EFFECT OF IRREVERSIBILITIES ESTIMATED WITH THE DIRECT METHOD ON THE STIRLING ENGINE PERFORMANCE

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Rezumat. Lucrarea prezintă o noua schemă de calcul a performanţele motorului Stirling, care a fost elaborată în cadrul Termodinamicii cu Viteză Finită și a Metodei Directe. Ea este bazată pe noua expresie a Principiului I al Termodinamicii pentru procese ireversibile cu viteză finita, care este integrată pe fiecare proces din ciclu, rezultând un model de calcul complet analitic al ciclului motorului Stirling. Se ia în considerare efectul ireversibilităţilor datorate pierderilor de presiune și regenerării imperfecte a căldurii in regenerator asupra performanţelor motorului, și anume putere și randament. Modelul include trei tipuri de pierderi de presiune care apar în cele patru procese ale ciclului motorului Stirling, datorate (1) vitezei finite a pistonului, (2) frecării mecanice, și (3) laminării gazului la curgerea prin regenerator. Efectul lor a fost pus în evidenţă succesiv pe fiecare dintre procese și apoi, separat și simultan, pe ciclu. Această nouă schemă de calcul a condus la obţinerea de rezultate foarte apropiate de datele experimentale a două motoare Stirling cu cele mai bune performanţe, aflate actualmente în funcţiune în lume.

Cuvinte cheie: Metoda Directă, Termodinamica cu Viteză Finită (TVF), motor Stirling, ireversibilităţi regenerator, optimizare

Abstract. The paper presents a new computation scheme of Stirling engine performance that was elaborated in the frame of Thermodynamics with Finite Speed and the Direct Method. Based on the new expression of the First Law of Thermodynamics for Irreversible Processes with Finite Speed that is integrated on each process of the cycle, a completely analytical model of the Stirling engine cycle is developed. It takes into account the effect of irreversibilities dues to pressure losses and imperfect regeneration on the engine performance, namely power and efficiency. The model considered three types of pressure losses occurring in the four processes of the Stirling engine cycle, dues to (1) finite speed of the piston, (2) mechanical friction, and (3) throttling of the gas flowing through the regenerator. Their effect was successively emphasized on each process and then simultaneously on the cycle. This new scheme yielded accurate results when compared with actual experimental data of two operational Stirling engines with best performance.

Keywords: Direct Method, Finite Speed Thermodynamics (FST), Stirling engine, regeneration irreversibilities, optimization

1. INTRODUCTION

Previous research relative to Stirling engine performance improvement focused on the study of the losses due to irreversibility of the cycle. In the frame of Finite Speed Thermodynamics, a scheme of computation and optimization of Stirling machine was elaborated [1-5] and validated by comparison with actual data of 14 operational engines [6-8]. It took into account internal and external irreversibilities of the Stirling engine based on the expression of the First Law of Thermodynamics for processes with finite speed. Then, analytical expressions of the engine performance were derived by using the Direct Method. In that scheme, the efficiency of the engine took into account irreversibilities on the whole cycle and two adjusting coefficient were used in order to achieve its validation.

The purpose of the present work is to emphasize effects of irreversibilities on each process and the whole cycle, on the engine's performance, namely power and efficiency, by considering them in the power output expression.

The study was conducted in three successive stages. Thus, the effects of the three types of irreversibility on the mechanical work on each process, then on the whole cycle were separately and globally studied, and also the effects of the regeneration degree of the heat on the engine's performance.

If pressure losses determined by the internal irreversibility on compression and expansion have been previously studied, the effects of internal irreversibility caused by the flow of gas in the regenerator (heating process, cooling process) are novelty elements in this work. Their integration in the study of irreversible cycle provided a new calculation scheme based on the Direct Method and the Thermodynamics with Finite Speed. This new scheme has been verified by comparison with actual performance of two of the most common Stirling engines.

2. THEORETICAL CONSIDERATION ON THERMODYNAMICS WITH FINITE SPEED AND THE DIRECT METHOD

Thermodynamics with Finite Speed and the Direct Method are respectively a new branch of Irreversible Thermodynamics and a new Method for computation of Performances (Efficiency and Power) of any real-irreversible cycle of thermal machine, which works with Finite Speed, by taking into account internal and external irreversibilities [1, 2, 4].

The Direct Method takes into account internal irreversibilities in thermal machines on a fundamental basis in the new expression of the First Law for Processes with Finite Speed [1, 2, 4]:

$$\mathrm{d}U = \delta Q - P_{m,i}\left(1 \pm \frac{a\,w}{c} + \frac{b\,\Delta P_{thr}}{2\,P_{m,i}} \pm \frac{f\,\Delta P_f}{P_{m,i}}\right)\mathrm{d}V \tag{1}$$

where: w is the average piston speed; a – coefficient which depends on the gas nature, $a = \sqrt{3k}$; b – coefficient related to throttling of the gas, $0 < b < 2$; c – average molecular speed, $c = \sqrt{3kT}$; k – ratio of specific heat at constant pressure and volume, $k = c_p / c_v$; ΔP_f = pressure drop due to friction between the moving parts of the machine; ΔP_{thr} – pressure drop caused by throttling; $P_{m,i}$ – instantaneous mean pressure of the gas; f – fraction of heat generated by friction between the moving parts of the machine, which remains in the system ($0 < f < 1$).

Actually, this expression combines the First Law with the Second Law of Thermodynamics in a unique equation that can be integrated for any irreversible cycle and get directly the Efficiency and Power as analytical expressions [1, 2, 4].

Irreversible mechanical work for processes with finite speed for closed complex systems such as the Stirling machine, is expressed as [1, 2, 4]:

$$\delta W_{irr} = P_{m,i}\left(1 \pm \frac{a\,w}{c} + \frac{b\,\Delta P_{thr}}{2\,P_{m,i}} \pm \frac{\Delta P_f}{P_{m,i}}\right)\mathrm{d}V = P_p\,dV \tag{2}$$

where P_p is the pressure on the mobile piston.

The + and – signs inside the parenthesis correspond to compression and expansion process respectively.

These expressions can be used for any irreversible cycle optimization providing the optimum value of the finite speed of the piston cycle in order to get the Maximum Efficiency or/and the Maximum Power output of the machine.

3. PRESSURE AND WORK LOSSES CALCULATION

3.1. Pressure losses by friction

The expression of the friction pressure losses, ΔP_f, was derived from Heywood's empirical formulas [10] for direct injection engine based on experimental measurements on internal combustion engines. It was transformed [2, 4] into a function of the piston speed, w, and stroke, z, instead of rotation per minute:

$$\Delta P_f = 0.75 + 0.46 \frac{3w}{100 \cdot z} \quad \text{[bar]} \tag{3}$$

Equation (3) is changed in comparison with the previous computation scheme due to high pressure level used in the calculation of the present work.

3.2. Pressure losses by throttling

For the throttling pressure losses ΔP_{thr} in the Regenerator a formula was derived [1, 2, 4] as function of the speed of the piston, w, and number of the screens, N_s, based on a figure from Organ's book [7] where a lot of empirical measurements of ΔP_{thr} have been synthesized as dependent on the Reynolds number (Re) of the gas passing through the Regenerator screens:

$$\Delta P_{thr} = \frac{15}{k} \frac{\rho_R \, w_R^2}{2} \cdot N_S \quad \text{[bar]} \tag{4}$$

where: ρ_R is the density of the gas in the regenerator; w_R – speed of the gas in regenerator.

3.3. Work loss on the isometric processes

Figure 1 illustrates the effects of the gas flow with finite speed through the regenerator which can be part of the displacer of the Stirling machine (D/R). Thus, two causes of irreversibilities, namely throttling and friction, are clearly suggested on the figure, while the third one, due to the finite speed of the piston, will be added analytically.

According to equations (2), for the isometric processes 2-3 and 4-1 from Figure 1, the irreversible work is expressed as [4]:

$$\delta W_{irrev} = \delta W_{lost} = \left(P_p' - P_p''\right) \mathrm{d}V \tag{5}$$

where: $\mathrm{d}V' = -\mathrm{d}V'' = \mathrm{d}V$.

By integrating eq. (5) from a to b position (see Fig. 1) for each irreversibility type one gets:

$$W_{irr,w} = -P_{m,i} \left(\frac{a\,w}{c'} + \frac{a\,w}{c''} \right) V_2 \tag{6}$$

$$W_{irr,thr} = -\Delta P_{thr} V_2 \tag{7}$$

$$W_{irr,f} = -\Delta P_f V_2 \tag{8}$$

Finally, the work lost during the isometric processes will be given by the sum of the above three expressions:

– for the heating process 2-3 [4]:

$$W_{heat,irr,w,thr,f} = -\left[P_{23,av}\left(\frac{a\,w}{c'} + \frac{a\,w}{c''}\right) + \Delta P_{thr} + \Delta P_f\right] V_2 \tag{9}$$

– for the cooling process 4-1 [4]:

$$W_{cool,irr,w,thr,f} = -\left[P_{41,av}\left(\frac{a\,w}{c'} + \frac{a\,w}{c''}\right) + \Delta P_{thr} + \Delta P_f\right] V_2 \tag{10}$$

where:

$$P_{23,av} = \frac{P_2 + P_3}{2}\ , \quad P_{41,av} = \frac{P_4 + P_1}{2} \qquad (11)\text{-}(12)$$

$$c' = c'' = \sqrt{3kT_{av}}\ , \quad \text{with} \quad T_{av} = T_{23,av} = T_{41,av} = \frac{T_H + T_L}{2} \qquad (13)\text{-}(14)$$

and with the throttling losses particularized to the heating (2-3) and cooling (4-1) process via the gas density.

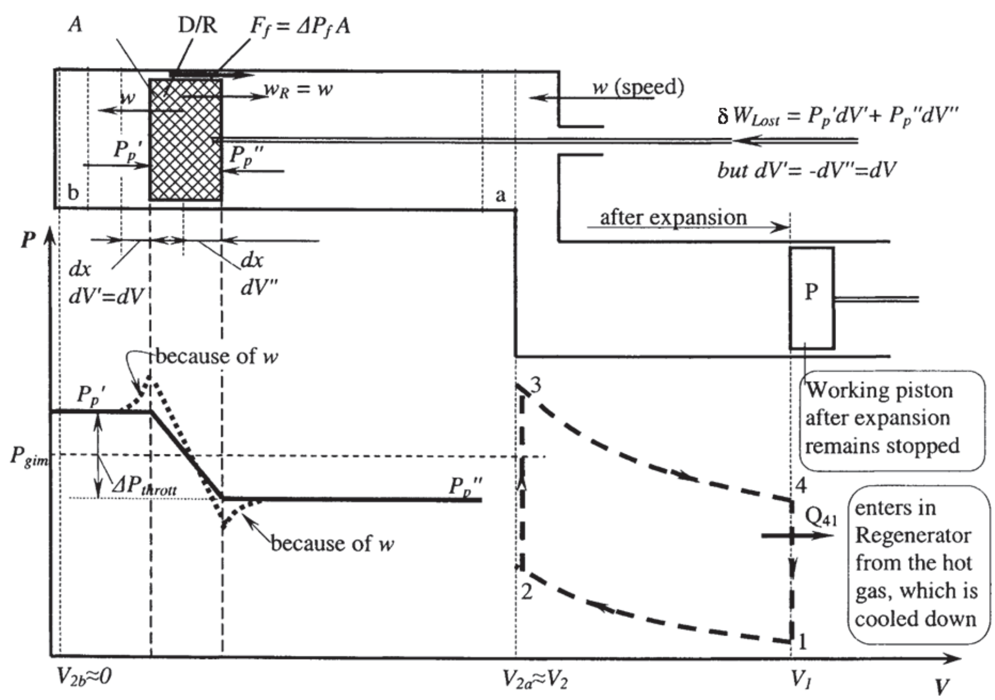

Fig. 1. Irreversibilities occuring during the flow of the gas through the regenerator [2].

The pressure losses due to friction are the same on the heating and cooling processes [2, 4]:

$$\Delta P_f = \frac{1}{4}\left[0.75 + 0.46\frac{3w}{100 \cdot z}\right] \quad \text{[bar]} \tag{15}$$

All terms in eqs. (9) and (10) are positive conveying to the evidence that when irreversibilities are considered during the isometric process the mechanical work is not zero, it is negative. Thus, it will reduce the work output of the cycle, and the results have shown that its weight is quite important.

4. MODELING OF THE STIRLING CYCLE WITH LOSSES CONSIDERED SEPARATELY AND GLOBALLY

The computational scheme aimed to emphasize the effect of each irreversibility on each process of the cycle, and then globally on process and cycle, respectively. All irreverisibilities due to pressure losses were introduced in the mechanical work (then power) expression unlike the previous scheme, where they were accounted for in the efficiency expression. Thus, non-dimensional coefficients were added to the expression of the reversible work each process, i.e. for the isothermal compression one gets:

$$B_{cpr,w} = \frac{a\,w}{c}, \quad B_{cpr,f} = \frac{\Delta P_f}{P_{12,av}}, \; B_{cpr,w,f} = 1 + B_{cpr,w} + B_{cpr,f} \tag{16)-(18}$$

$$W_{cpr,irr,w,f} = W_{cpr,rev} \cdot B_{cpr,w,f} = \left(P_1 V_1 \ln\frac{V_2}{V_1}\right) \cdot B_{cpr,w,f} \tag{19}$$

or for the isometric heating:

$$B_{heat,w} = \frac{a\,w}{c'} + \frac{a\,w}{c''}, \quad B_{heat,f} = \frac{\Delta P_f}{P_{23,av}}, \quad B_{heat,thr} = \frac{\Delta P_{thr}}{P_{23,av}} \tag{20)-(22}$$

$$B_{heat,w,f,thr} = B_{heat,w} + B_{heat,f} + B_{heat,thr} \tag{23}$$

$$W_{heat,irr,w,f,thr} = -P_{23,av} \cdot V_2 \cdot B_{heat,w,f,thr} \tag{24}$$

Note that pressure losses by throttling were not considered on the compression and expansion processes.

Similarly, mechanical work expression were derived for the isothermal expansion and the cooling isometric process, with losses taken successively and globally, so that the corresponding expression for the whole cycle when all irreversibilities are accounted for yields:

$$W_{cycle,irr,w,f,thr} = W_{cpr,\,irr,w,f} + W_{\exp,irr,w,f} + W_{hrat,\,irr,w,f,thr} + W_{cool,irr,w,f,thr} \tag{25}$$

The power output of the engine results as:

$$Power_{cycle,irr,w,f,thr} = \frac{W_{cycle,\,irr,w,f,thr} \cdot w \cdot \frac{30}{z} \cdot i}{2 \cdot 60} \tag{26}$$

where i is the number of cylinder.

The cycle efficiency takes into account the imperfect regeneration of heat in the regenerator by a coefficient of regenerative losses, X, as follows:

$$\eta_{cycle,irr,w,f,thr} = \frac{W_{cycle,\ irr,w,f,thr}}{z_Q \cdot Q_{34,rev} + X \cdot Q_{41}} \tag{27}$$

where z_Q is one of the *adjusting coefficient* of the model that accounts for the irreversibility of the isothermal heat transfer, its value equals 0.8 [1-4].

The coefficient of regenerative losses, X, depends on a large number of parameters and variables. Among these are the piston speed w, cylinder dimensions (diameter D_C and stroke z), regenerator dimensions (diameter D_R and length L) properties of the solid porous material in the Regenerator (density ρ_R, specific heat c_R), dimensions of the screens (d, b, N_S), gas properties (ρ_g, c_P, γ, R), and the range of operating conditions ($\varepsilon = V_1/V_2$, $\tau = T_H/T_L$, T_{av}, P_{av}). A relationship for X that includes the effect of these parameters has been evaluated using First Law and heat transfer laws [1-4]:

$$X_1 = \frac{1 + 2M + e^{-B}}{2(1+M)}, \qquad X_2 = \frac{M + e^{-B}}{1+M} \tag{28)-(29}$$

where:

$$M = \frac{m_g c_{v,g}}{m_R c_R}, \quad B = (1+M)\frac{hA_R}{m_g c_{v,g}} \cdot \frac{S}{w} \tag{30)-(31}$$

with the convective heat transfer coefficient expressed as:

$$h = \frac{0.395\left(4P_{av}/RT_L\right)w_{gR}^{0.424} \cdot c_P\left(T_{av}\right) \cdot \nu\left(T_{av}\right)^{0.576}}{(1+\tau)\left[1 - \frac{\pi}{4\left[(b/d)+1\right]}\right] D_R^{0.576} \cdot \Pr^{2/3}} \tag{32}$$

The computed values of X_1 and X_2 were found to accurately predict the values of X determined from experimental data available in the literature [8, 11-14] using the following equation:

$$X = yX_1 + (1-y)X_2 \tag{33}$$

where the *adjusting parameter y* is equal 0.36.

5. RESULTS AND DISCUSSIONS

The results provided by this new computational scheme used the following input data:

For Stirling Thermal Motor STM4-120

- Power (rated): 25 kW; 1800 rpm
- Number of Cylinders: 4, parallel square configuration
- Displaced Volume: 4 x 120 cm^3
- Bore: 56 mm
- Stroke: z = 48.5 mm
- Regenerators: Wire mesh,
- Working gas: Helium (hydrogen is alternative)
- Mean Gas Pressure (max): 12 MPa
- Gas Temperature (high):720°C
- Coolant Temperature (max): 45-70°C (taken 50)

For United Stirling 4-95 MKII

- Power (rated): 25 kW; 1800 rpm
- Number of Cylinders: 4, parallel square configuration
- Displaced Volume: 4 x 95 cm^3
- Bore: 55 mm
- Stroke: z = 40 mm
- Regenerators: Wire mesh, N_S = 200
- Working gas: Hydrogen (helium is alternative)
- Mean Gas Pressure (max): 20 MPa
- Gas Temperature (high):720°C
- Coolant Temperature (max): 50°C

Figure 2 and 3 illustrate the effect of irreversibilities gradually considered on the mechanical work. It results an important weight of the work losses during the two isometric processes (Fig. 3).

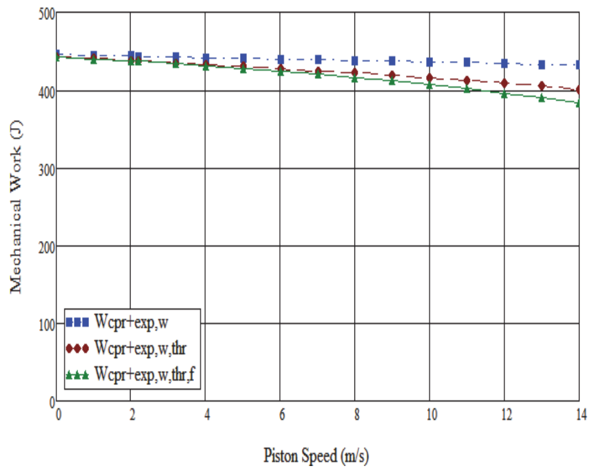

Fig. 2. Mechanical work on compression and expansion processes progressively reduced by irreversibilities.

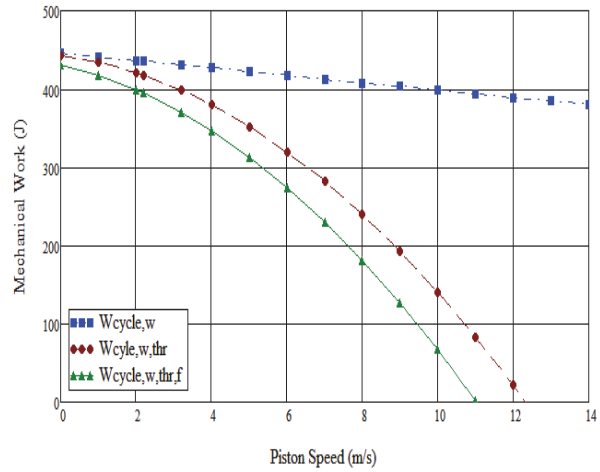

Fig. 3. Mechanical work output on the cycle with gradually added irreversibilities.

Figures 4 and 5 illustrate the effect of irreversibilities on the power output of the engine and on the efficiency. One may see that the power output continuously increases with the piston speed in the case of reversible cycle while its variation is completely different when taking into account (for an increase in piston speed from 0 to 12 m/s) the irreversibilities successively (w, w+thr and w+thr+f). Thus, in the first case, the power increases almost linearly from 0 to approximately 95 kW (for a piston speed variation from 0 to 12 m/s), then in the second, the power increases from 0 to a maximum value of about 40 kW (at a piston speed of about 7 m/s) and further decreases to 0 (at a piston speed of 12.5 m/s) and, in the third case, the power increases from 0 to a maximum value of about 34 kW (at a piston speed of about 6 m/s) and further decreases to 0 (at a piston speed of about 11 m/s).

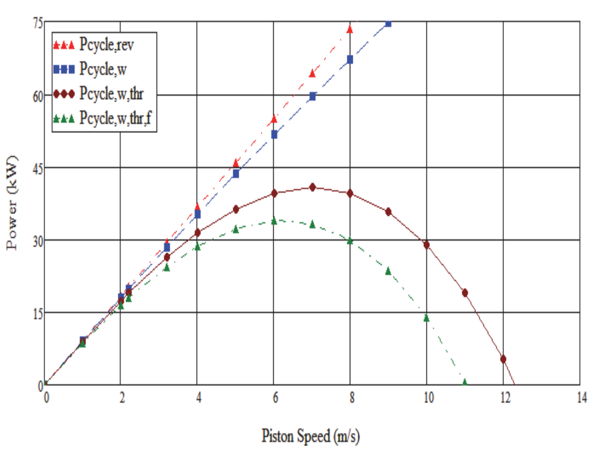

Fig. 4. Influence of the irreversibilities on the Power output of the engine.

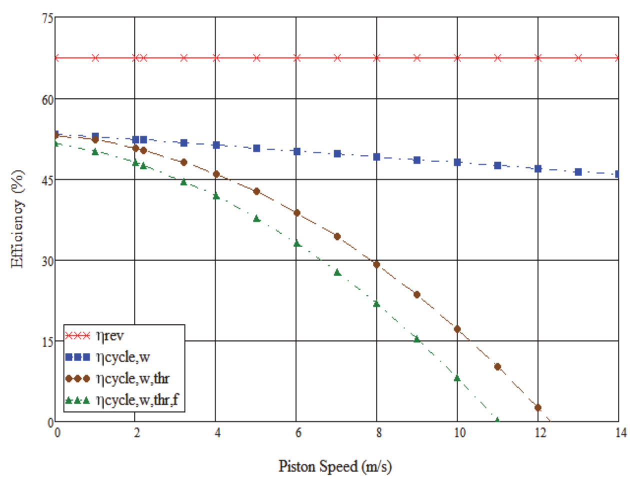

Fig. 5. Influence of the irreversibilities on the engine efficiency.

The efficiency is also affected by irreversibilities, so that it decreases almost linearly from about 55% to about 33% in the first case, drops from 55% to 0 in the second case, (for a piston speed of about 12.5 m / s) and also, decreases from about 52% to 0 (for a piston speed of 11 m / s) in the third case.

At a piston speed of 2.5 m/s, the efficiency of the engine considering the losses due to irreversibility, is about 44%.

Note that the actual efficiency of the engine (40 to 45 %) is reached for the same piston speed of 3.2 m/s that assures the actual power.

A comparison of the computed and actual performance of STM4-120 and 4-95 MK II Stirling engines is presented in Figure 6 and 7 and Table 1. For STM4-120 operating with He (Fig. 6), the rated power (about 25 kW) is achieved at a piston speed of 3.2 m/s, and the corresponding efficiency is about 43%. For 4-95MK II, which works with H_2, (Fig. 7), the rated power (about 25 kW) is achieved at a piston speed of 2.2 m/s, to which the efficiency is 28%.

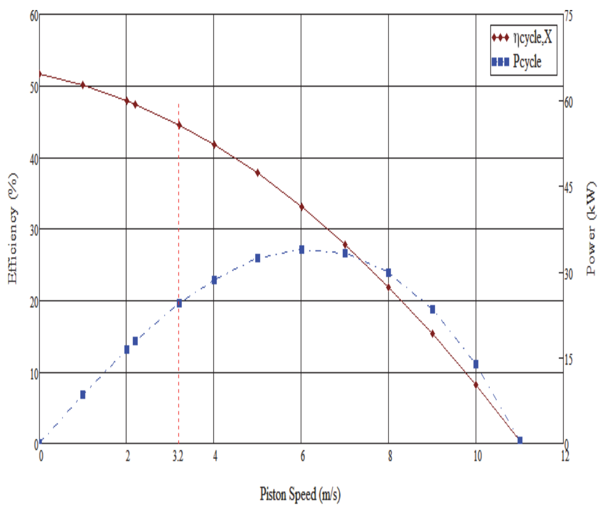

Fig. 6. Performance of STM4-120 Stirling engine with He as working gas.

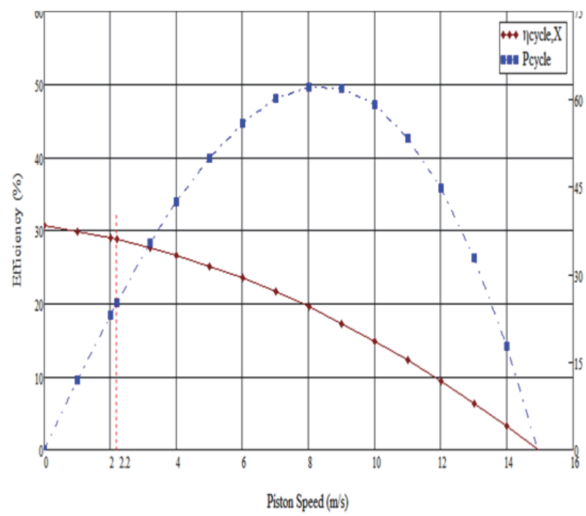

Fig. 7. Performance of 4-95 MK II Stirling engine with H_2 as working gas.

Table 1

Performance comparison for STM 4-120 and 4-95 MK II Stirling engines

Stirling Engine	p_{mean}	V_{max}	Gas	Power [kW]		η [%]	
	[bar]	[cm^3]	-	Actual	Calculated	Actual	Calculated
STM 4-120	120	120	He	25	25.01	40-45	42.7
4-95 MK II	200	95	H_2	25	25.20	41	27.6

6. CONCLUSION

The present paper highlighted the power output and efficiency reduction caused by 3 types of irreversibility found in Stirling machines, namely the pressure losses due to the finite speed of the piston, throttling, and friction. These irreversibilities were also considered in the analytical calculation of the gas flow in regenerator and it was shown that during the isometric processes (heating and cooling processes) the mechanical work is different from 0, namely negative. Thus, they reduce the mechanical work done on the cycle and so, the performance of the engine.

The results illustrated in the paper witnessed that the largest influence on mechanical work loss and efficiency is given by the throttling of the gas as it flows through the machine.

The model developed in the paper led to changes in the calculation scheme, developed so far without considering the irreversibilities on the isochoric processes. Hence, through the new proposed adjustment coefficient, a new calculation scheme yielded. Its results were confronted to actual performance of two of the most commonly used Stirling engines: STM4-120 and 4-95 MK II. For the first engines accurate performances (power and efficiency) compared to actual ones are obtained at a piston speed corresponding to the analytical calculation of about 3.2 m/s. For 4-95 MK II, at the piston speed for which the calculated power is equal to the actual power, the calculated efficiency is less than the actual one.

Further work aims to validate the new calculation scheme proposed here for the other types of Stirling engines.

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