THE TUNING OF THE PID AND PIDD² ALGORITHMS TO THE MODEL OBJECT WITH INERTIA SECOND ORDER AND TIME DELAY

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REZUMAT. În lucrare se analizează posibiltățile utilizării în stuctura regulatorului standard PID a unei componente adăugătoare, proporționale derivatei a doua, care corespunde accelerației abaterii mărimei reglate. Algoritmul standard PID şi modificat PIDD² sunt acordate la modelul obiectului cu inerție de ordinul doi şi timp mort după metoda gradului maximal de stabilitate cu iterații. Se efectuează o analiză comparativă a rezultatelor obținute la acordarea algoritmilor propuşi la obiectul dat.

Cosine cheie: obiect de reglare cu inerție şi timp mort funcție de transfer, algoritm PID, PIDD², parametrii de acord, acordarea algoritmului, metoda de acordare gradul maximal de stabilitate cu iterații .

ABSTRACT. In this paper was done the analyzation of possibilities the uses in the standard PID structure the additional component, the proportional second derivate, which corresponds with acceleration of the deviation of the control value. The standard algorithm PID and modified PIDD² are tuned to the model object with inertia (second order) and time delay using the maximal stability degree method with iteration. It was done the comparative analyzation of the obtained results of tuning the proposed algorithms to the model object.

Keywords: the control object with inertia second order and time delay, transfer function, algorithms PID, PIDD², tuning values, the tuning algorithm, transition process, the performances, the maximal stability degree method with iteration.

1. INTRODUCTION

At the present, the automation of various industrial and technological processes is based on the technology with microprocessors, which is able to realize the sophisticated control algorithms and, as result to obtain the high performance and good robustness of the control system.

In the automatic control systems of various industrial and technological processes is widely used the algorithm with proportional-integrative-derivative action (PID) [1, 2]. This type of algorithm has a simple structure of realization and allows to obtain for the control system the high performance and good robustness.

The object model with inertia second order and time delay is presented trough the following transfer function

$$H(s)=\frac{ke^{-\tau s}}{(T_1s+1)(T_2s+1)}=\frac{ke^{-\tau s}}{a_0s^2+a_1s+a_2}, \qquad (1)$$

where k is the transfer coefficient, T_1, T_2, τ - respectively the constants time and time delay, and a_0, a_1, a_2 - generalized coefficients of the control object.

For the model object (1) will be tuned the PID controller and PIDD² which are described with following transfer functions:

$$H_{PID}(s)=k_p+\frac{k_i}{s}+k_ds=\frac{k_ds^2+k_ps+k_i}{s}. \qquad (2)$$

$$H(s)=k_p+\frac{k_i}{s}+k_{d1}s+k_{d2}s^2=\frac{k_ps+k_i+k_{d_1}s^2+k_{d2}s^3}{s}. \qquad (3)$$

where k_p, k_i, k_d are the tuning parameters of the standard PID algorithm and k_p, k_i, k_{d1}, k_{d2} – tuning values of the modified PIDD² algorithm. In the modified algorithm was introduced the second derivative and with introducing the additional component in the algorithm the number of tuning values are increased and as result the calculations became complicated. The classical methods of tuning controller became unacceptable for this case.

There are many methods of tuning the standard PID algorithm to the model object with inertia [1, 2, 3]. Some of existing methods of tuning the PID controller have difficult procedures of tuning, others methods guarantee stability, but does not guarantee the quality of the operating mode.

For these reasons, in this paper is used the maximal stability degree method with iteration for tuning the controllers.

To highlight the effectiveness of this method it is necessary to realize the comparative analysis between tuning the standard PID controller and modified $PIDD^2$ to the model object with inertia and time delay.

It will be done the tuning the (2) algorithm and (3) algorithm to the model object (1) use the maximal stability degree method with iteration [3,4].

It will be analyzed the effectiveness of results in case of tuning the standard PID algorithm and the modified $PIDD^2$ algorithm and the case of variation the parameters of model object with ± 50% of the nominal values.

2. THE TUNING ALGORITHM OF CONTROLLERS

Assume, that the control system is formed by the object with transfer function $H(s)$, which is presented in relation (1), and transfer function of controller $H_R(s)$ with control laws PID, $PIDD^2$ (Figure 1).

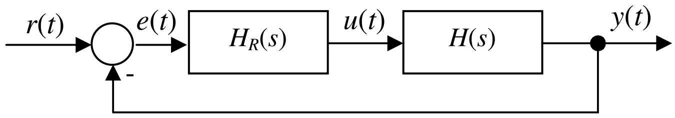

Fig. 1. The structure scheme of control system.

The maximal stability degree method with iteration it is presented in the restricted form. To tune the values of the PID controller is used the maximal stability degree method with iteration and is used the system of four algebraic equations with unknowns values J, k_p, k_i, k_d that were obtained in the author's works [4,5] and are presented in the following forms:

$$c_0 J^3 - c_1 J^2 + c_2 J - c_3 = 0, \tag{4}$$

$$k_p = \frac{e^{-\tau J}}{k}(a_0\tau^2 J^4 - J^3(\tau^2 a_1 + 5\tau a_0) + J^2(\tau^2 a_2 + 3\tau a_1 + 3a_0) - \tau J - 1); \tag{5}$$

$$k_i = \frac{e^{-\tau J}}{2k} J^3 (a_0\tau^2 J^2 - J(\tau^2 a_1 + 4\tau a_0) + \tau^2 a_2 + 2\tau a_1 + 2a_0), \tag{6}$$

$$k_d = \frac{e^{-\tau J}}{2k}(a_0\tau^2 J^3 - (\tau^2 a_1 + 6\tau a_0)J^2 + (\tau^2 a_2 + 4\tau a_1 + 6a_0)J - 2\tau a_2 - 2a_1), \tag{7}$$

where $c_0 = t^3 a_0$;

$c_1 = a_1 t^3 + 9t^2 a_0$;

$c_2 = a_2 t^3 + 6t^2 a_1 + 18t\, a_0$;

$c_3 = 3a_2 t^2 + 6t\, a_1 + 6a_0$.

Solved the expressions (4) it is determined the optimal value of stability degree J_{opt}, which presented the smallest positive and real root or the real positive part of the complex root.

From expressions (5), (6) and (7) were determined the optimal values of parameters k_p, k_i and k_d of PID controller. In this treatment of maximal stability degree method the parameters' values are taken the maximal values (view the curves from Figure 2). The performance of control system is verified by the computer simulation in the MATLAB (Figure 3) and the transient curve is presented in the Figure 4 (curve 1).

If the performances of control system are not satisfied the imposed requirements, when is using the maximal stability degree method with iteration, which is able to optimize the performance of control system. In thise cases the expresions (5)-(7) represent the functions of the $k_p=f(J)$, $k_i=f(J)$ and $k_d=f(J)$ and the known parameters of the control object and unknown value J . The variable J is varied from zero up to a certain value J_x (this value is chosen) and it is constructed the curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$. It is chosen the sets of suboptimal values of the J_i and at the respective slope of the curves are determined the suboptimal values of the tuning parameters values $k_{pi}=f(J_i)$, $k_{ii}=f(J_i)$, $k_{di}=f(J_i)$ of the PID controller, assuming that the value J_i is lesser or greater than J_{opt} value. For the chosen sets of tuning values of PID controller is simulated the control system and it is determined the highest possible performance of the control system.

To tune the values of the $PIDD^2$ controller is used the maximal stability degree method with iteration and is used the system of four algebraic equations with unknowns values J, k_p, k_i, k_{d1}, k_{d2}, which are presented in the following forms:

$$k_p = \frac{e^{-\tau J}}{k}(a_0\tau J^3 - J^2(3a_0 + \tau a_1) + J(2a_1 + \tau a_2) - a_2) - 3k_{d2}J^2 + 2k_{d1}J, \tag{8}$$

$$k_i = \frac{e^{-\tau J}\,{}^{3}}{k}(a_0 J^3 - a_1 J^2 + a_2 J) + k_{d2}J^3 - k_{d1}J^2 + k_p J, \tag{9}$$

$$k_{d1}=\frac{e^{-\tau J}}{2k}(a_0\tau^2J^3-J^2(6\tau a_0+\tau^2a_1)+ \\ +J(6a_0+4\tau a_1+\tau^2a_2)-2a_1-2\tau a_2), \quad (10)$$

$$k_{d2}=\frac{e^{-\tau J}}{6k}(\tau^3a_0J^3-J^2(9\tau^2a_0+\tau^3a_1)+ \\ +J(18\tau a_0+6\tau^2a_1+\tau^3a_2)-6a_0-6\tau a_1+3\tau^2a_2)). \quad (11)$$

To tune the modified algorithm PIDD^2 it is proceeding in the analogical mode as in the case of PID algorithm, but it is used the system of functions (8) - (11) – and is constructed the curves $k_p=f(J)$, $k_i=f(J)$, $k_{d1}=f(J)$, $k_{d2}=f(J)$ and chosen the sets of tuning values.

Below is presented the example of tuning parameters of PID algorithm and modified algorithm PIDD^2 to the model object (1).

3. APPLICATION AND COMPUTER SIMULATION

Assume that control object model (1) has the following values of parameters T_1=1,53 min, T_2=0.93 min, τ=0,2 min, a_0=1,4229, a_1=1,56, a_2=1. It is proposed to tune the PID parameters use the maximal stability degree method with optimal value and maximal stability degree method with iteration. Using the system of algebraic equations (4) - (6) are obtained the possible optimal values of the PID parameters, which are presented in the Table 1, line 1. According to the maximal stability degree method with iteration are obtained the curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ by the relations (5) - (7), which are presented in the Figure 2.

The chosen sets of values from the obtained curves are presented in the Table 1, rows 2-7. Computer simulation of the control system was made in MATLAB (Figure 3) and the obtained transient processes are presented in the Figure 4 (numbering of processes: 1 – control system with PID algorithm with optimal values, 2-7 correspond to the sets numbers 2-7 of the chosen values from the curves presented in the Figure 2) and the performance of control system are presented in the Table 2.

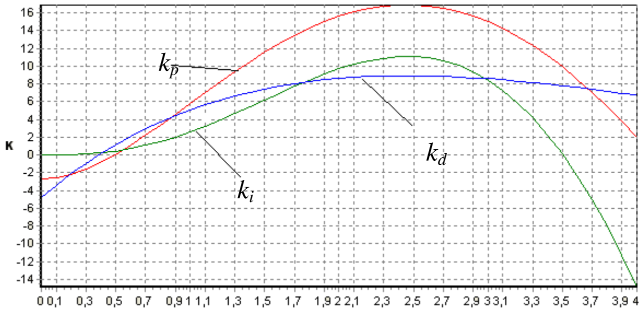

Fig. 2. The dependence PID controllers' parameters of the stability degree value.

For tune the modified algorithm PIDD^2 to the model object (1) is used the maximal stability degree method with iteration based on the algebraic equation (8) - (11) and was constructed the curves $k_p=f(J)$, $k_i=f(J)$, $k_{d1}=f(J)$, $k_{d2}=(J)$, which are presented in the Figure 5. The chosen sets of values from the obtained curves are presented in the Table 3, rows 1-7.

The computer simulation of the control system with modified algorithm PIDD^2 was made in MATLAB (Figure 3) and the obtained transient processes are presented in the Figure 6 (the numbering of processes: 1-7 correspond to the sets numbers 1-7 sets of the chosen values from the curves presented in the Figure 5) and the performance of control system are presented in the Table 4.

Table 1

The values of the PID controller's parameters

Nr. iter.	J	k_p	k_i	k_d
1	2.65	21.6183	14.6873	8.7518
2	1.2	9.7354	4.5329	5.1369
3	1.4	12.4647	6.3455	6.1895
4	1.6	14.9755	8.1864	7.0286
5	3.4	17.8053	8.6887	8.1438
6	3.5	16.7467	6.8621	7.994
7	**3.6**	**15.5651**	**4.7642**	**7.8240**

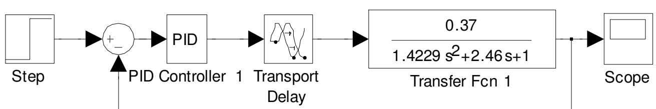

Fig. 3. Simulation diagrams of the control system.

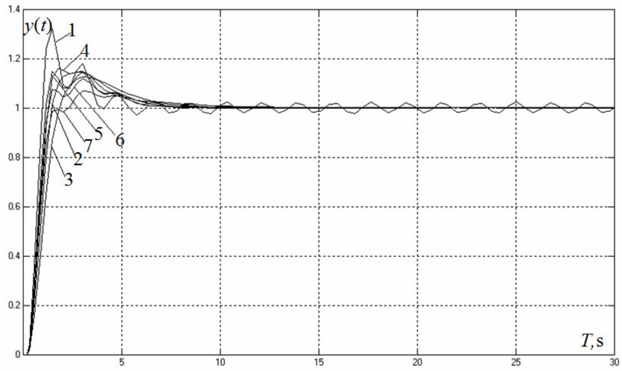

Fig. 4. The transient processes of control system with PID controller.

Table 2

The performance of control system with PID controller

Nr.	t_c	σ,%	t_r	λ
1	0.61	28.37	1.96	1
2	1.34	3.6	1.34	
3	0.81	12.5	2.4	1
4	0.98	7.7	2.49	1
5	0.69	14.6	1.87	1
6	0.72	10.5	1.63	1
7	**0.78**	**4.5**	**0.78**	

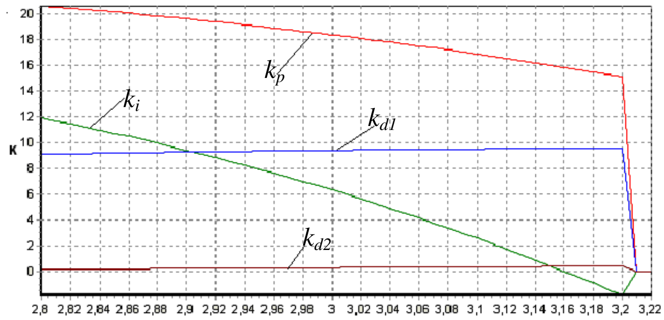

Fig. 5. The dependence of PIDD^2 controllers' parameters of the stability degree value.

Table 3

The values of the PIDD^2 controller's parameters

Nr. iter.	*J*	k_p	k_i	k_{d1}	k_{d2}
1	2,95	19,03	7,92	9,33	0,23
2	3	18,37	6,29	9.4	0,27
3	3,03	17,95	5,24	9,43	0,28
4	3,04	17,8	4,87	9,44	0,29
5	3,05	17,65	4,5	9,45	0,29
6	**3,06**	**17,5**	**4,12**	**9,46**	**0,3**
7	3,07	17,34	3,74	9,47	0,3

From analyzation the curves from Figure 2 and Table 1 it was observed that the maximal value of stability degree is J =2,65 and the tuning parameters have the maximal value.

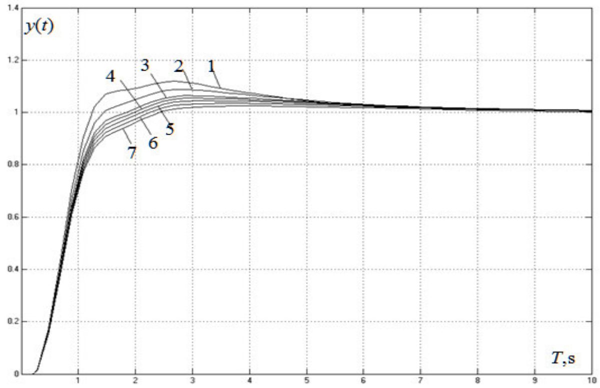

Fig. 6. The transient processes of control system with PIDD^2 controller.

Table 4

The values of the PIDD^2 controller's parameters

Nr.	t_c	σ,%	t_r	λ
1	0.65	9,76	1,53	1
2	0,67	6	2,1	1
3	0.69	3,7	0,69	
4	0,7	2,8	0,7	
5	0,7	2	0,7	
6	**0.71**	**1**	**0,71**	
7	0.72	0,5	2,93	

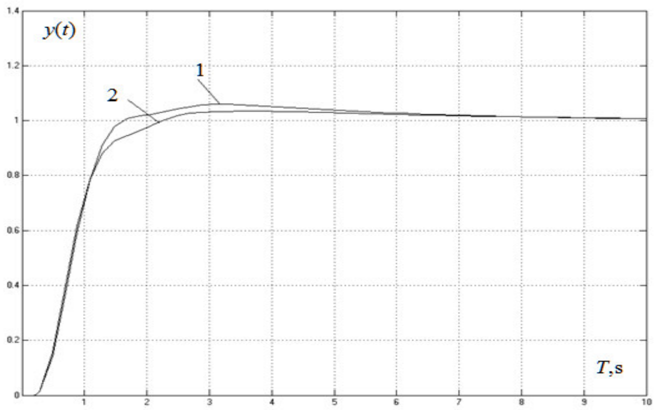

Fig. 7. The transient processes of control system with PID and PIDD^2 controller.

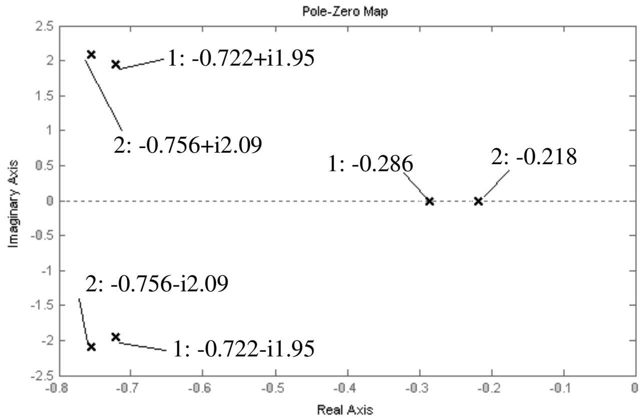

Fig. 8. The distribution of the characteristic equation's poles.

Analyzing the curves from Figure 2 and Table 3 the first row, it can be observed that the maximal value of the stability degree is J = 2.65 and the parameters of the controller have the maximal possibility values k_p=21.619, k_i=14.687 and k_d =8.752. The control system has the best possible speed according to the proposed method t_c = 0.61, but has the overshooting relatively so high σ = 28.37% and the control time t_r = 1.96.

Using the maximal stability degree method with iterations was tuned the values of the PID controller and obtained results are presented in the Table 1, rows 2-7 and analyzing them it was observed that the best performance was obtained for the value J = 3.6 and the parameters of controller have values k_p=15.57, k_i=4.76, k_d =7.82 for this case the performance of control system are the highest and the optimal in conformity with proposed method (view Table 2, line 7): T_c = 0.78, and the overshoot is relatively small σ = 4 , 5% and the control time t_r = 0.78. The speed is increased by the 60%, the overshoot is reduced from 28.37 to 4.5% and the control time is reduced with the 60%.

Analyzing the performance of the control system with modified algorithm PIDD^2 tuned by the maximal stability degree method with iteration it was observed that the best performances were obtained for the control system with controllers parameters from second row, Table 3 and the respectively performances from sixth row, Table 4, which are characterized with aperiodic process, increasing time t_c=0,71 and control time t_r = 0.71 (with 10% faster).

The control system with PIDD^2 algorithm has the higher performance (with 10% faster) than the control system with PID controller and the transient processes in the both cases are aperiodic.

In the Figure 9 is presented the distribution of control system's poles with PID algorithm - 1 and modified algorithm PIDD^2 - 2.

The highest reserve of stability has the control system PID algorithm.

It was applied unitary step disturbance what acting on the control system by the first-order inertia element with transfer coefficient equal with 0.5 and time constant equal with 1s and it was checked the reaction of control system with PID algorithm and modified algorithm PIDD^2, was obtained the transient processes and obtained performance are presented in the Table 5 (the numbering in the table are: 1 – control system with PID algorithm and 2 - the control system with PIDD^2 algorithm).

Table 5

The performance of control system at the disturbance action

Nr.	t_c	σ,%	t_r	λ
1	1.15		3.03	
2	4.53		4.53	

Analyzing the performance of control system with PID algorithm and control system with PIDD^2 under the disturbance action after the indicated performances in the Table 6, was observed that the performance are kept for the control system with PID controller (time setting is increased by the 3.88 times and in case of control system with PIDD^2 algorithm the control time is increased by the 6.4 times in comparison with nominal regime).

It was varied the object parameters with ± 50% from the nominal values: transfer coefficient k, constants time T_1 and T_2 and time delay τ and it was obtained the transient processes of the control system with modified parameters. It was analyzed the performances of control system, what are presented in the Table 6 – the control system with PID algorithm and Table 7 – the control system with decreasing parameters (sign -) and with increasing the performances (+ sign).

Table 6

Performance of control system with PID controller at the variation the parameters

Nr.	t_c	σ,%	t_r	λ
T_1-/T_1+	0,45/1,12	25/6,2	2,54/2,21	2/1
T_2-/T_2+	0,47/1,04	16,1/8,8	2,38/2,29	2/1
τ-/τ+	1,02/0,78	-/21,5	1,02/2,55	-/2
k-/k+	2,87/0,52	-/22,3	2,87/1,13	-/1

Table 7

Performance of control system with PID controller at the variation the parameters

Nr.	t_c	σ,%	t_r	λ
T_1-/T_1+	0.39/1.06	24.5/-	3,5/1.06	2/-
T_2-/T_2+	0,4/0,98	16,8/5,1	3,06/1.62	2/1
τ-/τ+	1,07/0.69	-/20,2	1,07/2,43	-/2
k-/k+	5.91/0,46	-/18,5	5,91/0,93	-/1

Analyzing the performances of control system with PID algorithm at the variation the object's parameters, it was observed that with reduction the T_1 and T_2, the performance is damaged and with increasing the T_1 and T_2 the performances are raised; at the increasing τ the overshoot is raised with 21.5% and control time by the 2.5 times in comparison with aperiodic regime at the decreasing the τ and at the increasing the constant time T_1 and T_2 the speed becomes lower, the overshoot is raised with 22.3% and reduced the control time with 2.5 times in comparison with decreasing the k.

Analyzing the performances of control system with PIDD^2 algorithm at the variation the object's parameters, it was observed that with reduction the T_1 and T_2, the performance is damaged and with increasing the T_1 and T_2 the performances are raised; at the increasing τ the overshoot is raised with 20.2% and control time by the 2.6 times in comparison with aperiodic regime at the decreasing τ and at the increasing the transfer coefficient k is raised the overshoot with 19% and reduced the control time with 6.4 times in comparison with performance of aperiodic regime at the decreasing the k. In generally can be made the conclusion that the performance of control system with PIDD^2 controller are higher than the performance of control system with PID controller.

4. CONCLUSION

Analyzing the obtained results it can be done the following conclusions:

- The control system with modified algorithm PIDD^2 has the performance of the control system better than the control system with PID algorithm: is increased the speed of system with 9%, decreased the overshoot with 22% and control time with 9%.
- Under the perturbation action the control system with modified algorithm PIDD^2 has the lower performance than the control system with PID algorithm: the speed is reduced with 4 times and the control time increase with 1.5 times.
- When it is modified the parameters of the control object, the control system with modified algorithm PIDD^2 has the performances better than the control system with PID algorithm: at the decreasing the parameters the speed of the system is reduced and the control time increase; at the increasing the parameters the speed is raised and the control time decreased.

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