ANALYSIS OF THE ELASTICITY AREA OF THE DEFORMATION-EFFORT DIAGRAM FOR TEXTILE MATERIALS KNITTED WITH ELASTOMER AND POLYAMIDE THREADS

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REZUMAT: Lucrarea prezinta modele de interpolare matematica pentru determinarea limitei de elasticitate, punctul E din diagrama efort deformatie. Mai multe variante de materiale textile, tricótate din fire poliamidice si elastomere au fost testate pe dinamometrul electronic, aliura acestor diagrame efort-deformatie fiind prezentata in figura 1. Aproximarea curbei PT trebuie să aibă în vedere în primul rând aproximarea cât mai bună a zonei de elasticitate, deoarece in proiectarea materialelor textile ce necesita elasticitate marita caracteristicile reologice ocupa un rol definitoriu.

Cuvinte cheie: materiale textile cu elasticitate marita, diagrama efort –deformatie, interpolara polinomiala Hermite

ABSTRACT:This paper presents mathematical interpolation models for determining the elasticity limit, point E in the effort-deformation diagram. Several variants of textile materials, knitted with polyamide and elastomer threads, were tested on the electronic dynamometer; the overlap of these effort-deformation diagrams is presented in figure 1. For the approximation of the PT curve must first be considered the best approximation of the elasticity area, since the increased elasticity and the rheological characteristics play a core role in the design of textile materials.

Keywords: high elasticity textile material, stress-strain diagram, Hermite polynomial interpolation

1.INTRODUCTION

The effort-deformation diagram is the graphical representation of the textile material's behaviour under stress until its rupture by the relationship force-elongation at a given moment.

The values of the indices/indicators of appreciation of the tensile properties, as well as the overlap of the diagrams, differ from one category of material to another depending on the fibrous composition, the parameters of the structure of the textile materials, the parameters of the processing and the finishing process applied to the materials (mechanical or chemical) [1,2].

Through the tensile stress method with constant deformation gradient, the sample is deformed gradually, uniformly and continuously until the moment of rupture. The increase in tensile forces over time depends only on the properties of the material being tested and on the deformation speed, thus comparisons between different materials and the efficient control of the determination conditions are possible [3, 4]. Textile materials with increased elasticity, like fabrics with elastane yarn, have a lower breaking force and higher breaking extension than ordinary fabrics, as well as a proportionally lower load and higher extension at the yield point.

Fabrics with elastane yarn have a wider region of time-dependent deformations or viscoelastic regions [5, 6]. The behaviour of the mechanical properties of the implemented textile yarns and structures under short-term stresses has been fairly well researched [7, 8, 9]. Study of materials viscoelastic behaviour is a subject of great importance from viewpoint of the theory of viscoelasticity originated from the material structure as well as from viewpoint of the material processing or its usage according to the specific purpose [10, 11]. To provide information about the viscoelastic behaviour of a polymeric material various experimental techniques have been used, among which stress relaxation, creep, and viscoelastic recovery after preceding sustaining at constant strain or at constant load are continually in common use [12, 13].

2. EXPERIMENTAL PART

2.1. Materials and methods

This paper presents the analysis of effort-deformation diagrams for knitted textile materials made of polyamide and elastane threads.

The textile materials subjected to research were tested on the electronic dynamometer, resulting in an effort-deformation diagram; their overlap is presented in figure 2.1. All the obtained diagrams had the form of figure 2.1, the difference between them being given by the coordinates of the points P,T and R.

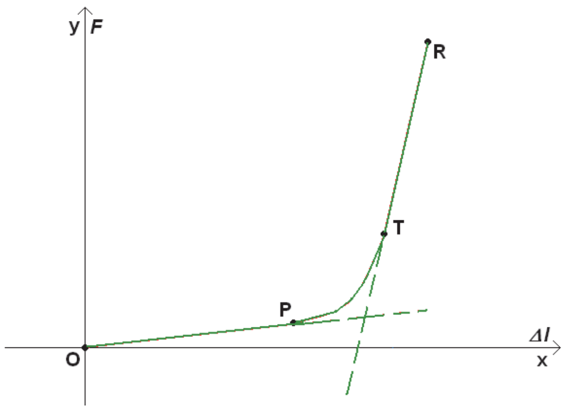

Fig.2.1 Overlap of effort-deformation diagrams for the variants of analysed textile materials

Graphical interpretation of the figure 2.1:

- The force graph starts from the origin of the coordinate system (zero elongation implies that no force acts on the specimen; and reciprocally).
- The OP area is a line segment and represents the Young proportionality area; point P represents the end of the Young proportionality area.
- The PT area implies a variation of the force depending on the absolute elongation at rupture (quadratic polynomial, or exponential etc.); this segment includes an elastic area in the first part.
- The TR area is a line segment in which the material behaves as a non-elastic solid under deformation until rupture; point T represents the beginning of this rupture area.
- The force graph ends at point R, the rupture point of the specimen.

Observations:

- The line described by the segment OP and going through the origin of the coordinate system O can be described by the equation: where represents the slope of the line (where is the angle made by the line with the axis , oriented anticlockwise).
- The line described by the TR segment has a slope greater than and can be described by the equation where the slope (that is); the coordinate points and are intersections of the line with the axes, respectively.
- The PT curve on the force graph has an upward (increases with elongation) and convex slope. The line is tangent to the curve at point P, and the line is tangent to the curve at point T. The convexity of the curve implies an increasing variation of the slope of the curve at its points, with the variation of these points from P to T. Specifically, the angles made by the tangents to the curve in its points vary incrementally from in point P to in point T.
- Determining the elasticity area PE: the point I of the intersection of the lines and is determined; then, the bisector at the angle is calculated; the line of the bisector intersects with the PT curve at point E.
- The oriented angle made by the bisector with the axis is the slope of the bisector is negative.

The location of the important points on the graph is shown in figure 2.2.

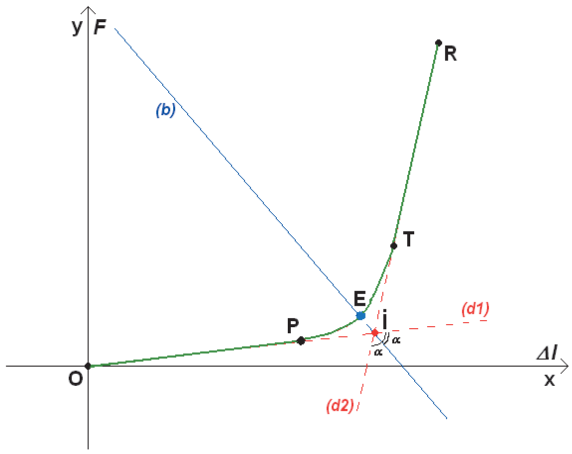

Fig.2.2 Location of the important points on the graph

2.2. Results and discussions

Problems that may arise for knitwear variants under study:

A. Measurement errors for the interpolation point coordinates - error propagation.

B. Solving the equation needed to find the point of elasticity E (in which the elastic area ends). This involves solving the equation:

$$f(x) = mx + n \quad (1)$$

where $f(x)$ is the function found by interpolation, and $y = mx + n$ is the bisector equation

- Polynomial interpolation: $f(x)$ is a polynomial;

Abel-Ruffini Theorem: an algebraic equation of degree ≥ 5 cannot be solved by radicals.

Consequently, $f(x)$ must be a polynomial of degree less than 5.

- Trigonometric interpolation or exponential: in this case, $f(x)$ is expressed as a linear combination of trigonometric ($\{\sin(kx), \cos(kx)\}$) or exponential ($e^{k_i x}$) functions.

For both situations, the equation it is not generally solvable, in the sense of determining the unknown x by formulas.

Polynomial interpolation; Lagrange; Newton

The theoretical basis of polynomial approximation is given by Weierstrass's Theorem.

- Polynomial interpolation: $(n+1)$ points determine a polynomial of degree n; the polynomial coefficients are obtained by calculating the specific determinants of the system of linear equations. The Abel-Ruffini Theorem forces the choice of a maximum of 6 interpolation points.

- Newton interpolation with divided differences: $(n+1)$ non-equidistant points determine a polynomial of degree n; the condition is that the differences between adjacent points must not be equal. An iterative method for determining the polynomial formula is given.

- Lagrange interpolation: same as Newton interpolation but using Lagrange polynomials (the method is not iterative).

- Interpolation with finite differences (forward, backward, central): $(n+1)$ equidistant points determine a polynomial of degree n;

Hermite interpolation

Knowing the value of $f(x)$ and $f'(x)$ for $(n+1)$ points produces a polynomial degree $2n+1$. In this case, 3 points produce a polynomial of degree 5.

Choosing 2 points (P and T) in Hermite interpolation produces a polynomial of degree 3.

- Advantages: the values on the graph in P and T are known, as well as the slopes of the tangents at these points (hence the values of the derivative of order 1).

- Disadvantages: the derivative of order 2 of the Hermite polynomial is a polynomial of degree 1, which can become zero between points P and T, violating the condition of convexity.

$$H(x) = c_3x^3 + c_2x^2 + c_1x + c_0 \text{,with } c_i \in \Re$$
$$i = 0..3$$

$$H'(x) = 3c_3x^2 + 2c_2x + c_1 \quad (2)$$

Obtaining a polynomial of degree 3 that does not satisfy the convexity condition on the first part of the PT curve is a big drawback, because this first part includes the elasticity area. The approximation of the PT curve must first consider the best approximation of the elasticity area, because the relative elongation of the knitted textile materials subject to research falls within the first 2 areas of the curve.

Therefore, we intend to obtain a polynomial of degree 2 which ensures the condition of convexity over the whole PT interval and which it is easy to work with as a mathematical model for said curve.

In the previous Hermite conditions, decreasing the degree of the polynomial by one implies renouncing a condition and, therefore, reducing the number of equations in the system. In Hermite interpolation through 2 points P and T, the condition that could be renounced without affecting the problem would be that of equality of derivability at point T, from which the deformation area before rupture begins.

Thus, the polynomial becomes:

$H(x) = c_2x^2 + c_1x + c_0$,with $c_i \in \Re$, $i = 0..2$

and we obtain: $H'(x) = 2c_2x + c_1$, $H''(x) = 2c_2$

The convexity condition implies:

$H''(x) > 0 \Rightarrow c_2 > 0$

The system is reduced to these linear equations:

$$\begin{cases} H(x_1) = y_1 \\ H(x_2) = y_2 \\ H'(x_1) = m_1 \end{cases} , \quad (3)$$

$$A = \begin{pmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ 2x_1 & 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ m_1 \end{pmatrix}, \quad (4)$$

where $m_1 = \frac{y_1}{x_1}$ is the slope of the line $(d1)$

We obtain: $\Delta = \det(A) = (x_2 - x_1)^2 \Rightarrow \Delta > 0$

And the coefficients:

$$c_2 = \frac{(y_2 - y_1) - m_1(x_2 - x_1)}{(x_2 - x_1)^2},$$

$$c_1 = \frac{m_1(x_2^2 - x_1^2) - 2x_1(y_2 - y_1)}{(x_2 - x_1)^2},$$

$$c_0 = c_2x_1^2 \quad (5)$$

The first coefficient may be written as:

$$c_2 = \frac{x_2}{(x_2 - x_1)^2}\left(\frac{y_2}{x_2} - \frac{y_1}{x_1}\right) \quad (6)$$

We observe that $c_2 > 0$, so the convexity condition is satisfied.

The PT curve will be synthesized by interpolation, thus obtaining a convex approximation function for the graph in question:

$y = H(x) = c_2 x^2 + c_1 x + c_0$, with $c_i \in \Re$, $i = 0..2$ and $c_2 > 0$

where:

$$c_2 = \frac{(y_2 - y_1) - m_1(x_2 - x_1)}{(x_2 - x_1)^2},$$

$$c_1 = \frac{m_1(x_2^2 - x_1^2) - 2x_1(y_2 - y_1)}{(x_2 - x_1)^2},$$

$$c_0 = c_2 x_1^2 \qquad (7)$$

The bisector required to find the point of elasticity was thus determined as follows:

$$y = mx + n,$$

$$\text{with: } m = \frac{1 - m_1 m_2 - \sqrt{1 + m_1^2} \cdot \sqrt{1 + m_2^2}}{m_1 + m_2} < 0,$$

$$n = y_I - m x_I \qquad (8)$$

where the coordinates of the intersection point

$$x_I = \frac{-n_2}{m_2 - m_1}, \quad y_I = \frac{-m_1 n_2}{m_2 - m_1},$$

with:

$$m_1 = \frac{y_1}{x_1}, \quad m_2 = \frac{y_3 - y_2}{x_3 - x_2}, \quad n_2 = \frac{x_3 y_2 - x_2 y_3}{x_3 - x_2} \qquad (9)$$

Determination of the point $E(x_E, y_E)$ involves solving the system:

$\begin{cases} y_E = H(x_E) \\ y_E = m x_E + n \end{cases}$, under the condition that $0 < x_1 < x_E < x_2$ which involves solving the equation:

$$H(x) - (mx + n) = 0 \Leftrightarrow c_2 x^2 + c_1 x + c_0 - mx - n = 0$$

or: $c_2 x^2 + (c_1 - m)x + (c_0 - n) = 0$

We obtain: $\Delta = (c_1 - m)^2 - 4c_2(c_0 - n)$

and: $x_{E\,1,2} = \dfrac{-(c_1 - m) \pm \sqrt{\Delta}}{2}$, corresponding to the points E' and E from figure 2.3

The point of elasticity sought is given by the second root of the equation (higher) $E(x_E, y_E)$:

$$x_E = \frac{-(c_1 - m) + \sqrt{\Delta}}{2},$$

With:

$$\Delta = (c_1 - m)^2 - 4c_2(c_0 - n)$$

$$y_E = H(x_E) \text{ (or } y_E = m x_E + n\text{)} \qquad (10)$$

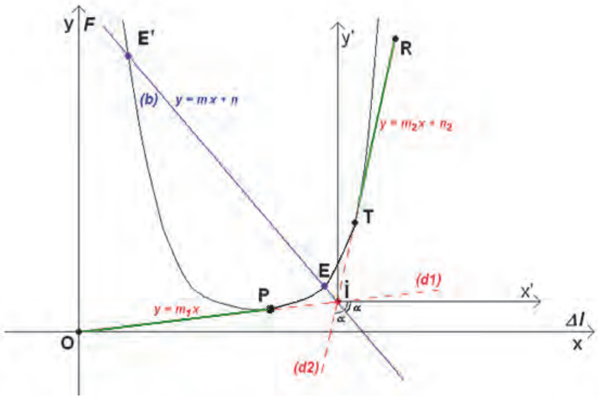

Fig.2.3 The intersection of the bisector line with the polynomial curve of degree 2 H (x)

3. CONCLUSIONS

The elasticity of the materials is characterized by the Hooke area delimited by the coordinate origin and the limit of proportionality P, the area characterized by the linear dependence between force and deformation and the elasticity area where the elastic deformations predominate and which manifest up to point E. The most accurate approximation of the PT area by interpolation is necessary when designing textile products for which the rheological properties play a functional role.

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