11}(k-i-1)r_1(i) + \Omega_{12}(k-i-1)[-Rr_1(i) + r_2(i)]\}, \end{aligned} \quad (31)$$

$$v(k) = \Omega_{22}(k)v(0) + \sum_{i=0}^{k-1} \Omega_{22}(k-i-1)[-Rr_1(i) + r_2(i)] \quad (32)$$

and $\quad v(0) = -\Omega_{12}^{-1}(k_f)\Omega_{11}(k_f)x(0) - \Omega_{12}^{-1}(k_f)e_\Sigma \quad (33)$

with e_Σ given by

$$e_\Sigma = \sum_{i=0}^{k_f-1} \{\Omega_{11}(k_f - i - 1)r_1(i) + \Omega_{12}(k_f - i - 1)[-Rr_1(i) + r_2(i)]\} \quad (34)$$

Finally, the optimal control is obtained as

$$u(k) = u_f(k) + u_s(k), \quad (35)$$

where $\quad u_f(k) = -P^{-1}[B^T \tilde{A}^{-T}(R - \tilde{Q}) + L^T]x(k) \quad (36)$

is the feedback component and

$$u_s(k) = -P^{-1}B^T \tilde{A}^{-T}[v(k) + Q_1 x_d] \quad (37)$$

is a supplementary component, depending on the vector $v(k)$, given by (31). This component contains a component depending on the initial state $x(0)$ and one depending on the exogenous vector *e*.

This last component can be computed only if $m(k)$ is beforehand known on the interval [0, k_f], because (32) and (33) can be computed only in this case. The problem can also be solved in the case when it is known the shape of $m(k)$, $k = 1, 2, ..., k_f$ and its magnitude is measured or estimated at the beginning of the optimization interval. The simplest case (but frequently met in electrical drives) $m(k) = m = constant$ was discussed in [12]. The paper deals with the case when the load torque has a step variation during the transient period, from a small (no-loaded) operation to another great value. Such situations are frequently met in different applications and, in many situations, it is possible to know beforehand the two values of the torque and the switching moment.

We shall suppose that the load torque is $m_1 = constant$ for $k = 1, 2, ..., k_\theta - 1$ and $m_2 = constant$ for $k = k_\theta, ..., k_f$. Consequently, the components of the vector *r* given by (16) will have the components r_{11}, r_{12}, and r_{21}, r_{22} on the mentioned intervals. The vector e_Σ given by (34) is computed with

$$\begin{aligned} e_\Sigma = &\sum_{i=0}^{k_\theta-1} \{\Omega_{11}(k_f - i - 1)r_{11} + \Omega_{12}(k_f - i - 1)[-Rr_{11} + r_{21}]\} \\ &+\sum_{i=k_\theta}^{k_f-1} \{\Omega_{11}(k_f - i - 1)r_{12} + \Omega_{12}(k_f - i - 1)[-Rr_{12} + r_{22}]\} \end{aligned} \quad (38)$$

For the mentioned two values of the load torque and tacking into account (28), the vector $v(k)$ can be computed with:

$$\begin{aligned} &v(k) = H_{22}^k v(0) + \sum_{i=0}^{k-1} H_{22}^{k-i-1}[-Rr_{11} + r_{21}], \quad for\ 0 < k < k_\theta \\ &v(k) = H_{22}^k v(0) + \sum_{i=0}^{k_\theta-1} H_{22}^{k-i-1}[-Rr_{11} + r_{21}] + \\ &\quad + \sum_{i=k_\theta}^{k-1} H_{22}^{k-i-1}[-Rr_{12} + r_{22}], \quad for\ k \geq k_\theta \end{aligned} \quad (39)$$

The supplementary component $u_s(k)$ of the control vector will be computed using the relations (33), (38) and (39).

Remark 1: The procedure indicated above can be extended for a drive system with a more general form of the variation of the load torque. In this case, it is possible to approximate this variation with a step function on several subintervals and the problem is solved in a similar manner.

Remark 2: The above presented relations are quite complicated, but the most part of the computing is performed off-line, in the stage of the controller design.

This stage implies to establish the solution to the discrete Riccati equation (25) (the command *lqrd* from Matlab can be used), and to compute the constant matrices and vectors.

The real-time computing implies to establish a usual feedback component $u_f(k)$ given by (36) and the supplementary component $u_s(k)$, given by (37), which depends on $v(k)$, given by (39). This last vector can be iteratively computed:

$$v(k) = H_{22}v(k-1) - (Rr_{11} - r_{21}), \quad for\ 0 < k < k_\theta$$
$$v(k) = H_{22}v(k-1) - (Rr_{12} - r_{22}), \quad for\ k \geq k_\theta$$

with $v(0)$ given by (33).

Remark 3: The adopted optimal control ensures that the desired value x_d will be reached at the final moment k_f. The behaviour after the moment k_f is not reflected by this control. Therefore one must adopt another control law in order to obtain a convenient behaviour of the drive system for $k > k_f$. A possibility in this direction can be found in [12].

4. SIMULATION RESULTS

A set o simulation test was performed for a drive system with the matrices in (2) having the values:

$$A_c = \begin{bmatrix} 0 & 20 \\ -3.6 & 19.4 \end{bmatrix}, \; B_c = \begin{bmatrix} 0 \\ 80 \end{bmatrix}, \; W_c = \begin{bmatrix} -35 \\ 0 \end{bmatrix}.$$

These matrices correspond to a drive system with a d.c. motor with rated data *U*=110V and *I*=3.3A. The sampling period is $\tau = 0.002s$ and the final time is 0.3s. The desired speed is $\omega_d = 25 rad/s$.

Fig. 1 shows the behaviour of the drive system for a step variation of the load torque from 0.2 Nm to 0.8 Nm (the rated value) at the moment $t = 0.1s$. Since the optimal control $u(t)$ is computed based on a beforehand computed „mean" value of the load torque, the optimal variation of the voltage and of the current are not affected by the step variation of the load torque. Only a small variation of the acceleration can be observed at the moment *t*=0.1s.

Fig. 2 and 3 indicate the effect of an erroneous estimation of the load torque and of the switching moment, respectively.

A comparison between the behaviour of the cases of correct estimation (continuous curves) and erroneous estimation (doted curves) is indicated in both cases.

In fig. 2, the result of estimation is considered the values 0.2 and 0.6 Nm (instead of real values 0.2 and 0.8). In Fig. 3, the switching moment was anticipated at 0.2s instead of 0.1s.

The variations of currents, voltages, velocities, energy losses and performance indices were compared for different estimation errors.

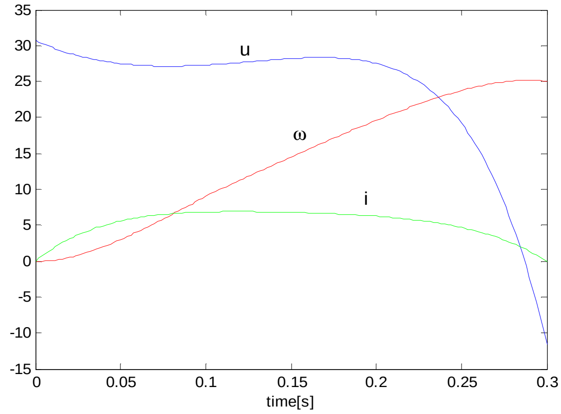

Fig. 1. Behaviour of the optimal system

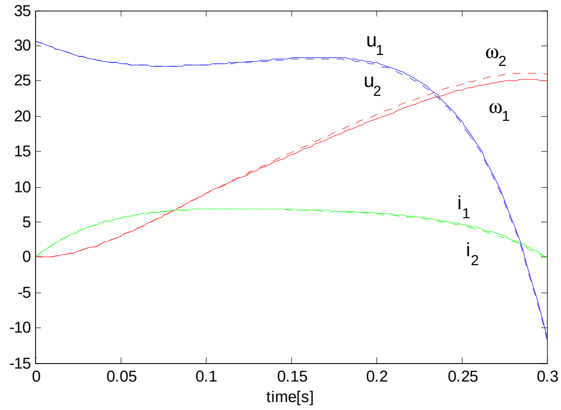

Fig.2. Behaviour in the case of an erroneous estimation of the load torque

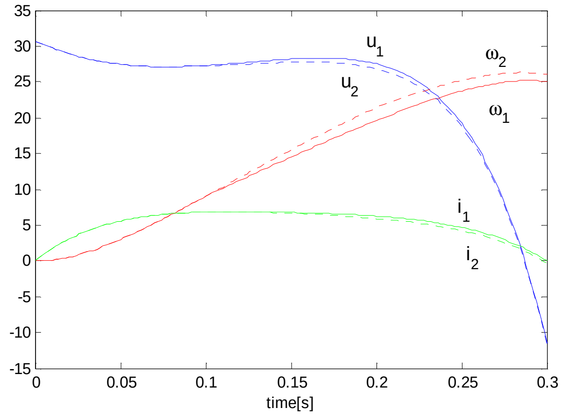

Fig. 3. Behaviour in the case of erroneous anticipation of the switching moment

The amplitude and sign of the mentioned variations are influenced by the sign and size of estimation errors. In any case, the variations of the mentioned entities around its optimal values are acceptable for reasonable size of estimation errors.

The analysis of the energy losses shows a small decrease in comparison with conventional cascade structure when the mean load torque is closely related to the rated one. But the reduction of the energy losses is up to 25% in the case of small mean load torque.

5. CONCLUSIONS

✓ A new method for the discrete optimal control problem with fixed end point for an electrical drive system with variable load torque is presented. The proposed algorithm can be easier implemented then the classical procedure for the LQ optimal control.

✓ The adopted criterion ensures a good behaviour for the system and a significant decrease of the energy consumption, especially in the transient states that appears at the changing of the imposed value of speed and the mean load torque is small in comparison with the rated one.

✓ The described optimal control is useful especially for the medium and high power electrical drives with frequent changing of the speed.

BIBLIOGRAPHY

[1] **Anderson**, **B.D.O.**, **Moore**, **J.B.**, *Optimal Control. Linear Quadratic Methods*. Prentice Hall, Englewood Clifs, New Jersey, 1990.

[2] **Kuo**, **B.C.**, *Digital Control System*. Saunders College Publishing, Philadelphia, 1992.

[3] **Mademlis**, **C**., **Margaris**, **N**., *Loss Minimization in Vector-Controlled Interior Permanent-Magnet Synchronous Motor Drives*. IEEE Trans. on Ind. Electronics, Vol. 49, No. 6, pp. 1344-1347, 2002.

[4] **Cavallaro**, **C**., **DiTommaso**, **A.O**., **Miceli**, **R**., **Raciti**, **A**., **Galluzzo**, **G.R**. and **Trapanese**, **M**., *Efficiency Enhancement of Permanent-Magnet Synchronous Motor Drives by Online Loss Minimization Approaches*. IEEE Trans. on Ind. Electronics, vol. 52, 4, pp. 1153- 1160, Aug. 2002.

[5] **Kurihara**, **K**., **Rahman**, **M.A**., *High-efficiency line-start interior permanent-magnet synchronous motors*. IEEE Trans. on Ind. Applic., vol.40, 3, pp. 789- 796, May-June 2004.

[6] **Botan**, **C**., **Onea**, **A**., *The Weight Selection in the LQ Optimal Control for an Electrical Drive System*. Proceedings of the 1997 IEEE Symposium on Industrial Electronics – ISIE'97, Guimaraes, pp. 1039-1043, 1997.

[7] **De Capua, C., Landi, C.**, *Measurement Station Performance Optimization for Testing on High Efficiency Variable Speed Drives*, IEEE Trans. on Instrumentation and Measurement, vol. 48, no. 6, pp. 1149-1154, 1999.

[8] **Lorenz**, **R.D**., *Future Motor Drive Technology Issues and their Evolution*. Proc. of 12th Int. power electronics and motion control conf. (EPE-PEMC 2006), Portoroz, Slovenia, CD-ROM, 2006.

[9] **Botan**, **C.**, **Onea**, **A**., *Discrete-Time Optimal Control of an Electrical Drive System*. Buletinul Inst. Politehnic din Iasi, Tomul XLV (IL), Fasc. 1-4, AC, pp. 35 – 42, 1999.

[10] **Botan, C**., **Horga, V**., **Ostafi, F**., **Albu**, **M**., **Ratoi**, **M**., *General Aspects of the Electrical Drive Systems Optimal Control*, 12th European Conf. on Power Electronics and Applications, Aalborg, Denmark, CD-ROM, Sept. 2007.

[11] **Botan**, **C., Ratoi, M., Horga, V**., *Optimal Control of Electrical Drives with Induction Motors for Variable Torques*, 13th Int. Power Electronics and Motion Control Conference, Poznan, Poland, CD-ROM, 2008.

[12] **Botan, C., Ostafi, F.**, *Discrete Free and Fixed End-Point Optimal Control Problems for Linear Electrical Drive Systems*. Proc. of 12th Int. Power Electronics and Motion Control Conf., August 30-September 1, Portoroz, Slovenia, CDROM, 2006.

[13] **Botan, C., Ostafi, F., Onea, A.**, *A Solution to the Fixed End-Point Problem*. Advances in Automatic Control, Ed.: M. Voicu, Kluver Academic Publishers, pp. 9-20, 2003.

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