PERFORMANCE PREDICTION FOR A COMPLEX INTERFERENCE PROBLEM

Assoc. Prof. Ph.D. Eng. **Doina CAȘCAVAL**

„Gheorghe Asachi" Technical University of Iaşi, Faculty of Industrial Design and Business Management, Iași, România

REZUMAT. Lucrarea tratează o problemă complexă de interferenţă a maşinilor, frecvent întâlnită în industria textilă, în care un muncitor deserveşte mai multe grupuri de maşini afectate de întreruperi accidentale. Într-un grup sunt reunite maşinile ce realizează procese identice. Pentru o astfel de problemă de interferenţă, chiar dacă se consideră un singur mod de defectare, se poate ajunge la modele Markov cu mii de stări. Lucrarea propune o metodă aproximativă în care maşinile dintr-un grup sunt modelate printr-un lanţ Markov echivalent redus. Cu această reducere, metoda de predicţie privind disponibilitatea maşinilor se simplifică considerabil.

Cuvinte cheie: Interferenţa maşinilor, Metode de predicţie, Lanţuri Markov, Reducerea stărilor.

ABSTRACT. This paper treats a complex machine interference problem, frequently encountered in medium-to-large textile mills, in which an operator serves many groups of machines affected by random interruptions. A group consists of machines that can be considered identical. For such a machine interference problem, even if a single failure mode is considered, the Markov model can reach thousands of states. The paper proposes an approximate method in which the machines in a group are modeled by a Markov chain reduced to only two states. With these reduced Markov chains, the application of the prediction method is greatly simplified.

Keywords: Machine interference problem, Prediction models, Markov chains, Equivalent state models

1. INTRODUCTION

In the textile processes, such as those of weaving and spinning, random stops occur especially due to the yarn breakages. In that case, an operator must intervene for remedying the broken yarn and restart the process again. When the number of the machines down is higher than the number of operators, a machine interference time occurs [1], [10].

The phenomenon of machine interference affects the availability of the machines and must be reduced as much as possible. For this purpose, the service capacity must be increased, but, in that case, the loading of the operators is lower. So, in the production organization, two contradictory aspects have to be balanced. To this end, a prediction problem needs to be solved in terms of machine availability and operator loading, depending on the machines assigned to it. In the literature dedicated to operational research, this is known as the 'machine interference problem' or the 'machine repair problem'. An excellent survey on this topic is presented in [6].

The machine interference problem is usually treated for the case encountered more frequently in practice in which the machines perform similar processes. For this simpler case, analytical results for evaluating the machine's availability and the loading of the operator depending on the number of the machines assigned to it are reported in many works (see, for example, [1], [2], and [5]). These analytical results are obtained under the hypothesis that all the primary random variables in the stochastic model have an exponential distribution law. This hypothesis is explained by the fact that the yarn breakages in the textile processes form Poissonian fluxes in the majority of cases.

The machine interference problem becomes much more complicated when the operator serves different textile processes, for which the yarn breakage rates or the remedying time of a broken yarn differ significantly from a machine to the other (for example, [3]). This is the case dealt with in this paper. Let us consider the more general case where the operator serves *k* groups of machines. The machines in a group perform similar processes and are considered identical.

When the machines ensure to some extent a yarn breakage tolerance or service priorities are required (see, for example, [4], [7], [8], [9]), the prediction problem is even more difficult to solve and this case is not considered in this study.

2. PROBLEM DESCRIPTION

Let us consider a machine affected by random breaks specific to the process that the machine achieves (a weaving or a spinning machine in our case). Take λ be the mean rate of these random breakages. On the average, the operator can remedy μ machines affected by broken yarns in the unit of time. The stochastic model for this process affected by accidentally breaks includes two primary random variables, namely:

- The Running Time of the machine until a break occurs (*RT*);
- The Time to Remedy a machine affected by an accidental interruption (*RT*).

For these two random variables, *RT* and *TR*, the mean values are $1/\lambda$ and $1/\mu$, respectively.

For the beginning, let us consider the simple case where an operator serves a single machine. In this case, when the machine is down the remedy of the broken yarn begins immediately (the interference phenomenon does not exist). The availability of the machine in such a case is given by the equation [1]:

$$A = \frac{\mu}{\lambda + \mu} \times 100\ (\%) \tag{2.1}$$

The prediction problem we consider is that, based on this primary stochastic model, to be able to estimate the availability of the machines depending on the number of machines assigned to the operator for remedying. Specifically, we consider the interference problem where the operator serves n_1 machines of type m_1, n_2 machines of type m_2, and so on, n_k machines of type m_k. For a machine of type $m_i,\ i = 1 \div k$, the mean yarn breakage rate is noted with λ_i, and the mean repair rate with μ_i. Take $n = n_1 + n_2 + ... + n_k$ be the total number of machines assigned to the operator.

To solve this prediction problem, analytical methods based on Markov chains or other elements of the theory of queues can be applied [1], [6], [10]. The methods based on Markov chains are applied when all the primary random variables have negative-exponential distribution laws. In this work we consider this particular case. The analytical method implies first to identify the states of the Markov chain (take *N* be the number of states), and then to build a matrix of the transition rates between states, $M = \left[r_{i,j} \right]_{N \times N}$ where $r_{i,j}, i \neq j$, represents the transition rate from state j to state i. For an element of the main diagonal (i.e., $i = j$), the value is calculated as a sum of all the other elements in the column taken with minus.

Based on this matrix of transition rates, the steady state probabilities $p_1, p_2, \ldots, p_N$ can be obtained by solving the following linear equation system [1], where $P = \left[p_1, p_2, \ldots p_N \right]^T$.

$$\begin{cases} M \times P = 0 \\ p_1 + p_2 + \cdots + p_N = 1 \end{cases} \tag{2.2}$$

With these steady state probabilities in hand, the availability of the machines in a group are then calculated as a sum of the probabilities of the states of success, where one or more machines in this group is operational. To illustrate this method, two examples are presented in the following sections.

3. EXAMPLE 1

Let us consider three different textile machines assigned to the operator. Each machine can be either operational (up) or unavailable (down). So a logic variable is appropriate to describe its state. For these logic variables, the same notations m_1, m_2, and m_3 are used. The interference problem as defined in the previous section can be solved based on a Markov chain. This analytical model includes 13 states, as presented in Table 1. This table comprises all the possible states of this model and also all possible transition rates from one state to the others. As notation, a machine down is described by a complemented variable, and the machine subject to remedy is indicated by using the symbol “ ′ ”.

In order to prioritize the remediation when two or more machines are down, let us consider that these three machines are numbered from 1 to 3 taking into account the remedying rate, so that $\mu_1 > \mu_2 > \mu_3$. This means that the machine m_1 has higher priority and the machine m_3, lower priority.

Based on the Markov model presented in Table 1, the transition matrix **M** with a dimension of 13×13 is generated directly. Specifically, a column i in matrix **M** includes the transition rates from state i to all the others states, as presented in Table 1, row i. The location on the diagonal is calculated as a sum of the others elements on column i, taken with minus. For instance, the column 1 in matrix **M** is

$$C_1 = \left[-\left(\lambda_1 + \lambda_2 + \lambda_3 \right) \ \ \lambda_1 \ \ \lambda_2 \ \ \lambda_3 \ \ 0 \ \ 0 \ \ \cdot \ \ \cdot \ \ \cdot \ \ 0 \right]^T \tag{3.1}$$

The steady state probabilities are obtained by applying equation (2.2). For a machine m_i, $i = 1 \div 3$, availability is calculated as a sum of the state probabilities, taking into account only the states in which the machine is operational.

Table 1. **The Markov model for Example 1**

#	State description	Expected transitions
1	$\langle m_1, m_2, m_3 \rangle$	$\lambda_1 \to 2; \lambda_2 \to 3; \lambda_3 \to 4$
2	$\langle \overline{m}_1', m_2, m_3 \rangle$	$\mu_1 \to 1; \lambda_2 \to 5; \lambda_3 \to 6$
3	$\langle m_1, \overline{m}_2', m_3 \rangle$	$\mu_2 \to 1; \lambda_1 \to 7; \lambda_3 \to 8$
4	$\langle m_1, m_2, \overline{m}_3' \rangle$	$\mu_3 \to 1; \lambda_1 \to 9; \lambda_2 \to 10$
5	$\langle \overline{m}_1', \overline{m}_2, m_3 \rangle$	$\mu_1 \to 3; \lambda_3 \to 11$
6	$\langle \overline{m}_1', m_2, \overline{m}_3 \rangle$	$\mu_1 \to 4; \lambda_2 \to 11$
7	$\langle \overline{m}_1, \overline{m}_2', m_3 \rangle$	$\mu_2 \to 2; \lambda_3 \to 12$
8	$\langle m_1, \overline{m}_2', \overline{m}_3 \rangle$	$\mu_2 \to 4; \lambda_1 \to 12$
9	$\langle \overline{m}_1, m_2, \overline{m}_3' \rangle$	$\mu_3 \to 2; \lambda_2 \to 13$
10	$\langle m_1, \overline{m}_2, \overline{m}_3' \rangle$	$\mu_3 \to 3; \lambda_1 \to 13$
11	$\langle \overline{m}_1', \overline{m}_2, \overline{m}_3 \rangle$	$\mu_1 \to 8$
12	$\langle \overline{m}_1, \overline{m}_2', \overline{m}_3 \rangle$	$\mu_2 \to 6$
13	$\langle \overline{m}_1, \overline{m}_2, \overline{m}_3' \rangle$	$\mu_3 \to 5$

Thus, for the three machines, m_1, m_2, and m_3, the availability is given by the following equations:

$$
\begin{aligned}
A_1 &= p_1 + p_3 + p_4 + p_8 + p_{10} \\
A_2 &= p_1 + p_2 + p_4 + p_6 + p_9 \\
A_3 &= p_1 + p_2 + p_3 + p_5 + p_7
\end{aligned} \tag{3.2}
$$

The load of the operator is obtained by the equation

$$O = (1 - p_1) \times 100 (\%) \tag{3.3}$$

4. EXAMPLE 2

Let us consider now an example in which the operator serves two weaving machines of type m_1, one of type m_2, and other one of type m_3 (i.e., $k = 3$, $n_1 = 2$, $n_2 = 1$, $n_3 = 1$, and $n = 4$). As in Example 1, we also consider that $\mu_1 > \mu_2 > \mu_3$. In this case, the Markov chain is more complicated being composed of 21 states. All these states and the transition rates between them are presented in Table 2.

The steady state probabilities are obtained by applying equation (2.2), and the availability for these three type of machines can be obtained by applying the following equations:

$$
\begin{aligned}
A_1 &= p_1 + p_3 + p_4 + p_9 + p_{11} + \\
&\quad + \frac{1}{2}\left(p_2 + p_6 + p_7 + p_8 + p_{10} + p_{14} + p_{17} + p_{21}\right) \\
A_2 &= p_1 + p_2 + p_4 + p_5 + p_7 + p_{10} + p_{13} + p_{16} \\
A_3 &= p_1 + p_2 + p_3 + p_5 + p_6 + p_8 + p_{12} + p_{15}
\end{aligned} \tag{4.1}
$$

Table 2. **The Markov model for Example 2**

#	State description	Expected transitions	#	State description	Expected transitions
1	$\langle 2m_1, m_2, m_3 \rangle$	$2\lambda_1 \to 2; \lambda_2 \to 3; \lambda_3 \to 4$	12	$\langle \overline{m}_1, \overline{m}_1', \overline{m}_2, m_3 \rangle$	$\mu_1 \to 6/8; \lambda_3 \to 18$
2	$\langle m_1, \overline{m}_1', m_2, m_3 \rangle$	$\mu_1 \to 1; \lambda_1 \to 5; \lambda_2 \to 6; \lambda_3 \to 7$	13	$\langle \overline{m}_1, \overline{m}_1', m_2, \overline{m}_3 \rangle$	$\mu_1 \to 7/10; \lambda_2 \to 18$
3	$\langle 2m_1, \overline{m}_2', m_3 \rangle$	$2\lambda_1 \to 8; \mu_2 \to 1; \lambda_3 \to 9$	14	$\langle m_1, \overline{m}_1', \overline{m}_2, \overline{m}_3 \rangle$	$\mu_1 \to 9/11; \lambda_1 \to 19$
4	$\langle 2m_1, m_2, \overline{m}_3' \rangle$	$2\lambda_1 \to 10; \lambda_2 \to 11; \mu_3 \to 1$	15	$\langle \overline{m}_1, \overline{m}_1, \overline{m}_2', m_3 \rangle$	$\mu_2 \to 5; \lambda_3 \to 19$
5	$\langle \overline{m}_1, \overline{m}_1', m_2, m_3 \rangle$	$\mu_1 \to 2; \lambda_2 \to 12; \lambda_3 \to 13$	16	$\langle \overline{m}_1, \overline{m}_1, m_2, \overline{m}_3' \rangle$	$\mu_3 \to 5; \lambda_2 \to 20$
6	$\langle m_1, \overline{m}_1', \overline{m}_2, m_3 \rangle$	$\mu_1 \to 3; \lambda_1 \to 13; \lambda_3 \to 14$	17	$\langle m_1, \overline{m}_1, \overline{m}_2, \overline{m}_3' \rangle$	$\mu_3 \to 6/8$
7	$\langle m_1, \overline{m}_1', m_2, \overline{m}_3 \rangle$	$\lambda_1 \to 13; \mu_1 \to 4; \lambda_2 \to 14$	18	$\langle \overline{m}_1', \overline{m}_1, \overline{m}_2, \overline{m}_3 \rangle$	$\mu_1 \to 14/17/21$
8	$\langle \overline{m}_1, m_1, \overline{m}_2', m_3 \rangle$	$\lambda_1 \to 15; \mu_2 \to 2; \lambda_3 \to 14$	19	$\langle \overline{m}_1, \overline{m}_1, \overline{m}_2', \overline{m}_3 \rangle$	$\mu_2 \to 13/16$
9	$\langle 2m_1, \overline{m}_2', \overline{m}_3 \rangle$	$2\lambda_1 \to 14; \mu_2 \to 4$	20	$\langle \overline{m}_1, \overline{m}_1, \overline{m}_2, \overline{m}_3' \rangle$	$\mu_3 \to 12/15$
10	$\langle m_1, \overline{m}_1, m_2, \overline{m}_3' \rangle$	$\lambda_1 \to 16; \lambda_2 \to 17; \mu_3 \to 2$	21	$\langle m_1, \overline{m}_1, \overline{m}_2', \overline{m}_3 \rangle$	$\mu_2 \to 7/10$
11	$\langle 2m_1, \overline{m}_2, \overline{m}_3' \rangle$	$2\lambda_1 \to 17; \mu_3 \to 3$	-	-	-

These two examples illustrate the efficiency of the Markov chain-based method in solving the prediction problem related to machine interference.

The Markov chain-based method is highly formalized and the results are accurate. However, the complexity of the Markov chain increases very strongly and for this reasons the method is difficult to apply for practical cases when more machines are allocated to the operator. For example, for $k = 3$, $n_1 = 2$, $n_2 = 2$, $n_3 = 2$, the Markov chain has no less than 60 states. But, in practice, an operator can serves up to 15 or more machines. In such a case, the Markov model can reach hundreds or even thousands of states. The question is how can the complexity of these large models be reduced? In the next section we try to answer this question.

5. REDUCED MARKOV MODEL

To cope with the complexity of the Markov models, we propose a two step approach. In the first stage, any group with two or more identical machines is treated separately, and the Makov model is reduced to an almost equivalent one consisting of only two states. For such a group, an average failure rate (denoted by λ_e) is determined.

Based on these simple Makov models, the interference problem is reduced to the case where k distinct machines are served by an operator. This simpler model is solved in the second stage. Thus, the application of the prediction method related to the interference problem is greatly simplified.

To illustrate this approximate method, let us resume the problem presented in Example 2, where the operator serves two identical machines of type m_1, one machine of type m_2 and one of type m_3. In the first step, we focus only on the group with two machines of type m_1. Assuming that the operator serves only these machines, the Markov model is presented in Fig. 5.1. Starting from equation (2.2), to determine the machine availability, the following linear equation system must be solved:

$$\begin{cases} -2\lambda p_1 + \mu p_2 = 0 \\ \lambda p_2 - \mu p_3 = 0 \\ p_1 + p_2 + p_3 = 1 \end{cases} \qquad (5.1)$$

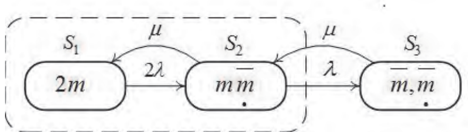

Fig. 5.1. Markov chain for two machines and one operator.

With the notation, $\rho = \dfrac{\lambda}{\mu}$, the system becomes:

$$\begin{cases} p_2 = 2\rho p_1 \\ p_3 = 2\rho^2 p_1 \\ p_1 = \dfrac{1}{1+2\rho+2\rho^2} \end{cases} \qquad (5.2)$$

Based on the steady state probabilities p_1 and p_2, the availability of the machines can be expressed with the equation:

$$A = p_1 + \frac{1}{2} p_2 \qquad (5.3)$$

Based on the equations (5.2), the machine availability results:

$$A = \frac{1+\rho}{1+2\rho+2\rho^2} = \frac{1}{1+\rho\dfrac{1+2\rho}{1+\rho}}. \qquad (5.4)$$

Suppose now that the states S_1 and S_2 in fig. 5.1 are merged into a single state where at least one of machines is operational. A reduced model with only two state is obtained, as illustrated in Fig. 5.2.

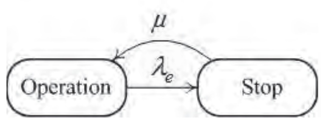

Fig. 5.2. Reduced Markov chain for a group of machines.

For this reduced model (see also equation (2.1)), the availability is given by the equation:

$$A = \frac{1}{1+\lambda_e/\mu} \qquad (5.5)$$

Now, the question is what should be the value for the average failure rate λ_e, so that the reduced model to be equivalent to the initial one? Starting from the equations (5.4) and (5.5), and imposing the condition

$$\frac{\lambda_e}{\mu} = \rho\frac{1+2\rho}{1+\rho} \qquad (5.6)$$

The following result is obtained:

$$\lambda_e = \lambda\frac{\mu+2\lambda}{\mu+\lambda} \qquad (5.7)$$

With this result, in a second stage we have to solve the interference problem in which an operator serves three distinct machines, with the mean rates of random stops $\lambda_1 = \lambda_e$, λ_2, and λ_3, respectively. But this problem is presented in Example 1. In this way, the complexity of the Markov model decreases from 21 states, as in Example 2, to 13 states, as in Example 1.

The method presented in this section for a group with two identical machines can be easily extended for groups with several machines: three, four, five and so on.

All the numerical results we have checked confirm a good accuracy for this approximate prediction method, in which the Markov model is reduced in a great extent.

6. CONCLUSION

An approximate analytical method based on Markov chains to solve a prediction problem related the machine interference is presented in this work.

The method we propose is applied in two steps. In the first stage, any group consisting of identical machines is treated separately, as if the operator should serve only these machines. For any such group of machines, a reduced Markov model with only two states is determined.

Based on these reduced models, in the second stage, a machine interference problem remains to be solved in which the operator serves several different machines. Thus, the prediction problem related to this general case of machine interference is greatly simplified.

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About the author

Assoc. Prof. Ph. D. eng. **Doina CAȘCAVAL**

„Gheorghe Asachi” Technical University of Iaşi, Faculty of Industrial Design and Business Management, Iaşi, România

He graduated from the Faculty of Electrotechnics of the Polytechnic Institute of Iaşi, specialization of Automatic Control and Computers. He is a teacher at the Faculty of Industrial Design and Business Management. He defended his doctoral thesis in the field of Industrial Engineering with the title "Contributions to improving real-time monitoring and control of processes in the textile industry”. Areas of interest: algorithmic and programming, discrete-event systems, stochastical processes, Markov models, Petri nets.

E-mail: cascaval@tex.tuiasi.ro.