STUDY OF A SINGLE PIPE BOREHOLE HEAT STORAGE BY MEANS OF AN ALTERNATE DIRECTION FINITE DIFFERENCE METHOD

Bogdan HORBANIUC[a], Andrei DUMENCU,[a] Michel FEIDT[b], Gheorghe DUMITRAŞCU[a],

[a]"Gheorghe Asachi" Technical University, 43, D. Mangeron Boulevard, Iași 700050, Romania
[b]Université de Lorraine, LEMTA, 2 Avenue de la Forêt de Haye-TSA 60604, 54518 Vandoeuvre Cedex, France

*bogdan_horbaniuc@yahoo.com

Abstract. Borehole thermal storage systems represent a promising candidate for storing heat in the ground, thus making it possible to increase the efficiency of heat pumps used for residential purposes. Such a heat storage should store heat during the warm season of the year, returning it back during the cold one. It is of highest importance to know the performance of the heat storage in terms of heat charging and discharging rates. For this reason, the study of the transient heat transfer into the ground can provide very useful information about heat propagation rates and temperature distribution across the storage domain, thus allowing to determine at any moment the stored heat which might be available for use when required. The paper deals with the use of finite difference methods as a tool in determining the temperature field across a domain subjected to conduction heat transfer. The alternate direction technique is used and the temperature distribution and the stored heat are computed at each time step across a long hollow cylinder representing the ground heat storage. The results are presented and discussed for the charging phase of the storage.

Keywords: borehole heat storage, single pipe, finite difference, alternate direction implicit scheme, cylindrical geometry

1. INTRODUCTION

In the case of thermal conversion of renewable energies from solar or wind energy sources, thermal storage is used to level the energy supply curve, which is subjected to important fluctuations, or to periodical or random cut-offs.

The main advantages of thermal energy storage (TES) are [1], [2]: reduced energy consumption, lower production costs, increased flexibility in operation, higher effectiveness in equipment operation, and last but not least, the use to a broader extent of renewable energy conversion systems, leading to a reduced ecological footprint of energy production.

The feature that makes thermal energy storage systems (TESS) a very promising option, consists of their capacity to increase the energy generation performance by fitting the supply and demand curves (it is known that only in certain circumstances a demand peak can be flattened by a corresponding surge in the supply, due to the fact that generally, energy generation systems run at constant output) [3].

There are three types of TESS [4]: i) sensible heat TESS, where heat is stored as heat capacity of the storage material (SM) – water, ground, rocks, aquifers, etc.; ii) latent heat thermal energy storage systems (LHTESS), where the latent heat of fusion of the SM (called phase-change material – PCM) is stored by making the substance melt (the most used are salts, paraffins, alloys, hydrated salts, and organic compounds such as fatty acids); iii) thermochemical heat storage, where the heat that is stored is represented by the reaction heat of reversible chemical reactions.

Ground thermal storage is a promising candidate due to some favorable features, such as fair thermophysical properties of the SM, availability, very low construction costs (only involving drilling and tubes positioning), and an almost unlimited storage capacity [5]. The main shortcoming of these systems is the high heat losses due to the fact that they cannot be thermally insulated [5].

The paper deals with the theoretical investigation of the performance of a single tube borehole seasonal thermal energy storage system (BTESS).

Analytical methods usually fail to provide solutions to the transient heat conduction equation, due to the fact that this partial differential equation that describes the heat diffusion phenomenon arises insurmountable mathematical difficulties. Only very simple cases (one-dimension heat conduction in bodies with simple

geometric shapes, constant thermophysical properties, and Dirichlet boundary conditions) can be approached by analytical techniques, but even in such cases the solution consists of complicated mathematical functions. For instance, one-dimension transient heat conduction in long cylinders leads to laborious solutions involving the use of Bessel functions which are very difficult to manipulate [6], [7].

Actual situations are generally described by two- or three- dimension transient conduction heat transfer with Neumann boundary conditions. Presently there are no analytical solutions for such cases and other approaches are necessary in order to solve the problem, i.e., to find the time-dependent temperature distribution across the body. In such situations, numerical techniques (such as the finite difference approach) are being used [8…10].

The finite difference technique turns partial derivatives into finite differences and thus linear algebraic equations result, which are easy to manipulate. The price to be paid is the accuracy of the result, which is affected by the truncation errors that are inherent to the finite difference hypothesis. Even so, when the heat conduction is two- or three dimensional, the approach becomes rather complicated.

There are two variations of the finite difference technique, depending on how one considers the present time step: the explicit approach, respectively the implicit one. The explicit finite difference method (EFD) involves using the three nodal temperatures in the finite difference equation at the past time step, thus allowing the calculation of the present time temperature in the current node. There is a limitation for the magnitude of the time step in this procedure, which is not allowed to exceed a maximum value imposed by the convergence and stability criterion of the scheme. The advantage of this approach is that the new nodal temperatures result in a sequential manner, "migrating" from a node to the next one. The implicit scheme (IFD – implicit finite difference) does not involve a time step constraint, since the three nodal temperatures are considered at the present time step. However, there is a price to be paid, consisting of the necessity to solve a set of linear algebraic equations corresponding to the total number of nodes [11].

The paper deals with the use of the alternate direction approach (ADI) of the IFD scheme applied to a two-dimension transient conduction heat transfer in cylindrical coordinates, as described further.

1. MATHEMATICAL MODEL

Two-dimension finite difference approaches are based on the assumption that the heat conduction equation can be formally expressed as a succession of two one-dimension heat conduction problems. This assumption is founded on a formal splitting of the two-dimension laplacian operator into two one-dimension laplacians.

Let us consider a long hollow cylinder of length L, inner radius R_0, and outer radius R_d. The initial temperature of the cylinder is uniform and equal to T_∞. Heat is transferred by convection from a fluid of temperature $T_H > T_\infty$ that flows along the central channel of the cylinder (heat transfer coefficient: k_r) and from another fluid of temperature $T_a < T_H$ (heat transfer coefficient: k_a) to the frontal surface $z = 0$ (see figure 1). The boundaries at $r = R_d$ and at the lower end ($z = L$) are adiabatic.

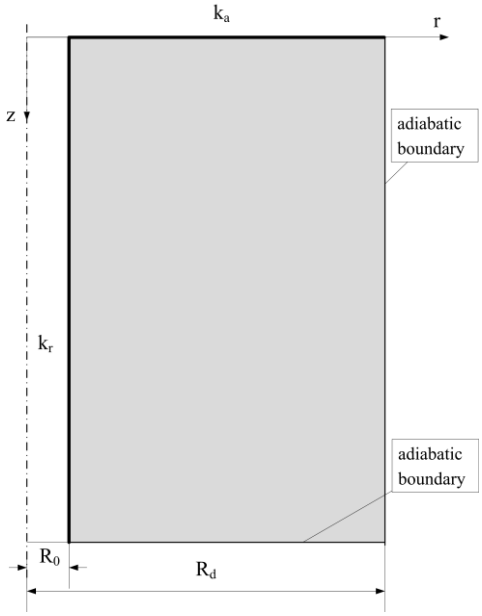

Fig. 1. Schematic of the cylindrical domain.

The conditions above correspond to a two-dimension transient heat conduction that is described by the heat conduction equation in cylindrical coordinates:

$$\frac{\partial\theta}{\partial\tau} = \frac{\lambda}{\rho c}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial z^2}\right) \tag{1}$$

where λ accounts for the heat conduction coefficient, ρ for the density, τ for time, c for the heat capacity, and θ for the dimensionless temperature defined as:

$$\theta = \frac{T - T_\infty}{T_H - T_\infty} \tag{2}$$

The boundary conditions are:

– at $r = R_0$:

$$k_r\left(1 - \theta_0\right) = -\lambda\left(\frac{\partial\theta}{\partial r}\right)\Bigg|_{r=R_0} \tag{3}$$

– at $z = 0$:

$$k_z\left(\theta_a - \theta_0\right) = -\lambda\left(\frac{\partial\theta}{\partial z}\right)\Bigg|_{z=0} \tag{4}$$

– at $r = R_d$:

$$\left(\frac{\partial\theta}{\partial r}\right)\bigg|_{r=R_d} = 0 \qquad (5)$$

– at $z = L$:

$$\left(\frac{\partial\theta}{\partial z}\right)\bigg|_{z=L} = 0 \qquad (6)$$

where θ_0 represents the dimensionless temperature of the cylinder's the inner surface.

3. FINITE DIFFERENCE APPROACH

A mesh has been attached to the cylindrical domain, consisting of N_r nodes of step h_r in the radial direction and N_z nodes of step h_z in the axial direction respectively (Fig. 2).

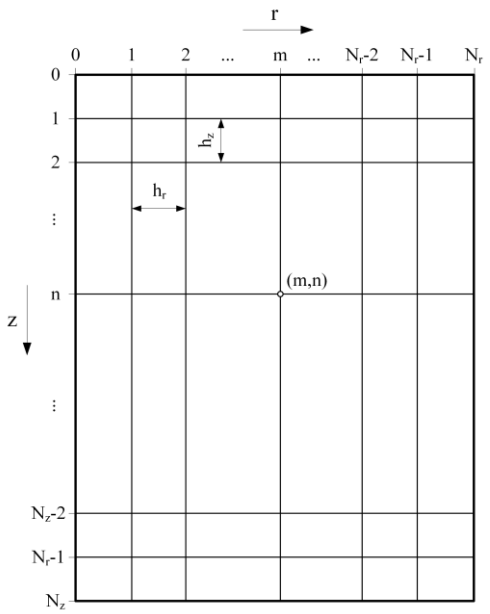

Fig. 2. The mesh attached to the domain.

The heat transfer equation (1) can be rewritten in finite difference form for a time step $\Delta\tau$, by using the finite difference operator D^2 as follows [11], [12]:

$$\frac{\theta^p_{m,n} - \theta^{p-1}_{m,n}}{\Delta\tau} = a\left(D^2_r\theta^p_{m,n} + D^2_z\theta^{p-1}_{m,n}\right) \qquad (7)$$

where a stands for the thermal diffusivity of the cylinder material, and p for the present time step number.

The essence of the alternate directions implicit method (ADI) consists of splitting the time step $\Delta\tau$ into two equal intervals of magnitude $\Delta\tau/2$, thus resulting two successive phases:

1. The intermediary phase, that begins at time $\tau_{p-1} = (p-1)\Delta\tau$ and ends at time $\tau_i = (p-1/2)\Delta\tau$. During this phase, the implicit approach in the radial direction is considered, whereas in the axial direction the explicit approach is taken into account. This means that in the finite difference equation (7), D^2_r is applied to the three consecutive nodal temperatures in the radial direction (presently unknown) at time τ_i, while D^2_z applies to the three nodal temperatures in the axial direction at the beginning (time τ_{p-1}), which are known because they have been already determined in the previous step of the procedure.

2. The final phase, that begins at time $\tau_i = (p-1/2)\Delta\tau$ and ends at time $\tau_p = p\Delta\tau$. In this phase, the approach is implicit for the axial direction and explicit for the radial one. In the latter, the intermediate nodal temperatures are known, since they have been determined for the previous phase, and thus the outcome of this phase is represented by the final temperatures for the current time step p.

Based on the considerations above, the finite difference equation (7) will be written in the two phases as follows:

− the intermediate phase (superscript i for the unknown temperatures):

$$\frac{\theta^i_{m,n} - \theta^{p-1}_{m,n}}{\frac{\Delta\tau}{2}} = a\left(\underbrace{D^2_r\theta^i_{m,n}}_{\text{implicit}} + \underbrace{D^2_z\theta^{p-1}_{m,n}}_{\text{explicit}}\right) \qquad (8)$$

− the final phase (superscript p for the unknown temperatures):

$$\frac{\theta^p_{m,n} - \theta^i_{m,n}}{\frac{\Delta\tau}{2}} = a\left(\underbrace{D^2_r\theta^i_{m,n}}_{\text{explicit}} + \underbrace{D^2_z\theta^p_{m,n}}_{\text{implicit}}\right) \qquad (9)$$

The expressions of operator D^2 in radial and axial directions respectively are:

$$D^2_r\theta_{m,n} = \frac{1}{h_r^2}\left(\frac{c_m - 1}{c_m}\theta_{m-1,n} - 2\theta_{m,n} + \frac{c_m + 1}{c_m}\theta_{m+1,n}\right) \qquad (10)$$

where:

$$c_m = 2\left(N_r\frac{R_0}{R_d - R_0} + m\right) \qquad (11)$$

$$D^2_z\theta_{m,n} = \frac{1}{h_z^2}\left(\theta_{m,n-1} - 2\theta_{m,n} + \theta_{m,n+1}\right) \qquad (12)$$

The ADI method derives from the Crank-Nicolson approach in one direction [13]: in each of the two phases there are three unknowns in the

implicit operator D^2: the nodal temperatures in three consecutive nodes (the central node temperature $\theta_{m,n}$ is present in both members of the implicit equation). The three temperatures that appear in the explicit operator D^2 are known since they have been determined in the previous phase. It results that for each phase of a time step, one must determine the solution of a set of linear algebraic equations written for each node (m,n), the unknowns of which are the nodal temperatures at the end of the respective phase.

By defining the convergence and stability criteria of the finite difference scheme:

$$\alpha_r = \frac{a\Delta\tau}{2h_r^2} \tag{13}$$

$$\alpha_z = \frac{a\Delta\tau}{2h_z^2} \tag{14}$$

and by performing a series of manipulations, equations (8) and (9) become respectively:

$$-\theta^i_{m-1,n} + \sigma_{r,m}\theta^i_{m,n} - \gamma_{r,m}\theta^i_{m+1,n} = \beta_{r,m} \tag{15}$$

$$-\theta^p_{m,n-1} + \sigma_z\theta^p_{m,n} - \theta^p_{m,n+1} = \beta_{z,n} \tag{16}$$

where:

$$\sigma_{r,m} = \frac{c_m}{c_m - 1}\left(2 + \frac{1}{\alpha_r}\right) \tag{17}$$

$$\gamma_{r,m} = \frac{c_m + 1}{c_m - 1} \tag{18}$$

$$\beta_{r,m} = \frac{c_m}{c_m - 1}\frac{\alpha_z}{\alpha_r}\left[\theta^{p-1}_{m,n-1} - \left(2 - \frac{1}{\alpha_z}\right)\theta^{p-1}_{m,n} + \theta^{p-1}_{m,n+1}\right] \tag{19}$$

$$\sigma_z = 2 + \frac{1}{\alpha_z} \tag{20}$$

$$\beta_{z,n} = \frac{\alpha_r}{\alpha_z}\left[\frac{c_m - 1}{c_m}\theta^i_{m-1,n} - \left(2 - \frac{1}{\alpha_r}\right)\theta^i_{m,n} + \frac{c_m + 1}{c_m}\theta^i_{m+1,n}\right] \tag{21}$$

The temperatures on the boundaries of the domain result from the boundary conditions written in the finite difference form.

4. Results and discussion

The numerical model presented above has been applied to a single borehole seasonal thermal energy storage system consisting of a 20 m long pipe 0.25 m across. The radius of the storage system boundary was R_d = 10.125 m. The temperatures were: T_H = 100°C, T_∞ = 10°C, T_a = 25°C. The considered thermo-physical properties of the soil are: λ = 0.25 W/mK, a = $0.18\cdot10^{-6}$ m^2s^{-1}. The convection heat transfer coefficients are: k_r = 1000 W/m^2K and k_a = 10 W/m^2K. The grid attached to the cylindrical domain had N_r = 50 and N_z = 50 nodes respectively.

A computer code was written the output of which were the nodal temperatures at the end of each time step of the procedure and the stored heat in the ground. The total charging time was 180 days. During a day, 10 hours of charging and 14 hours of standby have been considered (standby = heat diffusion in the soil without solar heat addition due to the fact that the system was supposed to store heat provided by a solar energy heat conversion system).

Figure 3 shows the 3-D plot of the temperature field after 90 days.

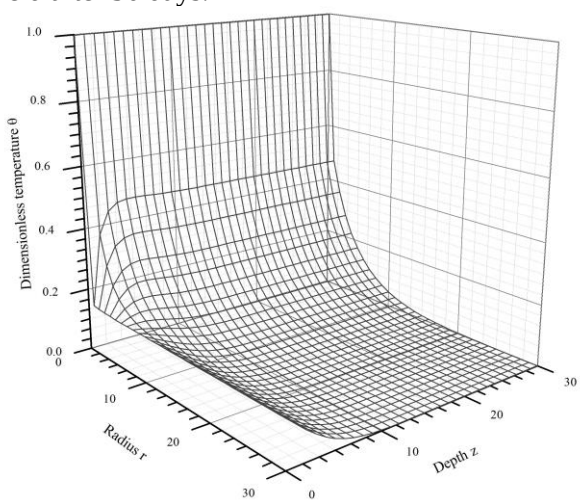

Fig. 3. 3-D plot of the temperature field, after 90 operation days.

It results that the heat propagation is very slow both in radial and in axial directions, which is why in the plot only 30 nodes have been considered as a reasonable penetration depth of the temperature perturbation, although the model considered 50 nodes. The reason why such high values for the domain radius and for the borehole pipe length have been selected is now obvious and it derives from the fact that actually, the considered domain is boundless in both radial and axial directions. On the other hand, it is impossible to apply the finite difference technique to infinite domains and the only acceptable option was to consider a finite domain, but with boundaries so far away that they become unreachable by the temperature

perturbation in a reasonable time. The adiabatic boundary condition applied to these boundaries was in fact a formal one, since the soil temperature at this distance stays unchanged.

Analysis of the output provided by the computer code showed that after 180 days, the penetration distance was of about 30 nodes in both radial and axial directions, which means 6 m in radial direction and 12 m in axial direction respectively (see also Fig. 4).

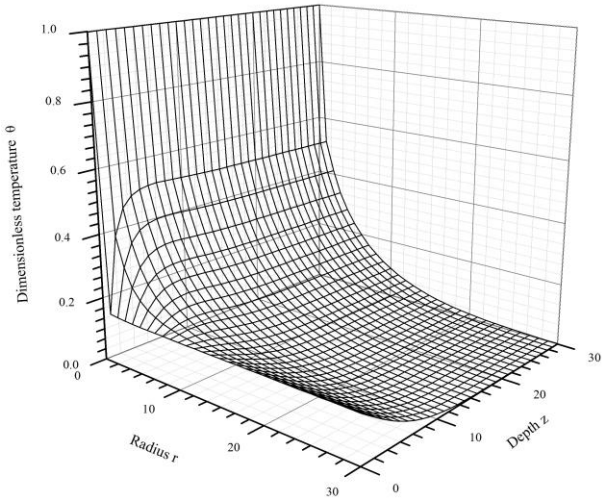

Fig. 4. 3-D plot of the temperature field, after 180 operation days.

A better picture of the temperature distribution can be obtained via a contour plot that shows the position of isotherms across the domain. Figures 5 through 7 illustrate the contour plots for 30, 90, and 180 days respectively.

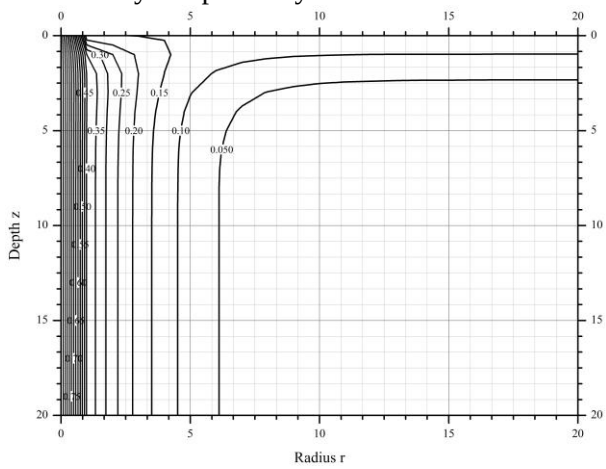

Fig. 5. Contour plot after 30 operation days.

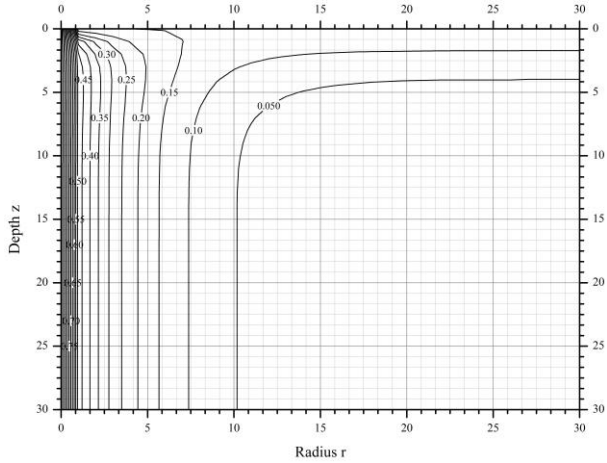

Fig. 6. Contour plot after 90 operation days.

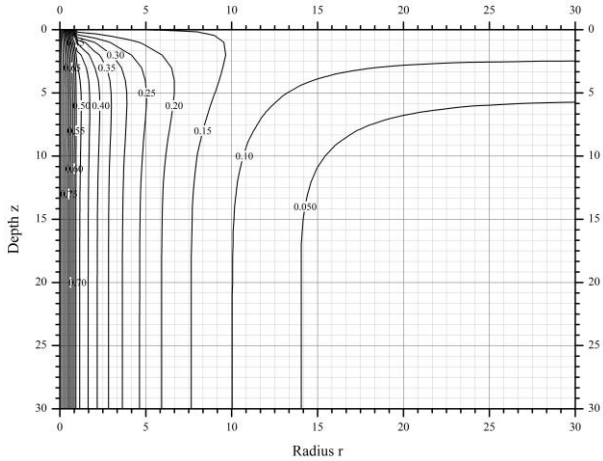

Fig. 7. Contour plot after 180 operation days.

The shape of the isotherms in Figs. 5 through 7 indicates that the heat transfer is actually two-dimensional for a depth *z* of less than 15 nodes, which is the distance until which the influence of the soil surface is present. At distances greater than 15 nodes in the axial direction, all the isotherms are parallel to the tube axis, which means that in this region the heat transfer is one-dimensional.

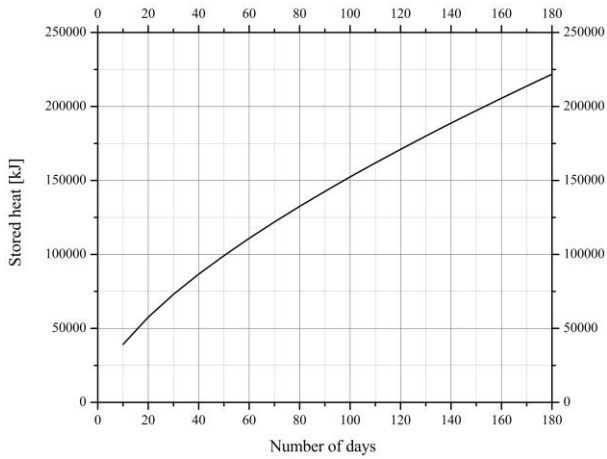

Fig. 8. The stored heat after 180 operation days.

The stored heat is plotted versus time in Fig. 8.

At the end of the 180 charging days, 221.872 MJ of heat have been stored in the ground. It is important to notice that the stored heat only represents a reduced percentage of the maximum amount that can be stored, and this is the price to be paid because of the rather poor heat storage properties of the soil (thermal conductivity and heat capacity).

In order to have a clearer picture of the performance of the single tube BTES in terms of actual stored heat versus the potential (maximum) storage capacity, an assessment has been made. As shown above, the penetration depth after 180 days of operation is about 30 nodes in both radial and axial directions. Actually (see Figs. 4 and 7), the distance measured from the tube wall in radial direction to which the temperature of the ground differs significantly from T_∞ is less than 10 nodes. Consequently, it is reasonable to consider as a comparison criterion, the maximum heat storage capacity of the mass comprised in a cylinder the radius of which is 10 nodes, that is heated from T_∞ to T_H. By dividing the actual stored heat by the maximum storage capacity, one obtains the percentage of stored heat versus time as shown in Fig. 9. The plot shows that at the end of the charging season, only about 22 percent of the maximum storage capacity has been obtained. On the other hand, it is obvious that the maximum heat storage capacity is not achievable for at least two reasons: firstly, the time necessary to get an isothermal storage is extremely long, and secondly, the heat losses prevent the temperature from becoming uniform across the storage material.

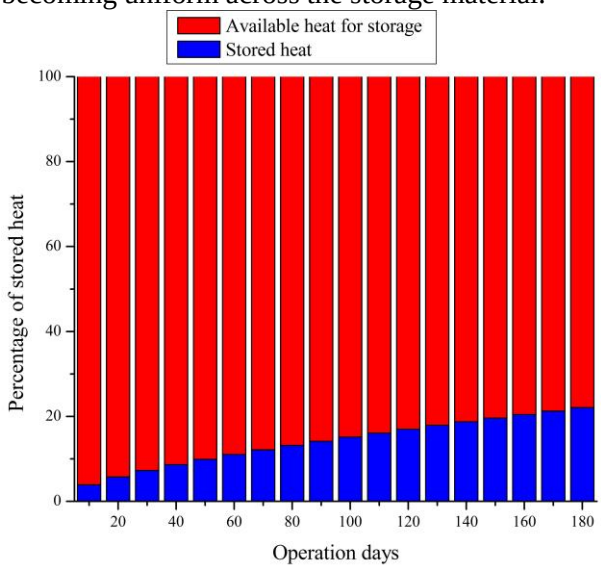

Fig. 9. Percentage of stored heat versus operation time.

5. CONCLUSIONS

Borehole thermal storage units are serious candidates for storing heat in the ground during the warm season in order to use it in heat pumps during the cold season.

The charging process of a single tube BTES has been studied by determining the temperature distribution and the stored heat during a 180 days period, by using an alternate direction implicit finite difference technique for the two-dimension conduction heat transfer.

The results show that for the adopted dimensions of the heat storage unit, due to the rather poor heat transfer characteristics of the soil, the penetration depth of the temperature perturbation is low, and the stored heat does not exceed about 22 percent of the maximum heat storage capacity.

Further study will be performed in order to determine the heat extraction rates during discharging and to optimize the operation of the heat storage unit.

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