COMPUTING ELEMENTS OF VISCOUS FRICTION BETWEEN THE ROTOR AND THE CASE TO A ROTATING MACHINE

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Abstract. The first part of the paper presents the constructive solution of a new type of a rotating working machine, with profiled rotors, which can circulate clean or with suspensions fluids. The present paper aims to assess the power consumed to overcome the viscous friction that occurs in two cases: a) on the radial surface between the top of the rotating piston and the case; b) on the front surface between the side surface of the rotor and the case walls. The variation of the consumed power function of the machine rpm for the two cases (a, b) is graphically presented. In the end of the paper the advantages of this new type of rotating machine with two profiled rotors are highlighted. The paper concludes with a selective list of references that reflects the current state of research in rotating machines with profiled rotors.

Keywords: rotating machine, profiled rotors, viscous friction.

1. INTRODUCTION

Currently, the research attention in the field of thermal and hydraulic machines goes to rotating machines because it has the advantages:

– it has no distribution elements: camshaft, valves, etc.;

– it has no crank drive mechanism.

The absence of admission and discharge valves, the crank drive mechanism, leads to an increase in the internal and mechanical efficiency of the rotating machine.

The power supplied to the machine shaft is almost entirely transferred to the fluid to increase the potential pressure energy of the fluid.

This is explained by the fact that the torque $\vec{M} = \vec{b}\cdot\vec{F}$ becomes $M = b\cdot F\sin\alpha$, where b is the force arm (F) and α – the angle between the force F and the force arm.

At the constructive solution presented in this paper, the force arm is exactly the rotor radius plus a half of the rotating piston height and the angle α, throughout a complete rotation, is 90°. From the rotating working machines category, in the paper is presented a rotating volumetric working machine with profiled rotors.

The "rotating machine" term relates to the fact that the presented constructive solution can be used as a blower or as a pump.

The experimental research will be conducted in the laboratory of Thermotechnics, Engines, Thermic and Refrigeration Plants Department, University „Politehnica“ of Bucharest.

2. THE CONSTRUCTIVE SOLUTION AND THE ROTATING MACHINE OPERATING PRINCIPLE

The machine has two profiled rotors which rotate in the opposite direction within a case (Fig. 1).

The synchronous rotation of the rotors (*3, 8*) is provided by two gearwheels, which form a cylindrical gear with straight teeth’s.

The gear wheels are mounted on the shafts (*5*) and (*9*) outside of the machine; during the rotational movement, the rotary pistons (*4*) enter into the cavities of the adjacent rotor.

The rotor profile shape has been established in [1, 2] and the manufacturing technology in [3, 4].

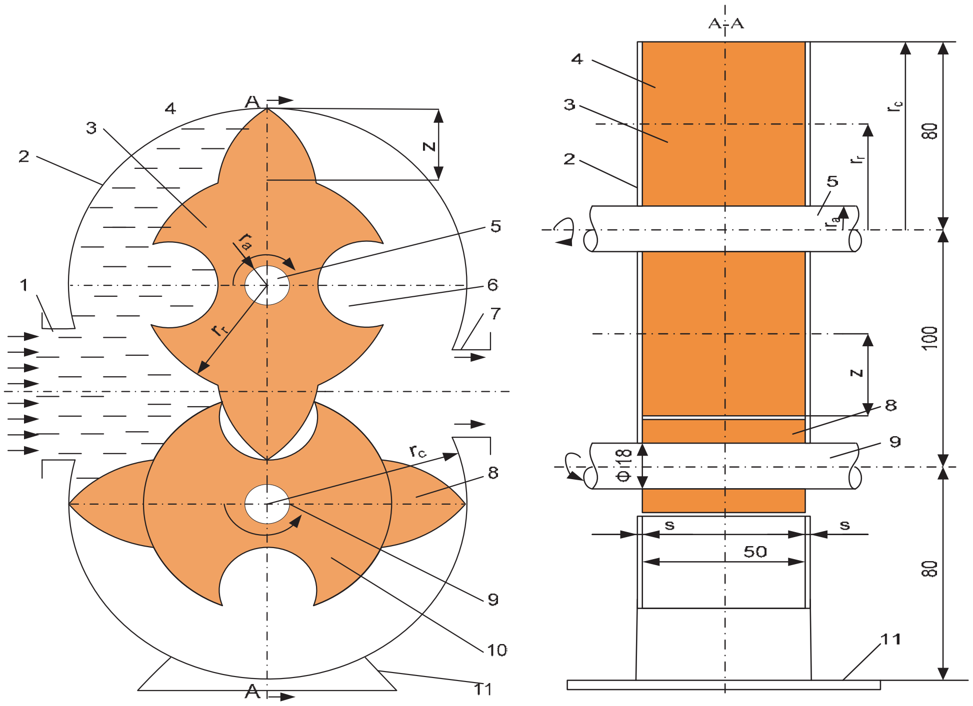
The fluid drawn into the connector (*1*) is transported to discharge (*7*) by the rotating pistons (*4*).

With the notations from Figure 1, the flow rate and the driving power of the machine calculation relations are [5, 6, 7]:

$$\dot{V} = \pi l z\left(2\cdot r_r + z\right)\cdot\frac{n}{30}[m^3/s] \tag{1}$$

$$P_m = \dot{V}_m\,\Delta p = \pi\cdot l\cdot z\cdot\left(2\cdot r_r + z\right)\frac{n}{30}\cdot\Delta p\left[W\right] \tag{2}$$

where: n is the the machine rotation; Δp – the increase in pressure between suction and discharge [N/m^2].



a b

Fig. 1. Cross section (a) and longitudinal section (b) through the blower:
1 – gas suction connection; *2* – upper case; *3* – upper rotor; *4* – rotating piston; *5* – driven shaft; *6* – cavity;
7 – gas discharge connection; *8* – lower rotor; *9* – driving shaft; *10* – contact surface between the rotor and the case wall;
11 – support; r_a – shaft radius; r_r – rotor radius; z – piston height.

3. THE DETERMINATION OF THE POWER CONSUMED BY VISCOUS FRICTION BETWEEN THE ROTATING PISTON TOP AND THE CASE

The calculation is performed for one rotor; during one rotation, in the semi-cylindrical case evolves one rotor piston.

The size of the gap between the top of the piston and the case is of sizes *x l* (fig. 2.*a*). The value of s is selected from $s = 0.01\cdot10^{-3}$ m [8], and the constructed rotor length is $l = 50\cdot10^{-3}$ m.

In this gap a linear fluid velocity distribution is adopted (fig.2.b).

The fluid velocity in the M point (fig. 2.*b*) will be:

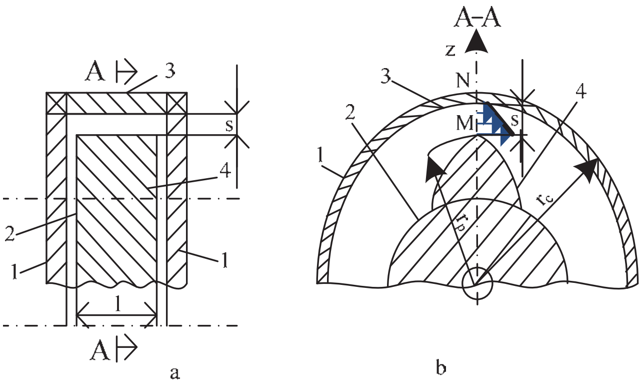
$$w = \omega \cdot r_p \ \text{[m/s]} \qquad (3)$$

where: ω – angular velocity [rad / s]; r_p – piston radius [m].

In the N point, the fluid velocity is zero. The viscous friction tangential effort is given by [9, 10, 11]:

$$\tau = \eta \frac{\partial w}{\partial z} = \eta \frac{w_M}{r_c - r_r} = \eta \frac{\omega r_p}{s} \qquad (4)$$

where: r_c – case radius [m]; η – dynamic viscosity of the fluid [Ns/m^2].



a b

Fig. 2. Computing notations
1 – case walls; *2* – rotor; *3* – case; *4* – rotating piston.

The viscous friction between the piston top and the case is given by [12]:

$$F_f = \tau \cdot S \ \text{[N]} \qquad (5)$$

where *S* is the friction surface in the radial direction which is equal to a cylinder half of length *l* and radius r_p:

$$S = \pi \cdot r_p \cdot l \ [\text{m}^2] \qquad (6)$$

Introducing the relations (4) and (6) into (5) it results:

$$F_f = \eta \frac{\pi \cdot \omega \cdot r_p^2 \cdot l}{s} \ \text{[N]} \qquad (7)$$

The friction torque between the piston top and the fluid is given by [11]:

$$M_f = F_f \cdot r_p = \eta \frac{\pi \cdot \omega \cdot r_p^3 \cdot l}{s} \text{ [N]} \qquad (8)$$

The mechanical power necessary to overcoe the viscous friction related to one rotor [13]:

$$P_1 = \omega \cdot M_f = \eta \frac{\pi \cdot \omega^2 \cdot r_p^3 \cdot l}{s} \text{ [W]} \qquad (9)$$

Because the machine has two rotors, for the entire machine, the mechanical power necessary to overcome the viscous friction on the radial surface will be:

$$P_{m,r} = 2P_1 = \eta \frac{2 \cdot \pi \cdot \omega^2 \cdot r_p^3 \cdot l}{s} \text{ [W]} \quad (10)$$

From equation (10) is observed that for a given constructive solution the value of $P_{m,r}$ is influenced by η and ω^2.

For a specific calculation, as the working fluid, oil is selected, having[14]:

$$\eta = 130.5 \cdot 10^{-3} \text{ Ns/m}^2$$

Is known:

$r_p = r_c - s = (50 - 0.01) \cdot 10^{-3} = 49.99 \cdot 10^{-3}$ m

Is selected: $n_r = 200$ rot/min, so:

$$\omega = \frac{2 \cdot \pi \cdot n_r}{60} = \frac{2 \cdot \pi \cdot 200}{60} = 20{,}93 \text{ rad/s} \qquad (11)$$

From the realtion (10) is obtained:

$$P_{m,r} = 130{,}5 \cdot 10^{-4} \frac{2 \cdot 3{,}14 \cdot (20{,}93)^2}{0{,}01 \cdot 10^{-3}} \cdot \frac{(49{,}99 \cdot 10^{-3})^3 \cdot 50 \cdot 10^{-3}}{0{,}01 \cdot 10^{-3}} = 22{,}415 \text{ W}$$

Similarly, the calculations are performed for n_r = 400, 600, 800, 1000 rot/min. The calculation results are shown in Table 1.

Table 1

Values of $P_{m,r} = f(n_r)$

n_r [rot/min]	200	400	600	800	1000
ω [rad/s]	20.933	41.866	62.799	83.680	104.660
$P_{m,r}$ [W]	22.417	89.671	201.760	358.23	560.392

Based on the results in Table 1, the curve $P_{m,r} = f(n_r)$ is plotted in Figure 3.

From Figure 3 it is noted that for a value of η = ct, $P_{m,r}$ increases as a parabola with the increased machine rpm.

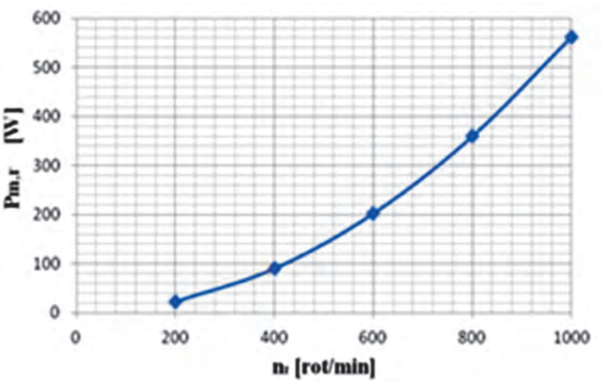


Fig. 3. $P_{m,r} = f(n_r)$ for oil.

4. THE DETERMINATION OF THE POWER CONSUMED BY VISCOUS FRICTION BETWEEN THE ROTORS AND THE SIDE WALLS OF THE CASE

Calculation hypotheses:

– the friction between the top of the piston and the case is neglected;

– the front surface of the piston completes the cavities created in the rotor, so the calculation area is from r_a to r_r (fig. 4);

– the fluid velocity at each point on the front surface of the rotor is equal to the rotor velocity;

– the velocity at a point on the surface of the disk will be in the range of:

$$w_a = \omega \cdot r_a \text{ and } w_r = \omega \cdot r_r \text{ [m/s]} \qquad (12)$$

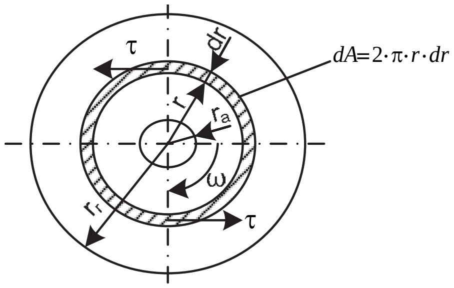


Fig. 4. Plan view of a portion of the rotor.

It is required to establish the power consumed by viscous friction between the front surfaces of the rotors and the case walls.

As initial data are known:

– the shaft radius on which the rotor is mounted: $r_a = 9 \cdot 10^{-3}$ m; the rotor exterior radius: $r_r = 50 \cdot 10^{-3}$ m;

– the angular velocity for a given rpm of the disk;

– the dynamic viscosity of the fluid at t = 20°C for oil: $\eta = 130.5 \cdot 10^{-4}$ Ns/m^2;

– the gap between the disk and the case walls is chosen equal to a numerical controlled center precision processing [8]: $s = 0.01 \cdot 10^{-3}$ m.

The calculation is made for one rotor. The elementary resistant torque due to viscous friction between the rotor and the two walls of the case will be [12]:

$$\mathrm{d}M_r = 2r \cdot \mathrm{d}F_f \tag{13}$$

where F_f is the viscos friction force.

$$\mathrm{d}F_f = \tau \mathrm{d}A \tag{14}$$

where: τ – tangential effort; $\mathrm{d}A$ – elementary surface area (Fig. 2):

$$\mathrm{d}A = 2\pi r \mathrm{d}r \tag{15}$$

The tangential tension (shear stress) due to fluid viscosity is calculated with Newton formula [10]:

$$\tau = \eta \frac{\mathrm{d}w}{\mathrm{d}y} \tag{16}$$

where the coordinate y is measured perpendicularly to the disk surface (Fig. 5).

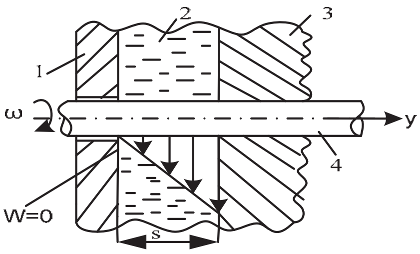


Fig. 5. Computing section:
1 – case; *2* – thin layer of fluid; *3* – rotor disk; *4* – shaft.

The velocity gradient for the boundary layer with "s" thickness, assuming a linear variation, has the expression:

$$\frac{\mathrm{d}w}{\mathrm{d}y} = \frac{\omega r}{s} \tag{17}$$

Equation (16) becomes:

$$\tau = \eta \cdot \frac{\omega r}{s} \tag{18}$$

Equation (14), taking into account equation (15) and (18) becomes:

$$\mathrm{d}F = \eta \cdot \frac{\omega \cdot r}{s} \cdot 2\pi r \mathrm{d}r = 2\pi r^2 \eta \frac{\omega}{s} \mathrm{d}r \tag{19}$$

A more exactly calculation for $\mathrm{d}F_f$ can be performed using the dynamic boundary layer theory [15, 16].

Form equation (13) and (19) is obtained:

$$\mathrm{d}M_r = 2r \cdot 2\pi r^2 \eta \frac{\omega}{s} \mathrm{d}r \tag{20}$$

$$\int_0^M \mathrm{d}M_r = \int_{r_a}^{r_r} \frac{4 \cdot \pi \cdot \omega \cdot \eta \cdot r^3}{s} \mathrm{d}r \tag{21}$$

$$M_r = \frac{\pi \cdot \eta \cdot \omega}{s} (r_r^4 - r_a^4) \; [\mathrm{N \cdot m}] \tag{22}$$

From mechanics is known the computing relation of the power consumed to overcome the viscous friction for one rotor [11]:

$$P_{1r} = M_r \cdot \omega \; [\mathrm{W}] \tag{23}$$

For the entire machine, the power consumed by viscous friction (P_m) will be:

$$P_{m,f} = 2 \cdot P_{1r} \; [\mathrm{W}] \tag{24}$$

Introducing the relations (22) and (23) into (24) it results:

$$P_{m,f} = 2 \frac{\pi \eta \omega^2}{s} \left(r_r^4 - r_a^4 \right) \; [\mathrm{W}] \tag{25}$$

From equation (25) is observed that for a given constructive solution the value of P_m is influenced by η and ω^2.

Assuming that the rotating machine is a pump that circulates oil for a rpm of 200 rot/min, as in equation (11) is obtained: $\omega = 20.93$ rad/s.

By replacing in (25) it results:

$$P_{m,f} = 2 \frac{3.14 \cdot 130.5 \cdot 10^{-4} \cdot (20.93)^2}{0.01 \cdot 10^{-3}} \cdot \left[\left(50 \cdot 10^{-3} \right)^4 - \left(9 \cdot 10^{-3} \right)^4 \right] = 22.417 \text{ W}$$

Similarly, the calculations of $P_{m,f}$ for other values of n_r are made, resulting the data in Table 2.

Table 2

Values of $P_{m,f} = f(n_r)$

n_r **[rot/min]**	200	400	600	800	1000
ω [rad/s]	20.933	41.866	62.799	83.680	104.660
P_{mf} **[W]**	22.417	89.671	201.760	358.23	560.392

Based on the results in Table 2, the curve $P_{m,f} = f(n_r)$ is plotted in Figure 6.

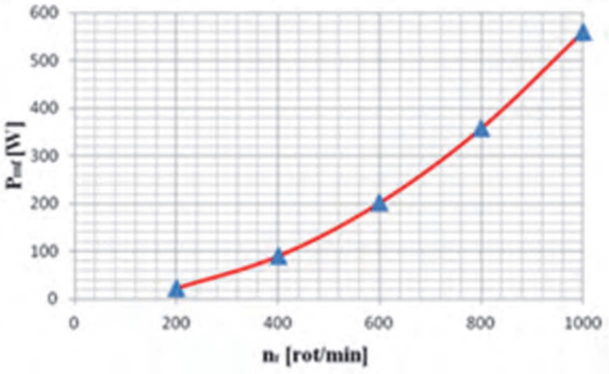


Fig. 6. $P_{m,f} = f(n_r)$, for oil.

If overlap in the same figure the graphs shown in Figure 3 and Figure 6, is obtained:

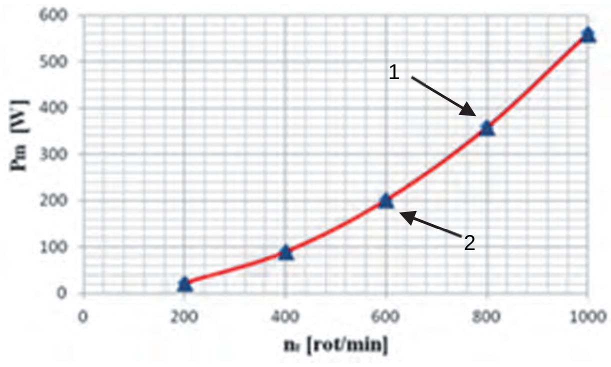


Fig. 7. 1-$P_{mr} = f(n_r)$; 2- $P_{mf} = f(n_r)$

From Figure 7 is observed that for the chosen constructive solution, the power consumed to overcome the radial and the frontal friction is approximately equal.

The total power necessary to overcome the viscous friction is:

$$P_m = P_{m,r} + P_{m,f} \quad [W] \quad (26)$$

By summing the data in Table 1 and 2, the data in Table 3 results:

Table 3

Values of $P_m = f(n_r)$

n_r [rot/min]	200	400	600	800	1000
P_m [W]	44.832	179.49	403.852	717.505	1122

Based on the results in Table 3, the curve $P_m = f(n_r)$ is plotted in Fig. 8.

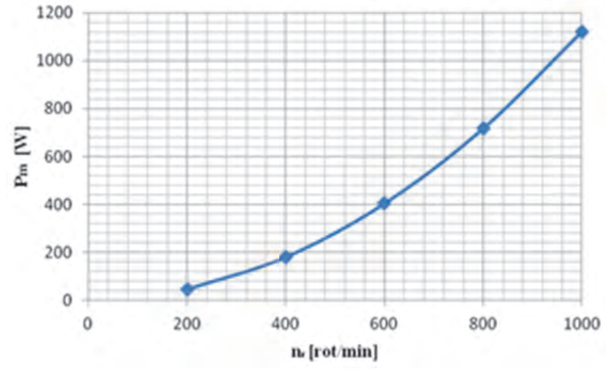


Fig. 8. $P_m = f(n_r)$

From Figure 8 is noted that, if the machine rpm increases, the power consumed to overcome the viscous friction will increase. This, "consumed power" has a small influence on the outside requested power, at the "backlash"regime, [17].

When the machine rpm increases, i.e., under "load" regime, this "consumed power" will increase.

5. CONCLUSIONS

1. For a given constructive solution, the power consumed to overcome the viscous friction depends on η and ω^2.

2. In order to reduce this power, the dynamic viscosity of the fluid must decline by adding additives; is obviously preferable that the value of ω to be as small as possible, this fact depends on the working regime of the rotating machine.

3. The rotor architecture is specified by r_p and l; because the viscous friction on the radial surface is of the same order of magnitude as the friction on the frontal surface, that means that the reduction of r_r and the increase of l reduces the power P_m.

4. Obviously, the gap (s) between the mobile rotors and the fixed case, strongly influence the value of P_m; this influence can be reduced by executing more precise the rotors and the casein a CNC.

5. We are concerned about finding new application and use ways of this new type of working machine which can circulate any fluid substance.

6. The machine driving power will depend on the fluid nature, air, water, oil.

7. This paper is a contribution to the theoretical and experimental research in the field of rotating machine with application to the study of profiled rotors.

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