APPROACING THE URBAN MOBILITY BY CATEGORY THEORY

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Rezumat. Teoriile şi modelele specifice fiecărui mod de transport îngreunează cooperarea intermodală, fragmentând lanţul logistic, precum și serviciile pentru călători. Pentru a utiliza excelența fiecărui mod de transport în zona sa de piaţă, se propune trecerea de la conceptele actuale de inter-, multi- şi co-modalitate, spre conceptul generalizator de cooperare a modurilor de transport. Rezultă necesitatea unor concepte, teorii şi modele generalizatoare care să aducă modurile de transport într-un spaţiu logistic echivalent în care sistemele de mobilitate sunt comparabile şi apte de cooperare pe diferite niveluri: firme, lanţuri logistice, coridoare urbane, poli de schimb etc. Se propune teoria categoriilor ca bază ştiinţifică în mobilitatea urbană, oferind o cale de identificare a invarianţilor logistici în tratarea unitară a sistemelor şi proceselor ce au configuraţii similare sau repetitive.

Cuvinte cheie: mobilitate urbană, teoria categoriilor, cooperarea multimodală.

Abstract. Theories and models tailored to each transport mode create difficulties against intermodal cooperation, fragmenting the freight logistic chain and services to passengers as well. Considering that transport modes are suited for certain market areas, it is proposed to shift from the existing concepts of inter-, multi- and co-modality to the generalizing concept of cooperation of transport modes. Thus, there is a need for generalizing concepts, theories and models in order to bring the transport modes into an equivalent logistics space where the mobility systems are comparable and capable of cooperation at various levels: companies, logistics chains, urban corridors, urban intermodal platforms etc. It is proposed the category theory as a scientific base for the development of new generalizing theories and models in urban mobility, offering a way for identifying the logistic invariants in unified treatment of systems and processes having similar and repetitive patterns.

Keywords: urban mobility, category theory, multimodal cooperation.

1. THINKING IN TERMS OF CATEGORY THEORY

The urban transport modes and the respective market share continue to be in the relative position of segregated items. The distinctions existing in the logistic area restrain the customers' ability to use them, fragmenting the freight logistic chain and services to passengers as well. A promising solution is to shift from the existing concepts of inter-, multi- and co-modality to the generalizing concept of cooperation of transport modes, using models transcending the transport modes and which must be based on structural and relational patterns valid in any transport mode. As a matter of fact, the transport of the loading units from the producer to the consumer, irrespective of the transport mode, includes the same logistic activities such as their loading-unloading into and from the vehicles, the transhipment, storage, consolidation and dismantling of the loads, the shipment-delivery formalities and customs operations. The category theory can be a scientific base for the development of new generalizing theories and models in urban mobility, offering a way for identifying the logistic invariants, i.e. similar and repetitive pattern of systems and processes [2].

The category theory is based on the idea of system of functions between certain objects. In a category **K**, an object A $\in$ Ob(**K**) is determined by its relations with other objects B $\in$ Ob(**K**). The analogy with the logistic chains in a set of logistic systems is obvious.
In a category **K**, there are given [5]:

- the class of objects A, B, C, ... , noted Ob (**K**);
- the set of morphisms from A to B:
  HomK (A,B), for any pair (A,B) $\in$ Ob(**K**) x Ob (**K**);
- the law of composing the morphisms:
  the application HomK (A,B) x HomK (B,C) $\to$ HomK (A,C), for any triplet (A,B,C) $\in$ Ob (**K**) x Ob(**K**); (u,v) $\mapsto$ v $\circ$ u, meaning v $\circ$ u : A $\to$ C, or
  vu $\in$ HomK (A,C).

We say that these data define a category K, if there are checked some axioms, like associativity, identity (self-identical morphism) etc. The enclosed diagram commutes.

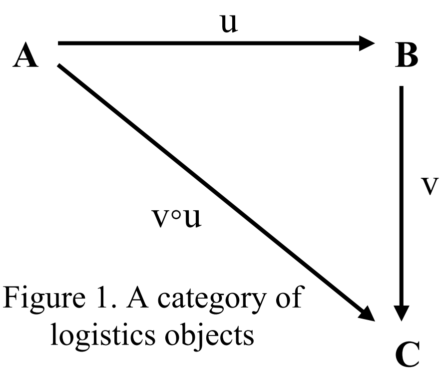

Figure 1. A category of logistics objects

If $\mathbf{K_1}$ and $\mathbf{K_2}$ are two categories, we say that it has been defined a covariant functor F from $\mathbf{K_1}$ to $\mathbf{K_2}$, F**:** $\mathbf{K_1} \to \mathbf{K_2}$, if:

1) it has been defined an application Ob($\mathbf{K_1}$) $\to$ Ob($\mathbf{K_2}$), which associates to any object A from $\mathbf{K_1}$ an object F(A) in $\mathbf{K_2}$;
2) for any pair (A,B) of objects from $\mathbf{K_1}$ it has been defined the operation
   F(A,B): Hom_{K1}(A,B) $\to$ Hom_{K2}(F(A),F(B))

such as, if instead of F(A,B)(u) we write F(u), we have:

$F(1_A) = 1_{F(A)}$ for any A $\in$ Ob ($\mathbf{K_1}$), F(v$\circ$u) = F (v) $\circ$ F (u),

for any morphisms u and v of the $\mathbf{K_1}$, for which, the compound v$\circ$u has a meaning.
The composition v$\circ$u is a kind of product of the functions u and v. A category is an algebra formed of the objects A,B,C... and morphisms u, v, f, g, h,... among the objects that meet certain conditions typical of the composition of functions. The category theory has been created as a modality of studying different types of structures in terms of their admissible transformations, which preserves the structures.
The transport modes are objects forming a separate topological space. Each transport mode is a topologic space. The current division into modes is a convention from the past, but on the future the transport modes could be unified or restructured in other topological spaces. The logistic objects may be aggregated by the relationships between them into more and more complex objects corresponding to the more and more complex requirements of the users. By the relationships between objects becoming morphisms from a class of objects to another one, the logistic systems may be created, modeled and managed. Logistic patterns may appear

from this, the systems' heredity can be studied and a genetic helix of each logistic system could be defined. The dynamics of the logistic objects consists in change the connections between objects, their intensity, the connection diagrams, and the structure of objects.
There are many reasons to use the category theory. Some of them are as follows:

- It puts the current transport and logistics concepts into a new perspective.
- It points out the unity of the logistic and transport concepts.
- It simplifies and standardizes the logistic and transport models.
- The results proven in the category theory generate automatically results regarding the categories in logistics and transport (mutatis mutandis).
- For each category there is a dual obtained by reversing the morphisms, which consequently suggests new logistic patterns.
- The problems in logistics and transport may be translated and solved into other field by using the functors that move the processes from a category to another.
- It specifies some notions that were quite vague: universality, accessibility, inter- and multi-modality, co-modality, inter-operability etc.
- By category theory, it is carried out the systemic treatment, there are modeled the relationships between a part and the whole, the analysis / synthesis ratio.
- The transport and logistics concepts can be generalized by category of categories.

Let's consider a category of transport modes (objects) being in intermodal relations (morphisms). It may be considered together with its dual. If the transport modes form an ordered set A where a, b $\in$ A, and a $\le$ b shows that the transport modes a and b are ordered according to the preference criterion, then A is a small category.
In a traffic network, there may be considered that the traffic flows represent a relationship over the group of nodes. If we consider, for instance, that the pallets arrived in B, coming from A, can be found in the containers flow that departs from B for C, then we are referring to the morphisms A$\rightarrow$B, B$\rightarrow$C, … between the objects of the category of nodes A, B, C, …

2. MULTIMODAL COOPERATION IN URBAN TRANSPORT

If A_1 and A_2 are objects of category **C**, then their product is the triplet (P, p_1, p_2), satisfying the following property, Figure 2: for any object B $\in$ **C** and $q_i \in$ C (B, A_i), there is a unique morphism u $\in$ C (B, P), so that $p_i \circ u = q_i$ for i = 1, 2.
That is, P is the terminal object of subcategory of the candidates B.

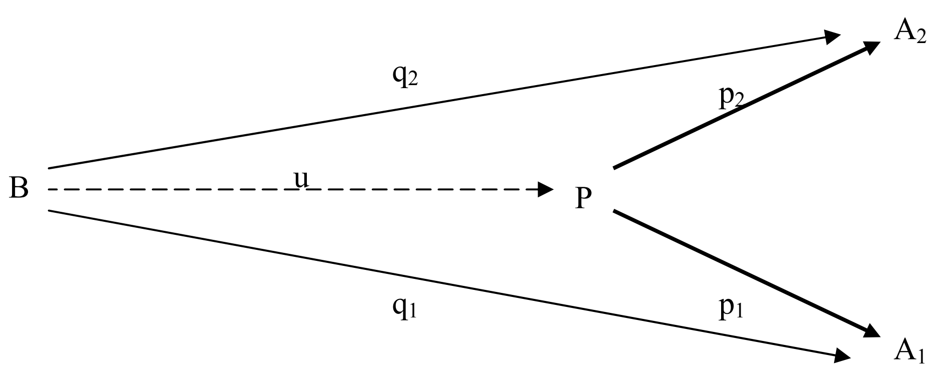

Figure 2. The product (P, p_1, p_2) of objects A_1 and A_2 from category **C**

One can say that (P, p_1, p_2), in particular $(A_1 \times A_2, p_1, p_2)$, is an optimal representative of the set (B, q_1, q_2) for the pair (A_1, A_2). If there is a product for a particular pair of objects of category **C**, it is unique. In the particular case where **C** = Set, each pair of sets (A_1, A_2) has a product, the Cartesian product $A_1 \times A_2$, with projections p_i.
If **C** is the category of transport flows processed on different modes, P could be generally bimodal transport and combined transport in particular. The objects A_1 and A_2 are modal transport flows, and morphisms p_1 and p_2 recover the transport modes as projections of intermodal cooperation that generated the bimodal transport P.
We can say that (B, q_1, q_2) and (P, p_1, p_2) are bimodal transport solutions networked by morphism $u: (B, q_1, q_2) \rightarrow (P, p_1, p_2)$, and (P, p_1, p_2) is the optimal choice.
In the general case, Figure 3, if one considers a family of objects $(A_i)_{i\in I}$ of the category **C**, then their product is an object $P \in$ **C** together with the family of morphisms $(p_i : P \rightarrow A_i)_{i\in I}$. The product $P = \Pi_{i\in I} A_i$, could be a model of multimodal cooperation in transport, and a model of a supply chain segments defined on a category of logistic segments or activities.

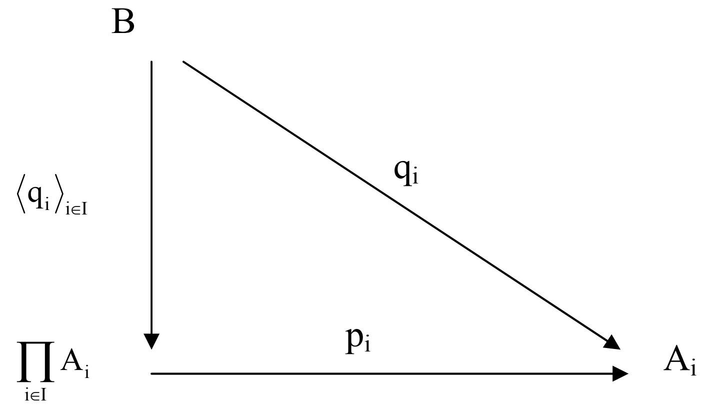

Figure 3. The product of a family of objects

Similarly, we obtain the dual object, co-product, in which P is the initial object in the subcategory of applicants.
Let's consider the notion of pullback of the pair of arrows f and g, that is the object P and the pair of arrows f 'and g', such as f' o g' = g o f'.
Then, if A and B are different modal markets represented by subsets of the set **C**, the combined transport market $A \cap B$ is a pullback, Figure 4.

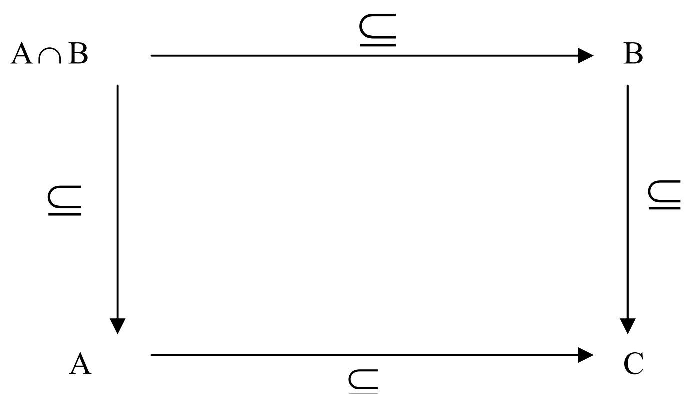

Figure 4. The combined transport market as a pullback

The bimodal transport can be also modeled and generalized by means of functors. It is considered that category $\mathbf{C_1}$ of the tramways, category $\mathbf{C_2}$ of the bicycles and category **C** of the bimodal transport (bicycles on tramway). The bifunctor B: $\mathbf{C_1} \times \mathbf{C_2} \rightarrow \mathbf{C}$, represents the bimodal transport system. For modeling the multimodal and intermodal structures and systems, there can be used also the concept of product of categories.

3. COMBINED TRANSPORT

The natural transformation concept (morphism of functors) can be useful for combined transport definition. For instance, let's consider the categories **A**- long range transport, **B**- short range transport, and **C**- transport relations (itineraries, O/D flows). Then the isomorphism:

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} \approx \mathbf{A} \times (\mathbf{B} \times \mathbf{C})$$

shows that the multimodal transport (combined system (**A**x**B**)) covering the demand **C** is equivalent to long distance transport **A** applied to logistics semi-products (**B**x**C**). That means that the long distance transport is applied to the demand **C** processed initially and finally by short distance transport **B**. There is a morphism between functors and it is not important who are the objects **A**, **B**, and **C**, but the relations between them.
Let's consider a category **C** with objects X, Y, and morphisms **C** (X, Y), Figure 5.
We consider the category of morphisms of the category **C**, denoted $\mathbf{C}^{\rightarrow}$, whose objects are triples (X, f, Y), for example, long-distance transport $f \in \mathbf{C}(X,Y)$ between nodes (terminals) X and Y, and as morphisms between (X, f, Y) and (X', f', Y'), the set $\mathbf{C}^{\rightarrow}((X, f, Y), (X', f', Y')) = \{(\alpha, \beta) \mid \alpha \in \mathbf{C}(X, X'), \beta \in \mathbf{C}(Y, Y'), f'\alpha = \beta f\}$, which could be equivalent combined transports between terminals X and Y'.

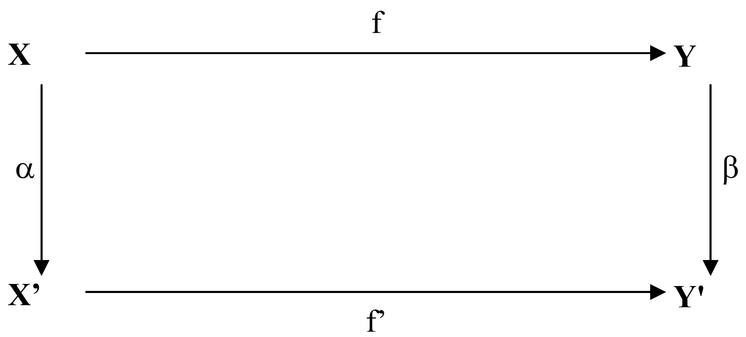

Figure 5. Equivalent combined transports between terminals X and Y'

4. INTERMODAL CHAIN

According to set theory, a relationship M from set X to set Y is a subset of the Cartesian product X x Y and contains pairs of the form $\langle x,y \rangle$.

In category theory [6], a relationship (R, p_1, p_2) is a set R endowed with two functions expressing projections p_1 and p_2 in Figure 6. Projections are monomorphism: there are no two elements $m, n \in R$, such that $p_1(m) = p_1(n)$, $p_2(m) = p_2(n)$.
By dropping the condition that the projections are monomorphism, we obtain a generalization (M, M_1, M_2), i.e. the notion of multirelation M from X to Y. Then there is a possibility of two elements $m, n \in M$, so that $M_1(m) = M_1(n)$ and $M_2(m) = M_2(n)$.

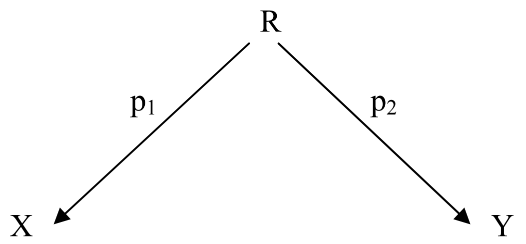

Figure 6. Relation R and its projections p_1 and p_2

Such set M endowed each with two functions M_1 and M_2, together form a bicategory. Returning to intermodal transport, we can say that we have a method of defining the bimodal transport M, based on modal transport systems X, Y.

To obtain an intermodal chain, which is on the next level of aggregation, it is to realize the composition of systems have made defined by multirelations M and N.

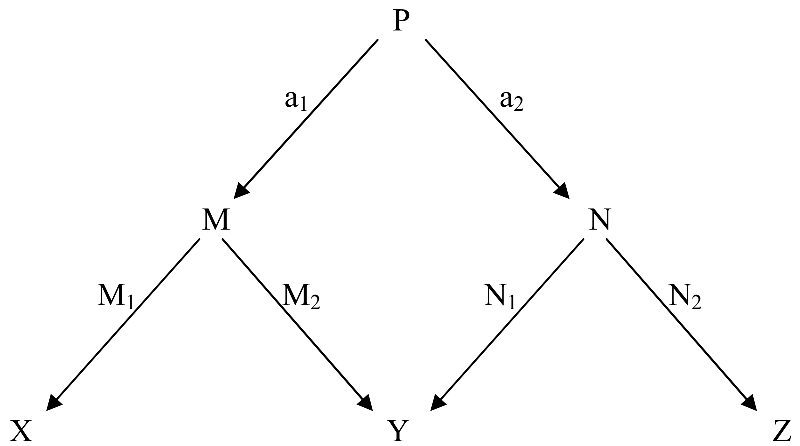

Figure 7. The composition of multirelations M and N by a pullback (P, a_1, a_2) of the M_2 and N_1

For the composition of two multirelations M and N, Figure 7, we resort to the notion of pullback ((P, a_1, a_2) of M2 and N1, resulting the multirelation (P, $M_1 \circ a_1$, $N_2 \circ a_2$) between X and Z. The elements of P are $(u, v) \in M \times N$ such that $M_2(u) = N_1(v)$. It follows, for example, that the transport modes X, Y, Z, contribute to the achievement of combined transports M and N, and then are made up in the intermodal chain P.

5. MULTIMODAL CHAINS IN THE CATEGORY OF TRANSPORT SYSTEMS

Starting from the definition of category, let's consider the set of morphisms Hom (A, B) from input A to output B of a system. Then, we can say that (A, Hom (A, B), B) is the category that defines the class of systems with input A and output B, and there unit morphisms 1_A and 1_B. Such a system could be a transport or any other urban system. Considering pairs of type Ob (K) x Ob (K) of a category **K** with Hom_K (A,B), we obtain a category of systems.

A morphism from system S_1 = (A, Hom (A,B), B) to the system S_2 = (C, Hom(C,D), D) is the pair (u, v), u: $A \rightarrow C$, v: $B \rightarrow D$, for every $f \in Hom(A,B)$ and $f' \in Hom(C,D)$. This morphism expresses relations between the urban systems S_1 and S_2. The composition of morphisms in the category of systems is associative:

$$(A, Hom(A,B), B) \xrightarrow{(u,v)} (C, Hom(C,D), D) \xrightarrow{(u',v')} (E, Hom(E,F), F)$$

6. TRANSFERING THE LOGIC OF A SYSTEM TO ANOTHER SYSTEM

Solutions of problems solved in a given system can be implemented in another system by using functors to move processes from one category to another. For example, the diagram representing the data transmission in a telecommunications system can be implemented in a logistics or transport system by a functor F, especially when they are isomorphic. It can be transposed structures, technologies, procedures, documents, etc., from a mode of transport to another or between different levels of complexity. Moreover, being known the results demonstrated in category theory, as the similarity identified with categories of logistics and transport, they can be transposed mutatis mutandis. Similarly, it can be transferred the organizational logic, functional and management systems of a specific mode of transport to another mode of transport.

A similar transfer is realized from a real system to its organizational and functional model (manager of the real system). For accuracy, the respective categorical models mapped from real system and from its managerial system must be isomorphic.

7. INTERMODAL SYSTEM DYNAMICS

For linear systems, the dynamics of the discrete-time system is:

$$x(t+1) = A\cdot x(t) + B\cdot u(t) + C\cdot f(t)$$

where the pair (X, A) is the dynamics of the system, X is state space, and A: X → X is a linear transformation which expresses changes in system states due to the u(k).

A dynamorphism [1] from the dynamics (X, A) to the dynamics (X ', A'), i.e.
h: (X, A) → (X ', A'), is a linear relationship h: X → X' that preserve the dynamics by making commutative its diagram, which implies h(A(x)) = A'(h(x)) for each x in X. Any urban system has transformations (morphisms) from one state to the other, including logistics chains, terminals, transport companies etc. The transformations are realized using different resources. It is to consider also the category of organizations having relations between them on urban environment.

To generalize, considering a category Kat, a dynamics of the system in Kat is the couple (X, A), X is an object Kat, and A: X → X' is a Kat-morphism. The morphism h of dynamics of the system, i.e. the dynamorphism, is a Kat-morphism that makes commute its diagram. A system in Kat is a 4-uplu S = (X, A, U, B) such that (X, A) is a dynamics of the system in Kat and B is a K-morphism of the type B: U → X.

There are models for categorical expression of the behavior of systems [1; 3], including continuous-time linear systems [4].

8. LOGISTIC PATTERN 'ORDER-DELIVERY-PAYMENT'

A method for simplifying the organizational and managerial models could be the use of the self-similarity of the services in the logistic chain. For instance, the order-delivery-payment sequence could be repeated mutatis mutandis as a pattern "Z" in the supply chain, between the consumer A and the service provider B, as shown in the commutative diagram below, Figure 8. The shorter the life cycle Δt of Z, the closer to the final consumer it is accomplished, i.e. it decreases from upstream to downstream, in the sense of time flow. The client and the supplier pass from the status (A,B) in the moment t of the purchase order for a certain service or logistic product to the status (A',B') in the moment (t+1) of payment the equivalent value for the service delivered.

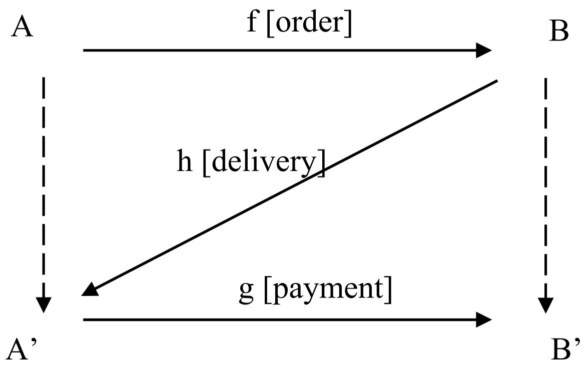

Figure 8. Dynamics of the logistic pattern "order-delivery-payment"

According to Zeiger's lemma, considering the sets A, B, A', B', if it is assumed that f is a surjection and g is an injection, then there is a unique application h so that the diagram remain commutative [7].

9. CONCLUSION

The clients' demands, the urban services, and the regulations are more and more sophisticated, which makes the management of the logistic chains be more and more based on knowledge. For this reason there is an increasing need for new theories and models. Using the category theory in modelling the urban mobility, as it was proposed in this paper, is at the beginning and has to be continued.

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