COMPARATIVE DISCUSSION ABOUT THE DETERMINING METHODS OF THE STRESSES IN PLANE SLABS

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Abstract. In this paper it is presented the necessity of intense developing of the finite differences method, highlighted by the authors since the first Congress of Numerical Calculus in Bruxelles 1992, at the determining of the stress state in plane structural elements by comparison with finite elements method. We determine the normal stresses in a point of an octogonal plane slab subjected to a plane system of forces by finite differences method and finite elements method. It is easily established, that the calculus volume using the first method is much diminished comparative with the one that is necessary for using the second method. The difference between values obtained by the two methods are under 10%. Furthermore, in finite differences method the accuracy of the results can be easily increased if are used relations for expressing the derivatives with improved finite differences without reducing the spacing of the discretization network, which is another important benefit of this method. The applying of the finite differences method in problems defined on double conex domains offers a wide field of using this method, being distinguished as an engineering method with advantages of rapidity in determining the stresses in plane structural elements.
Keywords: numerical methods, finite differences, finite elements, stresses, plane slab

1. INTRODUCTION

Numerical methods used in stress state determining of the structural elements may be classified in two main groups:

- methods based on the numerical integration of elasticity differential equations, using the domain discretization into small elements continuously connected one with the other and consequently the calculus approximation is purely mathematical;
- methods using on another physical model where the definition domain is divided into finite elements interconnected in certain points only and consequently the calculus approximation is of a physical nature.

In the both methods we arrive at the algebrical system of equations but in the first case, the calculus accuracy may be increased by the mathematical methods, while in the second case, this accuracy may be obtained increasing the number of discretization elements, and this fact can lead to a calculus difficulty.

Although, the second methods are much more known, the first methods, in the last time, begin to be more developed, at least, from two considerations:

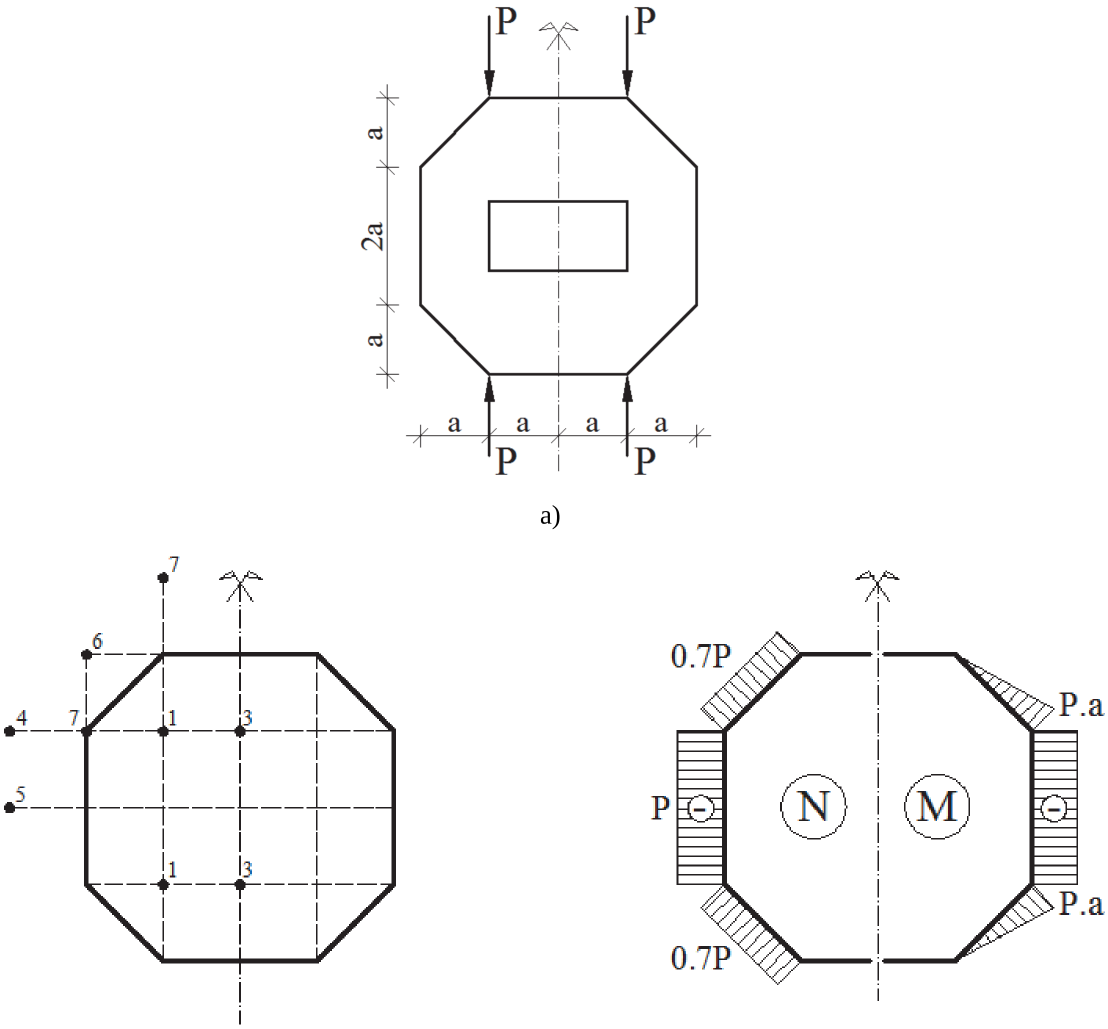
- the calculus accuracy is bigger because the physical model (based on the infinitesimal discretization of the definition domain), “describes” with fidelity the structure behavior;
- the calculus volume is, in many cases, lower, comparative with another procedure, which constitutes an important engineering advantage.

In this paper we present the stress determining by this two methods (finite differences method and finite elements method) for a plane structural element subjected to two concentrated forces (figure 1.a.)

2. CALCULUS USING THE FINITE DIFFERENCES METHOD

Using the domain discretization of the problem shown in the figure 1.b. the plane elasticity equation, written by this method leads to the equations:

$$-3F_1 + F_2 + F_3 = 0,325Pa$$
$$-8F_1 + 11F_2 + 2F_3 = -Pa$$
$$-8F_1 + 2F_2 + 11F_3 = Pa$$



a)

b) c)

Figure 1.

from where we obtain:

$$F_1 = -0,183 P \cdot a$$
$$F_2 = -0,227 P \cdot a$$
$$F_3 = 1,88 \cdot 10^{-3} P \cdot a$$

The stresses in point 1 are:

$$\sigma_{x_1} = \left.\frac{\partial^2 F}{\partial y^2}\right|_1 = \frac{-2F_1 + F_2}{a^2} = \frac{2 \cdot 0,183 - 0,227}{a} \cdot P = 0,14\frac{P}{a}$$

$$\sigma_{y_1} = \left.\frac{\partial^2 F}{\partial x^2}\right|_1 = \frac{F_7 - 2F_1}{a^2} = \frac{-1 + 2 \cdot 0,183 + 1,88 \cdot 10^{-3}}{a} \cdot P = -0,63\frac{P}{a}$$

For the numerical values:

P = 10^3 daN,

a = 50 cm,

the thickness of the diaphragm δ = 1,00 cm

it results:

$$\sigma_{x_1} = 0,14\frac{10^3}{50} = 2,8 \text{ daN/cm}^2$$
$$\sigma_{y_1} = -0,63\frac{10^3}{50} = -12,6 \text{ daN/cm}^2 \qquad (1)$$

3. CALCULUS USING THE FINITE ELEMENTS METHOD

As in the finite differences method, the reference system is orientated to symmetry axis of the slab, y axis to the direction of the external forces (Figure 2.a.). Having in view the double symmetry of the slab with the respect to the y and x axis, the calculus was conducted on the quarter of the element.

The discretization network is presented in the Figure 2.b;

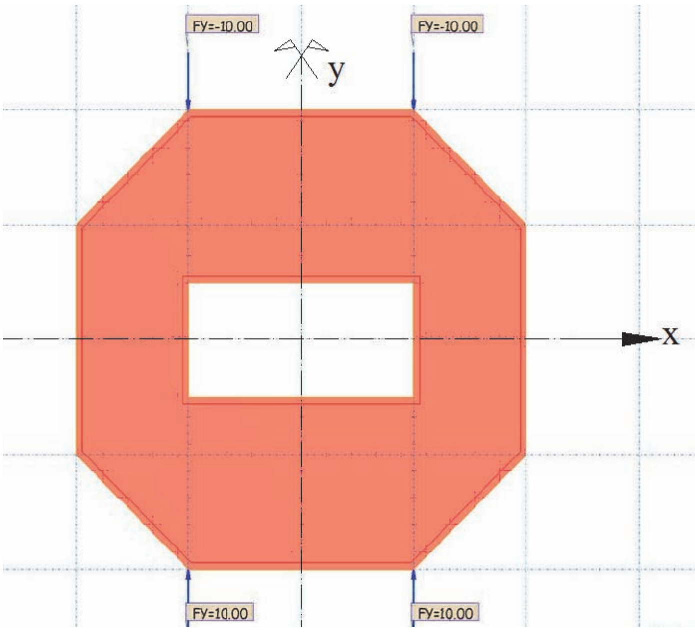


Fig. 2.a

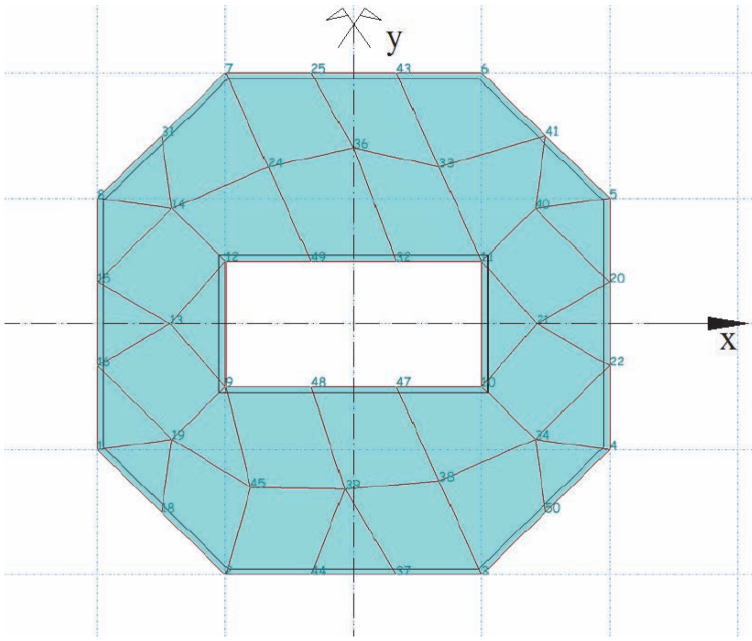


Fig. 2.b

Computation program is based on the following date:
- number of finite elements: 33
- number of nodes: 40
- modulus of elasticity: $2{,}1\times10^6$ daN/cm^2
- Poisson’s ratio: 0,3
- thickness of the diaphragm: δ = 1,00 cm
- external forces: P = 1000 daN = 10 kN

The stresses obtained by the computation program have the signification shown in the Figure 3 (in the center of the element the stresses are given to the reference system *x*, *y*, and on the boundary element to the normal and tangent at the side).

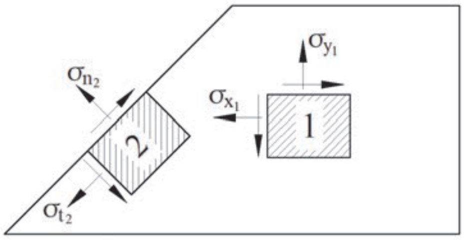
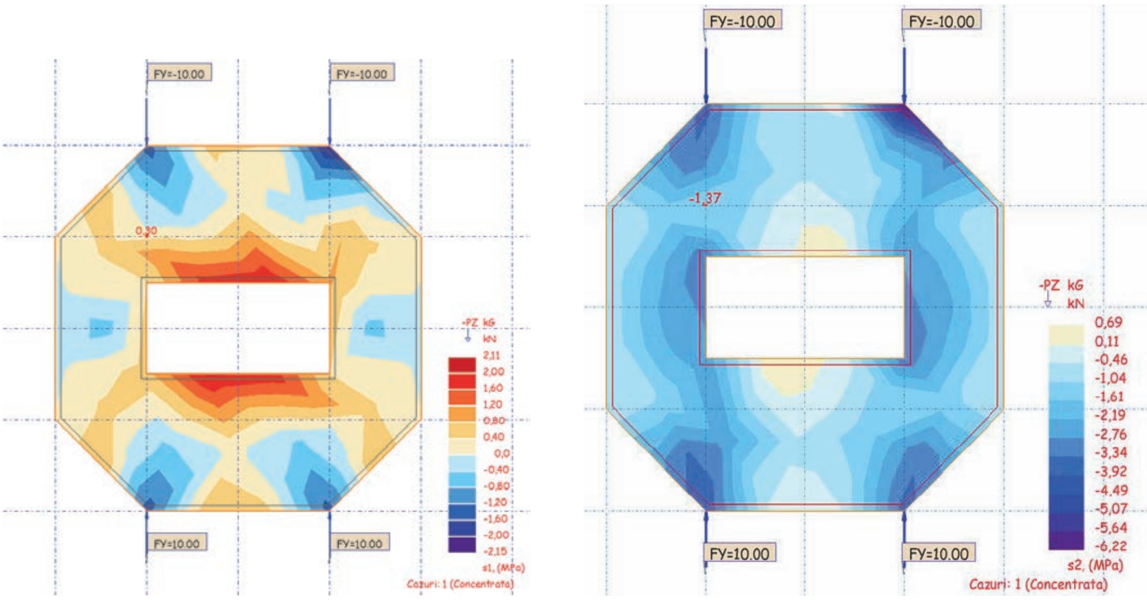


Fig. 3

But in this paper the stresses are determined only in the internal point 1 as in the previous method, the obtained values being:

$$\begin{aligned} \sigma_{x_1} &= 0,32 \text{ MPa} = 3,2 \text{ daN/cm}^2 \\ \sigma_{y_1} &= -1,37 \text{ MPa} = -13,7 \text{ daN/cm}^2 \end{aligned} \qquad (2)$$



a) b)

Fig. 4.

4. CONCLUSIONS

Comparing the relations (1) with (2) it results that between stresses values obtained by the two methods, the difference is of between 7-9 %, but the calculus volume in finite elements method is much more elaborate comparatively with the one that it is necessary in the finite differences method.

In finite differences method it is possible to obtain the results function of any parameters - in our case function of *P* and *a*; and consequently the results are applied to a very large domain, that is for any values of *P* and *a*, and that is an important advantage. This fact is not possible in finite elements method where, initially it is necessary to know all the numerical values of the respective parameters.

Finite differences method may be applied not only for the elements which satisfy the conditions of existence and uniqueness of the plane elasticity equation, but for any other elements if we use the fundamental equations of the elasticity theory for the continuous medium, too (equilibrium equations, continuity equations and physical law of the material).

The results obtained by finite differences method shown in this example are valid in the domain points of the problem excepted those situated around the gap for which a corresponding method will be presented in a future paper.

For the stresses values from the other points, it is evidently that the finite differences method is preferable with respect to the finite elements method, too

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DISCUȚII COMPARATIVE DESPRE METODELE DE DETERMINARE A STĂRII DE TENSIUNE DIN PLĂCILE PLANE

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Rezumat. În lucrare se prezintă necesitatea dezvoltării mai intense a metodei diferențelor finite, evidențiate de autori încă de la primul Congres de Calcul Numeric de la Bruxelles 1992, la determinarea stării de tensiune din elementele plane structurale prin comparație cu metoda elementelor finite. Se determină tensiunile normale într-un punct al unei șaibe octogonale acționate de un sistem plan de forțe. prin metoda diferențelor finite (MDF) și prin metoda elementelor finite (MEF). Se constată, cu ușurință, că volumul de calcul,în cazul folosirii primei metode, este mult mai redus în raport cu cel necesar în cazul folosirii elementelor finite. Diferențele de valori obținute prin cele două metode sunt sub 10%. Mai mult, în MDF precizia rezultatelor poate fi uşor mărită dacă se folosesc relații de exprimare a derivatelor cu diferente finite îmbunătățite fără a micșora pasul rețelei de discretizare, ceea ce este un alt avantaj important acestei metode. Aplicarea diferențelor finite la probleme definite pe domenii dublu conexe conferă un câmp larg de utilizare al acestei metode, evidențiindu-se ca o metodă inginerească cu avantaje de rapiditate în determinarea tensiunilor din elementele structurale plane.