EVALUATION OF THE WORK LOST DUE TO LEAKS THROUGH CYLINDER - DISPLACER GAP

Vlad Mario HOMUTESCU, Gheorghe DUMITRAŞCU, Bogdan HORBANIUC

TECHNICAL UNIVERSITY "GHEORGHE ASACHI" OF IASI

Rezumat. *Lucrarea prezintă un model fizico-matematic cu pierderi decuplate (bazat pe principiul suprapunerii efectelor) care evaluează pierderile de lucru mecanic produse de scăpările de agent de lucru prin interstiţiul dintre cilindru şi pistonul împingător al maşinilor Stirling de tip beta sau gama. Lucrul mecanic produs în cazul prezenţei scăpărilor de agent este calculat prin integrarea expresiei diferenţiale a lucrului mecanic de variaţie a volumului, prin luarea în considerare a schimbărilor legilor de variaţie a presiunii în camerele maşinii. Agentul de lucru este considerat gaz ideal. Modelul a fost aplicat pentru o maşină Stirling de tip beta şi permite evaluarea pierderilor de lucru mecanic produse de scăpările de agent prin interstiţiul dintre cilindru şi pistonul împingător.*
Cuvine-cheie: *motor Stirling de tip beta şi gama / pierdere de lucru mecanic / interstiţiu cilindru - piston împingător*
Abstract: *The paper presents an uncoupled model (based on the principle of effects superposi-tion) that assesses the lost work due to working agent leaks through the gap between the cylinder and the displacer of beta- or gamma-type Stirling machines. The work yielded when working agent leaks are taken into account is calculated by integrating the differential expression of the volume variation work, considering the pressure changes inside the chambers of the machine. The modeling hypotheses consider the ideal gas model and the influence of leaks. The model was applied to a beta-type Stirling engine. The model allows the calculation of the work lost.*
Keywords: *beta- and gamma-type Stirling motor / lost work / cylinder-displacer gap.*

1. INTRODUCTION

The Stirling machines (S.m.) are thermal machines with pistons, inside which a constant mass of working agent (usually a gas, air, helium or hydrogen) evolves in a theoretical thermodynamic cycle composed from two isothermal processes and two isochoric processes (*figure 1-a*). All S.m. functional units (equivalent to the monocylinder of the internal combustion engines) must have two pistons, one power piston and one displacer [Homutescu and al. (2003)], [Popescu (2001)], placed inside two cylinders (alpha- and gamma-type S.m.) or inside the same cylinder (at beta-type S.m.) - *figure 1*.

For the realization of the Stirling cycle (*figure 1-a*) inside a beta-type S.m. the working agent comes in contact alternatively with the high and low temperature sources. The displacer moves the working gas from the compression chamber to the expansion chamber and back. The power piston modifies the total volume occupied by the gas inside the machine. The isochoric processes of the theoretical Stirling cycle occur due to the intermittent movement of the power piston, that has two stations in BDC and TDC.

For the case of a beta- or gamma-type S.m. without piston rings, gas leaks appear through the gap between cylinder and displacer. A small amount of gas is leaking from the compression to the expansion chamber or back, depending on the displacer sense of movement and on the pressure difference between the two functional chambers (the pressure difference occurs due to the gas flow through the heat exchangers, a process that takes place with friction).

The blowby phenomenon – a small amount of gases leaks from the combustion chamber into the crankcase - appearing at the internal combustion engines [Heywood (1988)] is quite similar to the S.m. leakings. At S.m. the gas leaks from one functional chamber to another and thus it affects the thermodynamic cycle.

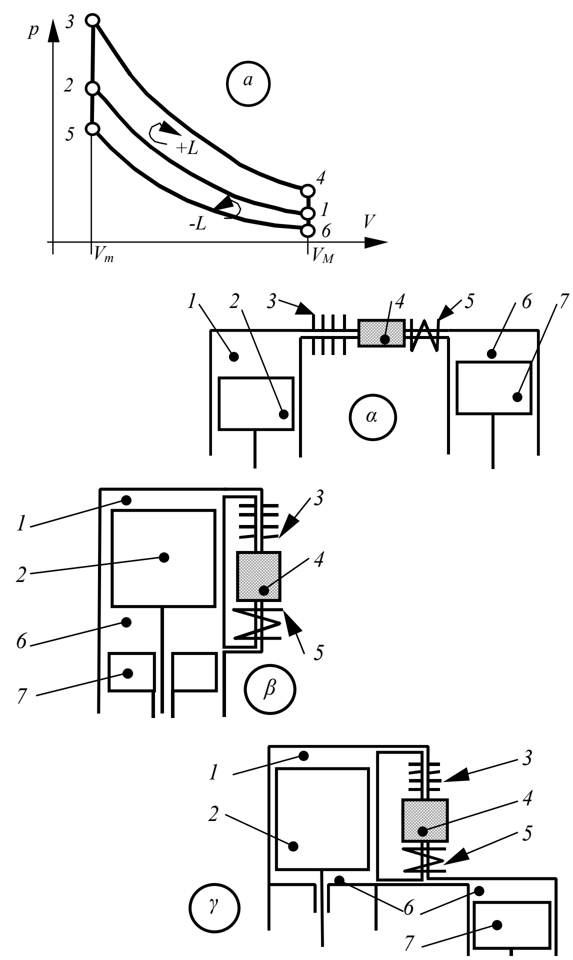

Fig. 1. Stirling thermodynamic cycle and constructive schemes for S.m.:
a – Stirling cycle; *1* – expansion chamber; *2* – displacer; *3* – heater; *4* – regenerator; *5* – cooler; *6* – compression chamber; *7* – power piston.

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2. PHYSICO-MATHEMATICAL MODEL

The flow through the annular gap between the cylinder (having the diameter D) and the displacer can be approximated as a planar flow between two surfaces placed at a distance δ. The two planar surfaces have finite width. The other two dimensions are considered infinite (the extremity effects are neglected). One of the surfaces is fixed and the other one moves with the displacer speed w_d. The flow regime is considered to be permanent.

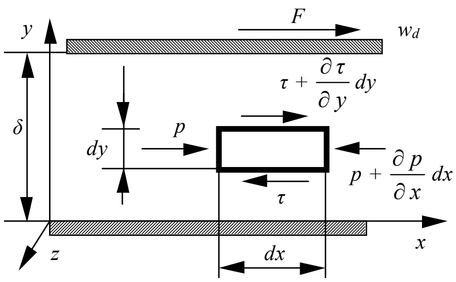

Fig. 2. Scheme for calculating the gas mass flow through the cylinder-displacer gap.

The force equilibrium for an elementary volume of gas is

$$p\,dy\,dz-\left(p+\frac{\partial p}{\partial x}dx\right)dy\,dz-\tau\,dx\,dz+\left(\tau+\frac{\partial \tau}{\partial y}dy\right)dx\,dz=0 \quad (1)$$

or $-\dfrac{\partial p}{\partial x}=\dfrac{\partial \tau}{\partial y}$. (2)

The tangential unitary effort between two moving layers of viscous fluid is, in conformity to Newton hypothesis,

$$\tau=\mu(\partial w/\partial y)\,, \quad (3)$$

where μ is the dynamic viscosity.

Introducing Eq. 3 in Eq. 2 and neglecting the partial derivatives (i.e. the pressure varies on the direction of x axis only) we obtain

$$\frac{d^2 w}{d y^2}=\frac{1}{\mu}\frac{d p}{d x}\,. \quad (4)$$

Integrating Eq. 4 two times and setting the boundary conditions $w(0) = 0$ and $w(\delta) = w_d$, the speed w is obtained as

$$w(y)=\frac{y\,w_d}{\delta}+\frac{1}{2\mu}\frac{d p}{d x}\left(y^2-\delta\,y\right). \quad (5)$$

The volumetric flow of gas leaking through the gap between cylinder and displacer is obtained by integration. Considering that the pressure varies linearly through the length L_d of the displacer and taking into account that the viscosity depends on temperature, after integration we obtain

$$\dot{V}_L=\pi D\delta\frac{w_d}{2}-\frac{\pi D\delta^3}{12\mu(T_L)}\frac{\Delta p}{L_d}=\dot{V}_1+\dot{V}_2\,. \quad (6)$$

The mass flow through the cylinder-displacer gap is calculated with

$$\dot{m}_L=\rho\left(\frac{\pi D\delta w_d}{2}-\frac{\pi D\delta^3}{12\mu(T_L)}\frac{\Delta p}{L_d}\right)=\rho\dot{V}_L=\dot{m}_{L1}+\dot{m}_{L2}\,, \quad (7)$$

where ρ is the volumetric mass of the working agent flowing through the cylinder-displacer gap. The term T_L is the temperature of the gas flowing through the cylinder-displacer gap. Subscript “L” refers to the leaks. The hypothesis that the sense of the gas flow through the gap is determined only by the pressure difference between compression and expansion chambers was made.

The term $\dot{m}_{L1}$ in Eq. 7 represents the mass flow through a fixed reference section (placed between the compression chamber and expansion chamber) due to the movement of the displacer only, without taking into account the pressure differences between the two sides of the displacer (when $\Delta p = 0$). The term $\dot{m}_{L2}$ in Eq. 7 represents the mass flow through a fixed reference section under the action of the pressure differences, the displacer being considered immobilized.

The working agent passing from the compression to the expansion chamber or back through the cylinder-displacer gap, bypassing the heat exchangers, is mixed with the agent inside these chambers. As a result of this mixing process the temperatures inside the compression and the expansion chambers change, and the pressures in these chambers change also.

The influence of the leaks over machine performances is taken into account using the superposition method, the various losses being considered to be produced by independent causes. The functioning cycle was divided into small time (or rotation angle) intervals. We used the following algorithm for each of these intervals:

- the mass of agent passing from one chamber to another through cylinder-displacer gap is calculated for each interval;
- the temperature inside each chamber is recalculated based on the energy balance equation;
- the new pressures inside compression and expansion chamber are calculated from the equation of state;
- the work lost due to the gas leaks is determined using the difference between the recalculated pressure and the theoretical adiabatic pressure. The theoretical adiabatic pressure is the pressure calculated for a Stirling machine considering that inside the compression and expansion chambers adiabatic processes take place and inside the heat exchangers isothermal processes take place. The adiabatic pressure was recalculated in order to taking into account the effects of the heat transfer taking place at finite temperature difference inside heater and cooler heat exchangers [Urieli and Berchowitz (1984)].

We made the hypothesis that the mass of working agent inside each chamber of the Stirling machine is equal to the mass calculated with a theoretical adiabatic model for each crankshaft rotation angle. For this we assume that the mass entering a chamber, bypassing the heat exchangers, pushes an equal mass of working agent into the nearby heat exchanger. The specific heats were assumed temperature-independent.

The work lost is calculated by summing the effects for each interval. The mass of agent leaks in one calculation interval is

$$\Delta m(\alpha) = \frac{1}{\omega \text{ step}} \int_{\alpha-\text{step}/2}^{\alpha+\text{step}/2} \dot{m}_L(\alpha_\alpha)\, d(\alpha_\alpha), \qquad (8)$$

where "step" is the value of an angular calculation interval and α_α is a local variable, also representing the rotation angle; ω is the angular frequency of the crank-shaft movement.

For the expansion chamber the energy balance equation for the mixing process is written as:

$$[m_e(\alpha) - \Delta m(\alpha)]T_e(\alpha) + \Delta m(\alpha)T_L(\alpha) = \\ = m_e(\alpha)T_{eL}(\alpha), \qquad (9)$$

and for the compression chamber is:

$$[m_c(\alpha) + \Delta m(\alpha)]T_c(\alpha) - \Delta m(\alpha)T_L(\alpha) = \\ = m_c(\alpha)T_{cL}(\alpha), \qquad (10)$$

where $T_{eL}(\alpha)$ and $T_{cL}(\alpha)$ are the new temperatures inside chambers after the mixing processes. The subscripts "c" and "e" refer to the compression and the expansion chamber.

The temperature T_L of the gas flowing through cylinder-displacer gap is a conditional function and the leaks have an associated sign (positive: the agent flows from compression to expansion chamber). We assumed that the masses passing into nearby heat exchangers have the temperature of the neighboring chamber with variable volume. Because T_k-T_c<<T_e-T_c and T_e-T_h<<T_e-T_c, the approximation is fair. Subscripts "k" and "h" refer to cooler and heater.

The new temperatures established in the expansion and compression chambers after the mixing processes are:

$$T_{eL}(\alpha) = T_e(\alpha) - \frac{\Delta m(\alpha)[T_e(\alpha) - T_L(\alpha)]}{m_e(\alpha)}; \qquad (11)$$

$$T_{cL}(\alpha) = T_c(\alpha) + \frac{\Delta m(\alpha)[T_c(\alpha) - T_L(\alpha)]}{m_c(\alpha)}. \qquad (12)$$

The mass inside in the variable volume chambers can be (for several calculation steps) smaller than the mass presumed to enter the chamber due to the leaks. In this situation the temperature of the mixture is considered equal to the temperature of the chamber from which the leaks came from.

The recalculated pressures in the two chambers are:

$$p_{eL}(\alpha) = \frac{m_e(\alpha)\, R\, T_{eL}(\alpha)}{V_e(\alpha)}; \qquad (13)$$

$$p_{cL}(\alpha) = \frac{m_c(\alpha)\, R\, T_{cL}(\alpha)}{V_c(\alpha)}. \qquad (14)$$

The work lost because of the leaks is calculated based on the difference between the pressure given by Eq. 13 and Eq. 14 and the theoretical adiabatic pressure p(α). For a complete rotation of the crankshaft, the equation takes the following expression:

$$\Delta L_L = \int_0^{2\pi} [p_{eL}(\alpha) - p(\alpha)] \frac{dV_e(\alpha)}{d\alpha} d\alpha + \\ + \int_0^{2\pi} [p_{cL}(\alpha) - p(\alpha)] \frac{dV_c(\alpha)}{d\alpha} d\alpha. \qquad (15)$$

3. EXAMPLE

In order to apply the presented physico-mathematical model for evaluating the lost work caused by the gas leaks through cylinder-displacer gap a beta-type Stirling motor is considered (*figure 3*). The Stirling motor is characterized by the following main dimensions: crank radius r = 0.0365 m, connecting rod length l = 0.15 m, distance between the lower yoke and the frontal surface of the displacer 0.8135 m, distance between upper yoke and the frontal surface of the displacer 0.363 m, displacer and power piston diameter D = 0.073 m, displacer stem diameter d = 0.02 m, displacer length 0.15 m, distance between the displaced in TDC and the cylinder head 0.002 m, ratio between the volume of the cooler and the volume swept by the displacer 0.1, ratio between the volume of the regenerator and the volume swept by the displacer 1.2, ratio between the volume of the heater and the volume swept by the displacer 0.1, cooler length 0.08 m, regenerator length 0.07 m, heater length 0.08 m, radial gap between cylinder and displacer $0.05\ 10^{-3}$ m, diameter of the cooler and heater pipes 0.002 m. The rhombic drive used for the beta-type engine is an offset crank gear. The distance between the axis of the crankshaft and the plane in which the yoke's pin moves is e = 0.046 m. The wire mesh of the regenerator have 400 wires of $0.0254\ 10^{-3}$ m in diameter per /inch.

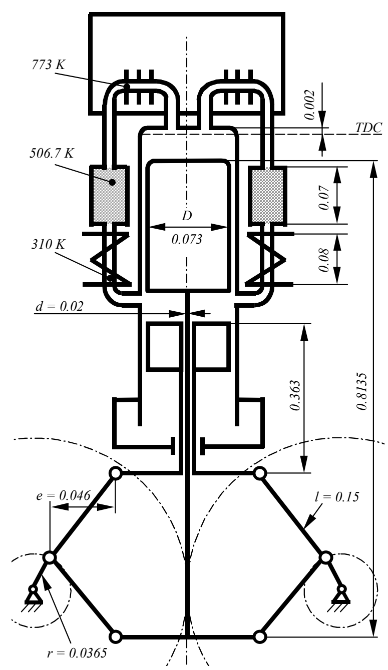

Fig. 3. Beta-type Stirling motor.

The functional parameters of the simulated working regime are: heater temperature 773 K, cooler temperature 310 K, logarithmic mean temperature of the regenerator 506.7 K, outside temperature 290 K, hydrogen mass 0.0025 kg, revolution speed 1000 ... 2000 rpm. For the numerical calculations were used for pressure, masses and temperatures the values determined with a theoretical adiabatic model of the S.m. [Homutescu and Bălănescu (2005)].

The pressure difference between expansion and compression chamber is determined by summing the linear and local pressure losses caused by the gas flowing with friction through heater, regenerator and cooler [Urieli and Berchowitz (1984)]. The pressure difference is considered positive when the pressure inside the expansion chamber is greater than the pressure inside the compression chamber. The pressure drop for regenerator matrix is calculated based on experimental data taken from [Walker (1980)]. The displacer speed results from the kinematic analysis of the machine and is considered positive if the displacer moves toward expansion chamber.

For the analyzed Stirling motor the pressure difference between the two chambers takes the shape presented in *figure 4*. The mass flow of gas leaking through cylinder-displacer gap is presented in *figure 5*. The mass flow is positive when the gas leaks from compression chamber to expansion chamber.

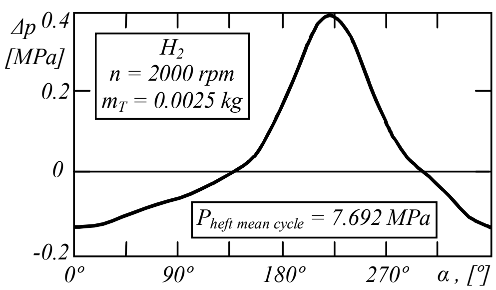

Fig. 4. Pressure difference between expansion and compression chambers.

In *figure 5* the variation of the mass flow for different rotation speeds of the Stirling engine is presented. The mass flow is more important for higher rotation speeds. It can be observed (*figure 6*) that the leaks caused by the displacer movements are smaller than the leaks caused by the pressure difference. The importance of the losses caused by the displacer diminishes when the rotation speed rises. The cyclic root mean square of the mass flow exchanged between expansion chamber and heater is $\dot{m}_{e\text{-}h\ RMS}$ = 0.0563 kg.s^{-1} at 2000 rpm. The RMS for the mass flow between compression chamber and cooler is $\dot{m}_{c\text{-}k\ RMS}$ = 0.0726 kg.s^{-1}.

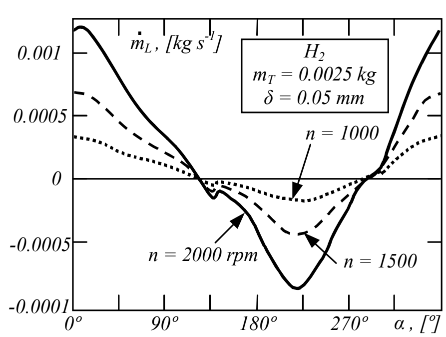

Fig. 5. Mass flow of gas leaking through cylinder-displacer gap.

On the indicator diagrams inside compression and expansion chambers presented in *figure 7* are emphasized graphically the work losses caused by the leaks of working agent through the cylinder-displacer gap. The recalculated (in order to take into account the effects of the heat transfer at finite temperature differences inside cooler and heater) theoretical adiabatic pressure (p_{heft}) and the pressures p_{cL} and p_{eL} for the compression and expansion chambers, adjusted with the influence of the leaks in conformity to the proposed model, were also represented.

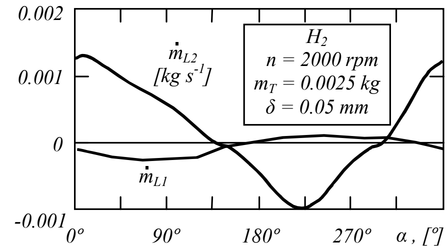

Fig. 6. Components of the mass flow through cylinder-displacer gap.

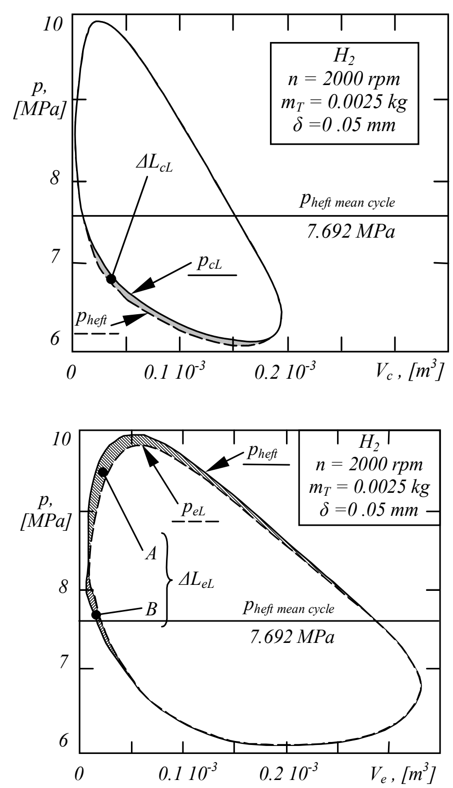

Fig. 7. Work losses caused by the leaks through cylinder-displacer gap.

The proposed physico-mathematical model takes into account the exchange of thermal energy accompanying the mass exchange. When the gas leaks from the expansion to the compression chamber, inside the compression chamber a mass of hotter agent is entering, i.e. the leaking agent is bringing thermal energy with it. After the mixing process the temperature inside the compression chamber rises, becoming greater than the recalculated adiabatic temperature, and the pressure inside this chamber rises too. A process equivalent to a thermal compression is obtained, so the work spent for the compression process diminishes and an apparent "saving" compared to the theoretical adiabatic case is obtained.

When the gas leaks inside the expansion chamber, the pressure and temperature inside the expansion chamber diminish (*figure 7*). The "lens" A represents the lost work inside the expansion chamber. On the indicator diagram inside the expansion chamber a surface B also appears, placed before the minimum volume of this chamber. The area B represents a positive work, generated by the diminished pressure at the end of the compression process. So, the compression process taking place inside the expansion chamber will spend less work, because the agent is cooler than in the theoretical adiabatic case.

The total area of the three "lenses" on *figure 7* is proportional with the work lost due to working agent leaks. The numerical calculations show that the analyzed S.m. produce 460.3 J of work cyclically. The work lost due to the leaks are 10.1 J, i.e. 2.2 % from the work produced by the analyzed S.m. at 2000 rpm, for a 0.05 mm gap between cylinder and displacer.

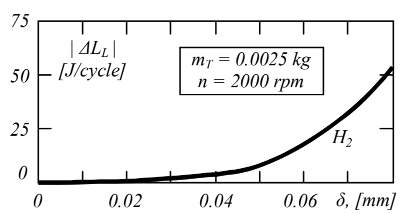

Fig. 8. The influence of the cylinder-displacer gap over the lost work.

The influence of the cylinder-displacer gap over the lost work is presented in *figure 8*. It can be observed that for smaller gaps the losses are very small. This observation shows the fine correlation with the equation of the leaked mass flow (Eq. 7). The factor δ^3 is the one that defines the importance of the losses.

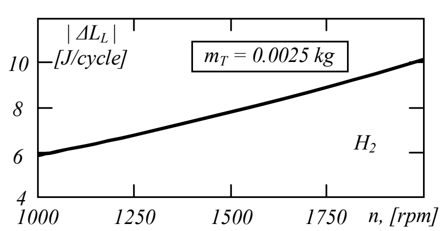

Fig. 9. The influence of the rotation speed over the lost work due to the leaks.

The work lost raises with the rotation speed, as it can be observed in *figure 9*.

4. CONCLUSIONS

The physico-mathematical model for evaluating the work lost due to the leaks through cylinder-displacer gap takes into account the gas leaks due to displacer movements and the gas leaks due to the pressure difference between compression and expansion chamber (pressure difference caused by the gas movement with friction through Stirling engine's heat exchangers).

As a consequence of the gas leaks through the gap between cylinder and displacer a reduction of the work produced by the motor occurs.

The representation of the work losses due to the leaks through cylinder-displacer gap as areas in the indicator diagrams inside compression and expansion chambers is very suggestive.

The reduction of the area of the indicator diagram inside the compression chamber shows that the leaks lead to a pressure rise inside this chamber during the compression stroke, process equivalent to a thermal compression. The reducing of the area of the indicator diagram inside the expansion chamber is caused by the pressure diminishing inside this chamber during the expansion stroke.

The work losses due the cylinder-displacer gap leaks rise with the rotation speed and with the gap dimension.

The model allows for the analysis of the influences that some constructive and functional factors have on the work losses caused by the gap between cylinder and displacer.

Nomenclature

d – displacer stem diameter [m]; D – displacer diameter [m]; e – distance between the axis of the crankshaft and the plane in which the yoke's pin moves [m]; F – force [N]; l – rod length [m]; L – work [J] or length [m]; $\dot{m}$ – mass flow rate [$kg.s^{-1}$]; n – rotation speed [rpm]; p – pressure [Pa]; r – crank radius [m]; V – volume [m^3]; $\dot{V}$ – volumetric flow rate [$m^3.s^{-1}$]; x – coordinate [m]; y – coordinate [m]; w – speed [$m.s^{-1}$]; α – crankshaft position angle [°]; δ – distance between cylinder and displacer [m]; Δ – difference; μ – dynamic viscosity [Pa.s]; τ – tangential unitary effort [$N.m^{-2}$]; c – compression; d – displacer; e – expansion; h – heater; heft – heat exchanged at finite difference of temperature; k – cooler; L – lost or leaked; m – minimum; M – Maximum; T – total; 1 – refer to the leaks through a fixed reference section (placed between compression and expansion chambers) due to the movement of the displacer only ($\Delta p = 0$ between the two sides of the displacer); 2 – refer to the leaks through a fixed reference section (placed between compression and expansion chambers) under the action of the pressure differences only (for an immobilized displacer)

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