ON THE USE AND USEFULNESS OF SOME FUNDAMENTAL MATHEMATICAL CONCEPTS IN ENGINEERING EDUCATION

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Abstract: In this communication we present several aspects regarding the ways and characteristics we communicate as well as considerations about the need to systematically use certain fundamental mathematical concepts in engineering education. We insist on the dissimilar significance of notions and the necessity of avoiding the subjective character of communication. Moreover we express the point of view that it might be useful to reveal more thoroughly the mathematical concepts implied in the description and characterization of physical realities. We suggest the need for mastering of certain fundamental concepts, clearly understood, for long-lasting learning and the importance of making a clear difference between physical realities and their mathematical models. Last but not least, we present several aspects regarding mathematical vocabulary and the tools used to describe and characterize physical realities.
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1. INTRODUCTION

Choosing the most appropriate educational methods in engineering is a subject that was, is, and surely will be discussed in the future, taking into consideration the great diversity of educators and students as well as the evolution of the teaching methods. In [1] Carl Jung introduced and developed the concepts of sensing and intuition as basic tools through which people sense the world and learn respectively. Being conscious and investigating these two components as well as their weights for a person or a group of persons can be an efficient mean for education in general and in engineering education in particular. On the other hand, as shown in [2], the main part of students in engineering are sensitive to visual information, prefer sensing, are inductive and more or less active, some of the most creative students being global. In the same time, engineering education is often auditive, abstract, deductive, passive and sequential. Thus there are often incompatibilities between the student and the educator style. Of course, in his equation the motivation of the learning and teaching – aspects about one can talk a lot - should be taken into consideration as well. Since fitting to each student's style is impossible, the educator's art is to find an optimal solution and to make, when feasible, a differentiated education. In what follows we will make several considerations regarding some elementary mathematical concepts whose importance in engineering education of motivated, creative and active students is sometimes less valorized. Thus, we discuss on the one hand the necessity of being conscious of the subjective character of educational communication and, on the other hand, we propose a more thorough use, where appropriate, of fundamental mathematical concepts involved in engineering.

2. ABOUT THE WAY WE COMMUNICATE

We start from the elementary observation that, in order to communicate, people use languages. In general, a language can be any form of communication, not only written and/or spoken ones but also drawings, sketches, music, dance, recorded sounds and images, programming languages, mathematical symbols and concepts (mathematical language) etc. Moreover, communication is not necessary inter-human but also between man and computer, between computers etc. Languages are useful for describing and characterizing objects and phenomena, concepts, for expressing ideas, states of spirit etc., some being more appropriate than others, according to the concrete situation. It often happens to simultaneously use more languages like the written one, drawings, sketches, audio-video recordings and last but not least, the mathematical language.

The great majority of languages, starting with written/spoken ones and continuing with those related to arts, are more or less subjective, the communication exhibiting thus a loose and qualitative character. These characteristics are acceptable in certain limits even in engineering – a domain characterized mainly by precision – anytime when accurate descriptions are either futile or impossible. Often subjective or qualitative characterizations like useful, appropriate, or small, big, weak, strong etc. are utilized. In such cases people that communicate give rather similar significances to the notions, i.e., they have a common language. To understand each other, any communication should be based on a "raw material" unanimously accepted, consisting of known notions, facts and ides as well as previous explanations. On the other hand, in order to define notions/words we should use other notions/words perhaps doubled by drawings, sketches, photos, audio-video recordings etc. Fortunately there exist enough notions with sufficient precise significances and practically unanimously accepted. They can be called primary notions and are not defined but only exemplified. Examples of primary notions are: element, set, object, phenomenon, procedure, time, space, reality, swiftness etc. Indeed, in an explicative dictionary the above notions are … explained but these explanations basically consist of using of synonyms like rapidity, speed, fastness, velocity (or even derivative for mathematicians) etc. for swiftness.

3. CONVENTIONS OF TERMINOLOGY

In teaching engineering sciences, and not only, one should be aware that the significance of a word can vary according to the context and/or the knowledge and educational background of those involved in communication – conventions of terminology. An important convention of terminology is, in our view, that of associating the concept of description with notions like (almost) complete or detailed and that of characterization with partial or rough. Thus, the variation in time of an electric voltage can be described by the function $u(t)=U_m\cos(\omega_0 t+\varphi)$ and characterized by the amplitude (maximum value) U_m, angular speed ω_0 or initial phase φ. A physical reality can be described by a linear differential equation and characterized by a dominant time constant. The symbols used for denoting remarkable physical quantities or functions are also conventions of terminology.

Using primary notions and/or conventions of terminology one can introduce definitions for notions whose significances can be related to other known notions (primary or previously defined). The boundary between conventions of terminology and definitions is not precise. It is convenient to consider the conventions of terminology related especially to ambiguities of significance removal while definitions related to introducing of concepts directly associated to other with known significance. For instance, the concepts of signal, system, and circuit have various significances and, in an engineering text should be disambiguated through conventions of terminology. On the other hand, speed is "by definition" the space derivative with respect to time. Several conventions of terminology related to engineering education, in particular in the domain of electronics, which, according the author's experience proved to be helpful to be systematically used refers to the separation between physical realities denoted by *physical* systems and signals and their mathematical models for which we prefer the name of systems and signals respectively.

Modeling, i.e., going from physical realities to mathematical models (primary, secondary and relations between them) is a combination of science and art, the reverse process being the so-called implementation.

The mathematical description of physical realities starts with the measurements which subsequently allow descriptions and characterizations by means of numbers and relations between numbers, i.e., mathematical methods. Mathematics is a precise, concise, flexible and universal language. In science and technique it allows the objective and precise of the physical realities and their interdependencies and is useful not only for calculations but also, maybe more importantly, for structuring and systematizing our way of thinking. Analysis is another notion which is sometimes ambiguously used either with respect to physical realities or to models – its aim is to determine the so-called behaviors. Synthesis, the opposite of analysis, refers also ambiguously sometimes, to the way from imposed behaviors either to models or to physical realities – this last case being better characterized as design.

Using the primary notions, conventions of terminology and definitions, we can put into evidence motivations and cause – effect relations offering thus explanations for the discussed notions, ideas and phenomena. Explanations consist of using logical judgments to put into evidence relations between causes and affects. Understanding explanations depends both on have a common language as well as understanding previous explanations. In presenting any engineering subject one considers that some facts are "given" – the so-called fundamental truths – that can only be communicated or exemplified in the sense that it cannot or they are not worth being explained in detail. With the tremendous progresses in science and technique the necessity of adopting more and more accurate descriptions and characterizations grew up. The solution was to use the mathematical language. In fact the mathematical language became mandatory with the first measurements of physical properties, i.e., when they were characterized by numbers. Soon people discovered dependencies between measured values and thus, instead of being measured certain properties and behaviors could be computed from knowing other ones.

4. SEVERAL FUNDAMENTAL MATHEMATICAL CONCEPTS USED IN MODELING

Obviously, mathematical modeling implies the use of mathematical apparatus. On the other hand, often the student is not very happy to hear that math is absolutely necessary. This is why sometimes the educator avoids using mathematical concepts and tries to explain things more or less intuitively. This also happens because fundamental mathematical concepts are underestimated or forgotten such that one cannot count on them. Moreover, as already mentioned, a typical attitude is that of regarding math mainly as a tool and not necessarily as a way of thinking. In other words, there is a popular belief that the usefulness of mathematical concepts refers mainly to the possibility of computing, i.e., to a toolbox helpful for finding solutions to various quantitative problems. Of course, this basic role cannot be denied at all but the possibility of regarding math as a unifying point of view, useful to understand and systematize concepts and later as a computing tool might be an interesting alternative for the bright and motivated student.

Often, because of lack of time or from psychological reasons the educator avoids to systematically use fundamental notions like element, set, relation. Maybe it is worth to frequently stress that the "raw material" of mathematics consists of elements and sets which are primary notions [3]. It would be useful to also underline that fact that starting from one or more sets one can invent new sets. Moreover, combining elements and sets one can further invent other elements and other sets among which relations, applications and mathematical structures. Thus, the elements that compose the new sets can be (among) the initial sets like in the case of set union and intersection or elements of a different nature as in the case of the Cartesian product; as it is well-known, the Cartesian product **A×B** of two sets **A** and **B** is the set of ordered pairs of elements, the first from **A** and the second from **B**. The relations are subsets of the Cartesian product (i.e., sets of ordered pairs as well) and reflect the idea of correspondence between elements belonging to the two sets (in particular **A=B**): (certain) elements from the set **B** correspond to (certain) elements from set **B**. Applications are particular relations; their specificity is that to an element from **A** it correspond only on element from **B**. Last but not least, operations are applications from the Cartesian product of two sets in one of them.

An essential observation that led to a tremendous progress in mathematics is that relations as well as applications can be considered themselves elements of sets of relations or applications. The idea of using all concepts introduced in relationship to ordinary sets for sets of relations and applications (functions) was really a great step forward.

Structures are sets with elements among which operations have been defined. Roughly, there are three types of structures: algebraic, topological and measures.

An algebraic structure is a set endowed with at least one internal operation. Structures can be defined with one or more operations on one or several sets.

Topological structures generalize the notions of distance, neighborhood and interval from Euclidian geometry to arbitrary sets and allow to define how close are elements and if they can get as close as possible to certain elements called limits. The simplest way to define a topology is by using the concept of metric (distance)

Last but not least, a measure is a generalization of the concept length, surface volume, weight, time duration, electrical charge etc. In particular, a remarkable measure is the probability.

Coming back to applications, according to the nature of the domain and codomain (numbers, n or functions, f), roughly the mathematical instruments used in engineering can be functions (n→n), parameterizations (n→f), functionals (f→n) or operators (f→f).

Among the most used operators for describing physical realities, are equtions which can be algebraic or differential, linear or nonlinear. We consider as being very important to stress the types of relationship between the amplitudes (a.) and speeds of variation (s.) of the physical quantities representing causes and effects. These two characteristics can be either low (.l) or high (.h). Thus, the equations that describe physical realities can be algebraic linear for (a.l, s.l), algebraic nonlinear (a.h, s.l), differential liniar (a.l, s.h) or differential nonlinear (a.h, s.h). Of course, when the quantities involved depend on spatial coordinates as well, the discussion can be extended to partial differential equations.

We express the opinion that having the above aspects clearly understood would significantly help understanding engineering topics.

5. CONCLUDING REMARKS

In this communication we have discussed several aspects regarding the way we communicate in engineering education and the role of regular use of fundamental concepts from mathematics can have in improving understanding. As a conclusion, we consider that durable education can be achieved among others by a systematic and thorough utilization of fundamental mathematical concepts which, in fact are often doubled by simple intuitive interpretations.

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CÂTEVA CONSIDERAȚII PRIVIND UTILITATEA ȘI UTILIZAREA UNOR CONCEPTE FUNDAMENTALE ÎN EDUCAȚIA INGINEREASCĂ

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Rezumat: În această lucrare facem câteva consideraţii legate de modalităţile de comunicare şi de caracteristicile acestora precum și de utilizarea sistematică a unor concepte matematice fundamentale în educația inginerească. Între altele, insistăm asupra diversităţii semnificaţiilor noţiunilor și necesitatea evitării caracterului subiectiv al comunicării inclusiv prin utilizarea limbajului matematic. De asemenea exprimăm punctul de vedere conform căruia poate fi util să atragem atenția mai consecvent asupra conceptelor matematice implicate în descrierea și caracterizarea realităților fizice. Sugerăm necesitatea stăpânirii unor concepte fundamentale înțelese temeinic pentru a realiza o învățare durabilă și importanța stabilirii unei deosebiri clare între realitățile fizice și modelele matematice ale acestora. Nu în ultimul rând, prezentăm o serie de aspecte legate de terminologia matematică şi de instrumentele de descriere și caracterizare a realităților fizice.