

# OPTIMIZATION OF FLYING ROBOTS GROUP BASED ON UNMANNED AERIAL VEHICLES

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**REZUMAT.** Este propusă o metodologie de conversie a sistemului compus dinamic într-un sistem discontinuu cu dimensiunea vectorilor de stare și control care se schimbă în momentele de schimbare structurală.

**Cuvinte cheie:** sistem dinamic, optimizare, traiectorie.

**ABSTRACT.** The method of converting the compound dynamic system to the branching system with transient size of state and control vectors at a moment of structural changes is proposed.

**Keywords:** dynamic system, optimization, trajectory.

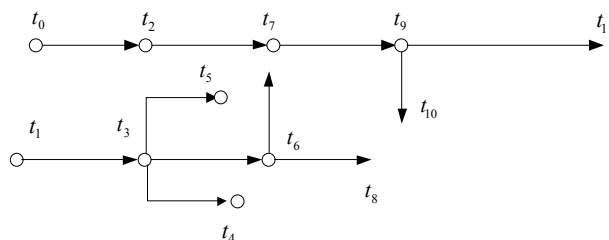
The importance and complexity of the application of robotic systems, including unmanned aerial vehicles (UAVs) in many areas of science, technology and industry caused primarily by the fact that they can be used where human activity is difficult or impossible, for example, radioactive or chemical contamination zones, under battle conditions, during underwater or space research.

At the same time as the experience of the application in the designated areas, a single UAV, no matter how intelligent it may be, can be used only to solve some particular problems, or to perform relatively simple operations, because it usually has a relatively small capability to accomplish the problem.

The obvious solution for the problems mentioned above is deployment of multiple UAVs, i.e. UAV group to solve complex problems.

To use the group of UAVs decide a single large task, they need a certain way to interact with each other so that as efficiently as possible to solve the problem in a complex environment when the situation can change suddenly. To achieve this goal, a group of UAVs will act as something whole, and the actions of each individual UAV should be aimed to achieve the largest group effect. For this purpose, it is proposed to consider the group of UAVs that perform a single task as a compound dynamic system [1, p.91; 2, p.53; 3, p.74].

The deployment effectiveness of these systems depends on the best and optimal choice of coordinates and the time of compound dynamic system (CDS) partition, as well as the optimal method of CDS movement to partition point and optimal movement of the subsystems to targets along paths after partition.



**Fig. 1.** Example of a branching path:  
 $t_1, \dots$  – moments of the CDS structural transformation;  
The CDS subsystems can be grouped and separated many times during its motion. The arrows indicate symbolically the movement direction of the CDS subsystems.

Motion of the subsystems along the CDS path is described by the differential system [4, p. 16]

$$\dot{x} = f(x, u; y, v; t), \quad t \in [t_0, t_f], \quad (1.1)$$

where:  $x \in E^n$ ,  $u \in \cap \subset E^m$ ;  $y$  – phase coordinates,  $v$  – control of other subsystems from the CDS affecting the motion of the subsystem;  $t_0$ ,  $t_f$  – the moments of beginning and end of the motion subsystem on the path branches.

The scalar restrictions of the below form impose the subsystem path (1.1)

$$g_i(x(t_0), y(t_0), t_0; x(t_f), y(t_f), t_f) \begin{cases} = 0, & \overline{i=1, k_g} \\ \leq 0, & \overline{i=k_g+1, n_g} \end{cases}; \quad (1.2)$$

$$q_i(x(t), u(t), t_0; y(t), v(t); t) \begin{cases} = 0, & \overline{i=1, k_q} \\ \leq 0, & \overline{i=k_q+1, n_q} \end{cases}, \quad (1.3)$$

where  $t \in [t_0, t_f]$ .

Criterion for assessing the CDS performance is described by the expression

$$P = \bar{I}(\cdot) + \rho \Sigma \rightarrow \min, \quad (1.4)$$

where:  $P(\cdot)$  – criterion terminal component, depending on the subsystems phase coordinates in times of CDS structural transformation and its time points;  $\rho \Sigma$  – criterion integral component consisting of the sum of particular integral component of the form

$$\rho = \int_{t_0}^{t_f} h(x(t), u(t); y(t), v(t); t) dt, \quad (1.5)$$

corresponding to the individual branches of the CDS path.

Thus, the problem (1.1) - (1.5) of the CDS path optimization is to find the optimal controls and paths of subsystems along branching path, minimizing the criterion (1.4), as well as finding the optimal time and phase coordinates, in which the CDS structural changes occur.

It is supposed to accomplish this task in three stages: first to make the transition from the state of a dynamical system to a discontinuous dynamical system with variable size of the control state vectors; then to optimize a discontinuous system, and finally back to the original task, expressing the result of a

$$I = S(X_{1i}(t_0^+), t_0; X_{1i}(t_1^-), X_{2i}(t_1^+), t_1; X_{2i}(t_2^-), X_{3i}(t_2^+), t_2; \dots \dots; X_{ii}(t_i^-), X_{i+1i}(t_i^+), t_i; \dots; \dots; X_{Ni}(t_N^-), t_N) + \sum_{i=1}^N \int_{t_{i-1}^+}^{t_i} \widehat{O}(X, U, t) dt \rightarrow \min, \quad (1.6)$$

$$G_i(X_{1i}(t_0^+), t_0; X_{1i}(t_1^-), X_{2i}(t_1^+), t_1; \dots; X_{Ni}(t_N^-), t_N) \begin{cases} = 0, & \overline{i=1, K_G}; \\ \leq 0, & \overline{i=K_G+1, N_G} \end{cases}; \quad (1.7)$$

$$Q_{ij}(X_{ii}(t), U_i(t), t) \begin{cases} = 0, & \overline{j=1, K_{Q_i}}; \\ \leq 0, & \overline{j=K_{Q_i}+1, N_{Q_i}} \end{cases}; \quad (1.8)$$

$$\dot{X}_i = F_i(X_i, U_i, t), t \in [t_{i-1}^+, t_i], i = \overline{1, N} \quad (1.9)$$

and  $X_i \in E^{n_{\Sigma_i}}, U \in \Omega_i \subset E^{m_{\Sigma_i}} U_i(\cdot) \quad (1.10)$

piecewise continuous.

Here,  $X_i, U_i$  – advanced phase state and control vectors corresponding to the  $i$  time interval between the CDS structural transformations of dimension  $n_{\Sigma_i}$  and  $m_{\Sigma_i}$ ;  $\Omega_i$  – bounded set of space  $E^{m_{\Sigma_i}}$ ;  $S(\cdot),$

$G_j(\cdot) (j = \overline{1, N_G})$  – smooth of  $E^{2\sigma} \times E^{N+1}$  ( $\sigma = \sum_{i=1}^N n_{\Sigma_i}$ ) scalar functions of variables for

discontinuous system optimization through the notation of the task original formulation.

The transformation method of a compound dynamical system in a discontinuous dynamical system with changing state and control vectors at structural changes is as follows [4, p.16]:

- the branching diagram is plotted based on physical considerations of the CDS, the trajectory equations of motion subsystems along the path branches are formulated, the restrictions operating continuously for subsystem and at boundary points are recorded, the criterion is formulated;

- the chronological sequence of time structural changes of the CDS is set;

- the extended state  $X_i$  and control vectors  $U_i$

( $i = \overline{1, N}$ ) are introduced in the time intervals between the CDS structural changes, where  $N + I =$  number of the CDS structural transformations taking into account structural transformation associated with the beginning of the CDS movement ( $i = 0$ ), consisting of state and control vectors of dynamic subsystems (blocks), which are moved along the path of branches in the time interval.

As a result of this method, the following formulation of optimization of the discontinuous system with variable size of state and control vectors [5, p. 223; 6, p. 284; 7, p. 21; 8, p. 21] is achieved.

$X_1, \dots, X_N, t_0, \dots, t_N; Q_j(\cdot) (j = \overline{1, N_G})$  – continuous of  $E^{n_{\Sigma_i}} \times E^{m_{\Sigma_i}} \times E^1$  together with the first derivatives with all arguments as scalar functions for which the correlations (1.8) satisfy the condition of community of position [64], i.e. the vectors

$\text{grad}_{U_i} Q_j(X_i(t), U_i(t), t) \quad (j = \overline{1, K_{G_i}})$  and  $\text{grad}_{U_i} Q_\gamma(X_\gamma(t), U_\gamma(t), t) \quad (\gamma \in I_\gamma; I_\gamma)$  – the set

of all indices from  $j = \overline{K_{G_i}+1, N_{Q_i}}$ , for which  $Q_j(\cdot) = 0$  are linearly independent;  $F_i(\cdot)$  – continuous with mapping of matrix derivatives  $E^{n_{\Sigma_i}} \times \Omega \times E^1 \rightarrow E^{n_{\Sigma_i}}$ ;  $K_G, N_G, K_Q, N_Q, N$  – given whole numbers,  $0 \leq K_G \leq N_G, K_G \langle \sum_{i=1}^N (2n_{\Sigma_i} + 1) + 1;$   $0 \leq K_{Q_i} \leq N_{Q_i}; K_{Q_i} + K_\gamma \leq m_{\Sigma_i}$ , where  $K_\gamma$  – number of  $\gamma$  indices.

It should be noted that only a reduction of the optimization problem of the CDS branching path to optimization of a discontinuous system path with variable size of the state control and vector (1.6) - (1.10) allows us to formulate the theorem on the base of which the CDS path optimization with arbitrary branch circuit is performed. Otherwise, each new branch circuit of the CDS path requires to perform all procedures of proof considering features of the path.

As discussed above, the third stage of solving the optimization problem of the CDS branching path is return to the terms of the original formulation of the task.

Bringing the conditions for optimal control and trajectory discontinuous dynamic system to optimality conditions CDS, which is the equivalent of a formal breaking system implemented by decomposition of extended state and control vectors, constraints and boundary conditions, support functions and variables used in the application of optimization method, in reverse sequence source transformations that led to the transition from the CDS to a discontinuous dynamic system by the rule of transition to the original terms.

For the most typical cases of this transition will be considered in subsequent publications in the form of the consequences of the main theorem, which formulates the result of solving the task (1.6) – (1.10).

## CONCLUSIONS

In this paper we solve the task of the compound dynamical system path optimization consisting in finding the optimal controls and paths of the subsystems along branching paths that minimize the given criterion, as well as finding the optimal time

and phase coordinates, in which the CDS structural transformation are performed.

The method of converting a compound dynamic system to a branching dynamical system with transient size of state and control vectors at a moment of structural changes is proposed also.

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