SOME ASPECTS REGARDING STRESS WAVE PROPAGATION IN TRONCONICAL SHORT BARS WITH VARIABLE ANGLE

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Abstract: In the first part of the paper an algorithm of calculus, using FEM, developed by the authors in a previous paper, is presented. This computational algorithm was applied in the study of stresses wave propagation produced by an axial impact in a tronconical short bar. Calculation methodology allowed determination of σr and σθ stresses distribution, produced by the wave-front in different cross sections of the bar. Calculation methodology developed was validated experimentally using three investigative techniques combined: dynamic photoelasticity, electro-resistive strain gauge technique and laser-photomultiplier technique. In the second part of the paper this calculation methodology is applied to investigate the particularities involved in stresses distribution in the cross sections of short conical bars, with variable angle peak, subjected to an axial impact. The results of performed simulations are analyzed comparatively, settling influence of reflected waves on the stresses distribution in the cross sections of bars.

Keywords: stress waves, photoelasticity, electrical stress measurement, finite element method

1. THEORETICAL CONSIDERATIONS

As is well known an exact theory of stress wave propagation in conical bars is not available. Landon and Quinney [3] were among the first to investigate the stress pulse propagation in cilindro-conical steel bars of small apex angle. They derived an elementary approximate theory applicable for the general engineering use considering that:

a. the cone angle is small;

b. the stress waves are spherical or plane and the stresses are distributed uniformly across the section;

c. the motion occurs only in the axial direction.

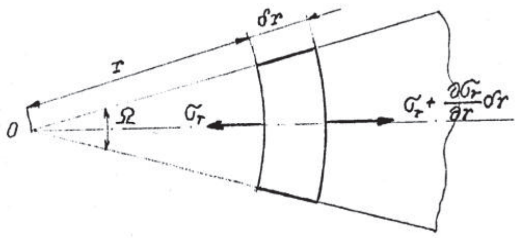


Fig. 1. Spherical wave front calculation model.

Assuming a spherical wave front (Figure 1) and using the one-dimensional constitutive equation:

$$\sigma_r = E\varepsilon_r = E\frac{\partial u_r}{\partial r},$$

they arrived at the equation:

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{2}{r}\frac{\partial u_r}{\partial r} = \frac{1}{c_0^2}\frac{\partial^2 u_r}{\partial t^2}, \tag{1}$$

where σ_r and ε_r are axial stress and strain components, u_r is the axial displacement in direction r, $c_o = \sqrt{E/\rho}$ is rod wave velocity and E is Young`s modulus.

Expresion (1) represents the equation of a spherical stress wave which deplaces along a semi-infinite cone with an small apex angle. The general solution of this equation is:

$$u_r = \frac{1}{r}\left[f\left(r - c_o t\right) + F\left(r + c_o t\right)\right], \tag{2}$$

where f and F represent two spherical waves which propagate in opposite sense along the bar.

For a wave travelig away from the cone apex, eq.(2) reduces to relation:

$$u_r = \frac{1}{r}\left[f\left(r - c_o t\right)\right]. \tag{3}$$

Since

$$\varepsilon_r = \frac{\partial u_r}{\partial r},$$

the strain is given by following expresion:

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = \frac{1}{r}\left[f\left(r - c_0 t\right) - \frac{1}{r} f\left(r - c_0 t\right)\right] \tag{4}$$

Using eq.(4) the strain history in conical bars with small apex angle can by determined if the boundary conditions are know. Kenner and Goldsmith [4], derived a numerical solution to observ the particular strain input in a 5.38° cone.

This study was concerned with one dimensional aspects of stress wave propagation in conical bars, without providing information on the stress distribution in different cross section of bar. In order to elucidate all these aspects concerning stress waves propagation in truncated conical bars with different apex angle, a numerical analysis, using finite element method on the three dimensional models was undertaken in this paper.

2. FINITE ELEMENT MODEL

Numerical analysis was carried out on truncated short bars with different apex angles (5°, 10°, 20°,30°, 40°). The three dimensional calculus model was meshed with solid and tetrahedral elements, with variable dimensions, as it can see in the Figure 2.

For a fidelity reproducing of the experiment, a metalic cilinder of 12.7 mm diameter was placed on the head of the truncated cone. The base of the model was clamped. The stress longitudinal waves were generated by shuting the small end of the bars with a 12.7 mm diameter metalic ball with 29.2 m/s velocity.

The numerical simulations reagrding the waves stress propagation, along the truncated bars were done using the LS-Dyna program.

3. NUMERICAL ANALYSIS RESULTS

In order to put in evidence the propagation process of the stress waves, along the model and to determine the stress distribution in different cross sections of the bars, a special post procesor was used. The wave front propagation in the three dimensional model of the bar with apex angle of 20° is presented in Figure 3.

Intersection of the three dimensional image of the wave front with longitudinal plane of the model, at three time intervals after impact is presented in Figure 4.

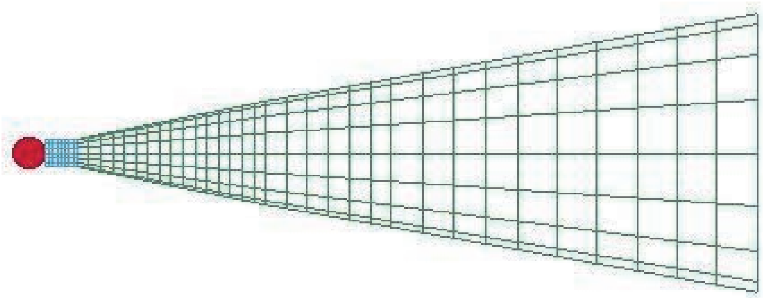
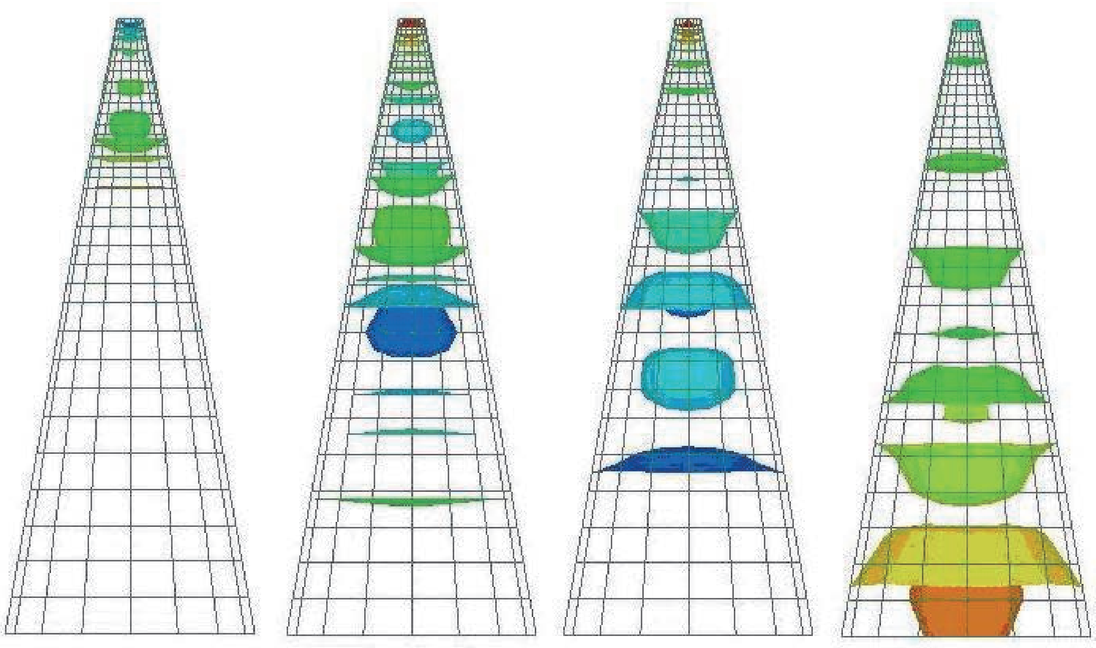
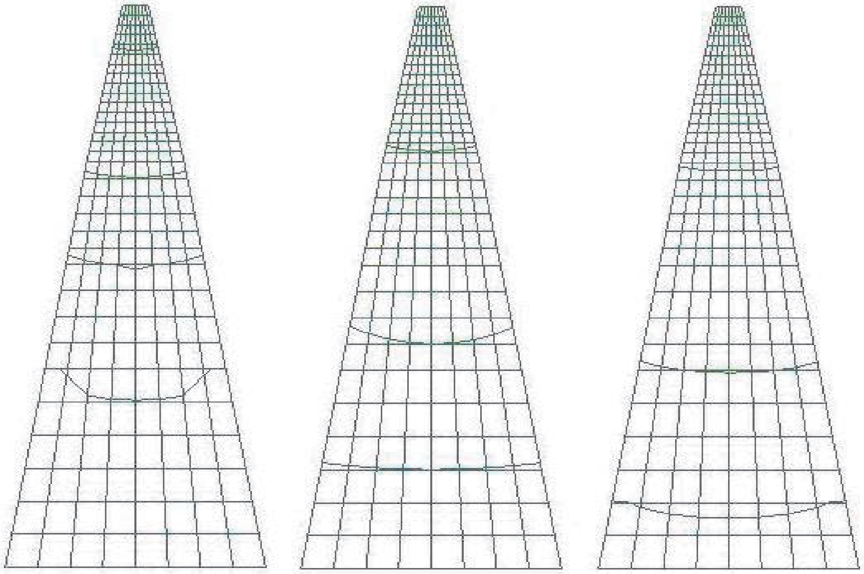


Fig. 2. Finite element model.



50 µs 100 µs 150 µs 200µs

Fig. 3. Stress wave front in the bar with 20° apex angle at different time intervals after impact.



100 µs 200µs 220µs

Fig. 4. Stres wave propagation in the longitudinal plane of the bar with 20° apex angle.

As can see, in figures 3 and 4, the stress-wave front is spherical, with the maximum stress at the central axis of the cone. It may also be observed that the wave front radius increases as pulse travels towards the large end of the conical model.

A close examination of these figures shows that the stress wave front is followed by a more complex waves which seems to represent a trailing portion of the pulse.

Using numerical analysis data, the stresses σ_y in the three different sections in the longitudinal plane of the bar with 20° apex angle, are ploted in the Figures 5 and 6.

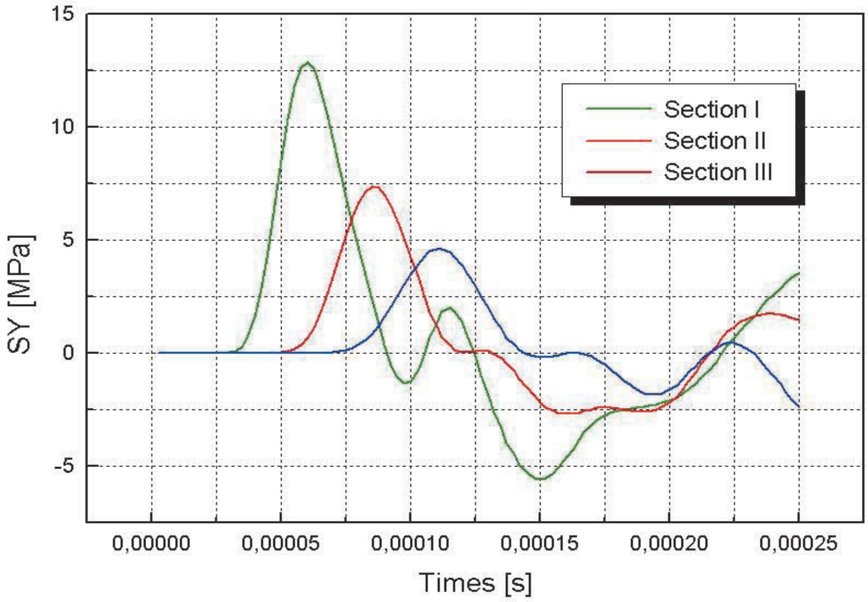


Fig. 5. Variation of stress σ_y in the three sections for bar with 20° apex angle.

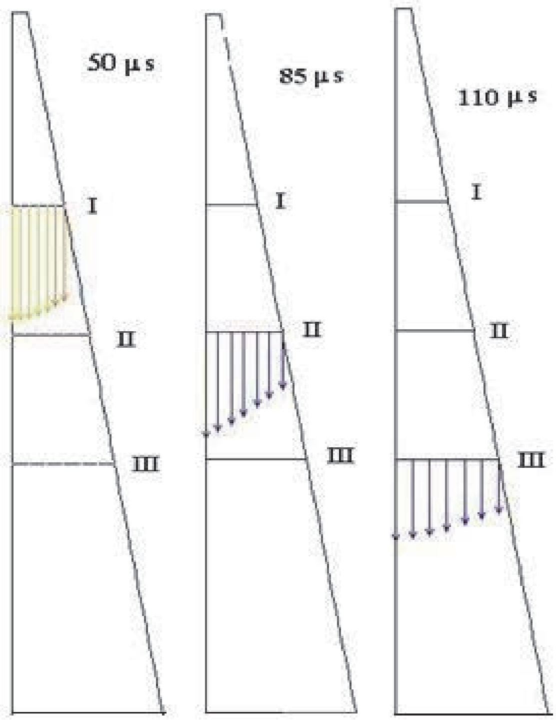


Fig. 6. The stress σ_y variation in the three cross sections for bar with 20° apex angle.

From these diagrams it is evident that initial pulse propagates with a great attenuation. The attenuation of the stress level between section I and section III is about 33.4% for a length of 100 mm.

The diagrams presented in Fig. 5 also shows that the negative taile of the pulse develops as the wave travels down the cone. Similar simulations were done for the other beams with apex angle of 5°, 10°, 20°,30° and 40°.

In Figure 7, the varation curves of σ_y stress vs apex angle, for three different cross section (A, B, C) are ploted. These diagrams indicate that the stresses σ_y decrease as the apex angle increase. The grater diference between values of these stresses occure for angles between 10 to 30 degrees.

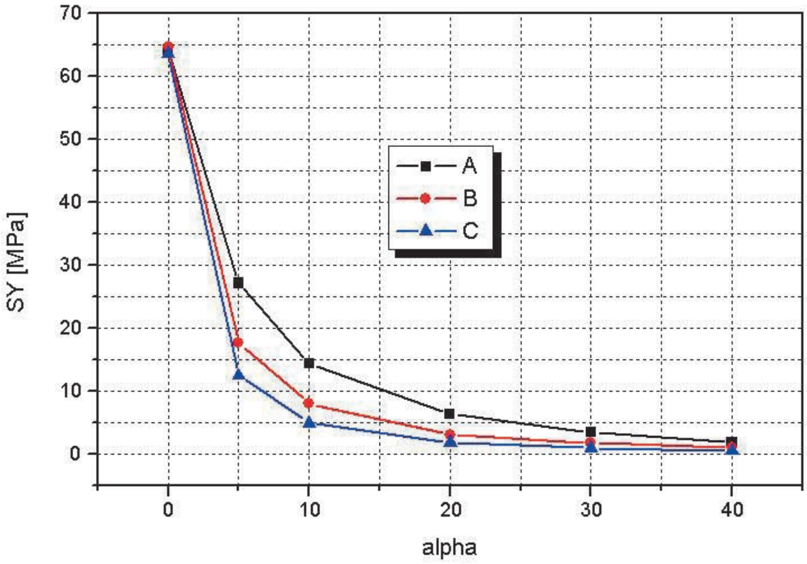


Fig. 7. Variation curves of σ_y stress vs. apex con angle.

4. CONCLUSIONS

The distribution of stresses σ_y is not uniform in the cross section of the bars, as the one-dimensional theory predicated. The stresses have the maximum values at the central axis of the cone and they decrease in value with increasing distance from the impact end (Fig. 4 and Fig. 6).

Dispersion of the longitudinal waves is greater at surface of the bars then of the axis due to lateral inertia (Fig.3, 4 and 6). That shows the lateral inertia has to be taken into account in the one-dimensional constitutive equation for the case of the bars with large apex angle.

The stresses σ_y decrease as the apex angle of bar increase. The grater diference between values of these stresses occure for angles between 10 to 30 degrees.

REFERENCES

[1] Brillhart, L.V., and Dally, J. W. *A Dynamic Photoelastic Investigation of Stress-wave Propagation in Cones,*. Experimental Mechanics, 1968, pp.145-153.

[2] Iliescu, N., *The Study of Stress Wave Propagation in a Tronconical Bar by Dynamic Photoelasticity, Symposium on Experimetal techniques* in Applied Mechanics, Bucharest, 1-3 November 1972.

[3] Kenner, V. H., and Goldsmith, W., *Elastic waves in Truncated Cones,* Experimental Mechanics, 8, 1968.

[4] Landon, I. W. and Quinney, I., *Experiments with Hopkinson Pressure Bar,* Proc.Roy.Soc. London, 1943.

[5] Năstăsescu, V., *Metoda elementelor finite,* Editura Academiei Tehnice Militare, Bucureşti, 1995.

UNELE ASPECTE PRIVIND PROPAGAREA UNDELOR DE TENSIUNE ÎN BARELE TRONCONICE SCURTE CU UNGHI VARIABIL

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Rezumat: În prima parte a lucrări este prezentat un algoritm de calcul cu MEF, elaborat de autori, într-o lucrare anterioară. Acest algoritm de calcul a fost aplicat pentru studiul propagării undelor de tensiune, produse de un impact axial, într-o bară tronconică scurtă. Metodologia de calcul a permis determinarea distribuţiei tensiunior σr şi σ_θ , produse de frontul de undă în diferite secţiuni transversale ale barei. Metodologia de calcul elaborată a fost validată experimental folosind trei tehnici de investigaţie combinate: fotoelasticitatea în regim dinamic, tensometria electro-rezistivă şi tehnica fascicul laser-fotomultiplicator. În partea a doua a lucrării această metodologie de calcul este aplicată pentru investigarea particularităților care intervin în distribuția tensiunilor din secţiunea transversală a barelor tronconice scurte, cu unghi variabil la vârf, supuse unui impact axial. Rezultatele simulărilor efectuate sunt analizate comparativ, stabilindu-se influenţa undelor reflectate asupra distribuţiei tensiunilor din secţiunile transversale ale barelor.

Cuvinte cheie: unde de tensiune, fotoelasticitate, tensometrie, metoda elementelor finite