OPTIMIZATION OF DIAMETER RATIO FOR ALPHA-TYPE STIRLING ENGINES

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Abstract: The problem of the constructive dimensioning and optimization of the Stirling machines is still insufficiently studied. The paper analyzes the alpha-type Stirling engine from the point of view of the constructive optimization, by choosing the optimum ratio between the diameters of the two cylinders in the machine composition. This theoretical analysis showed that, by analyzing the functioning of the engine with either an isothermal model or the adiabatic model without or with losses taken into consideration, an optimum value of the diameter ratio exists, for which the work produced by the engine takes a maximum value. The optimum diameter ratio is calculated for a numerical example.
Key words: alpha-type Stirling engine, diameter ratio optimization, adiabatic model

1. Introduction

The thermodynamic Stirling cycle [6] is composed of two isothermal processes (that take place at the minimum and maximum temperatures of the cycle, T_m and T_M) linked by two isochoric processes (that took place at the minimum and maximum volumes occupied by the gas inside the machine, V_m and V_M), as on Fig. 1.

Irrespective of the architecture and the motive drive type, all Stirling engines include in their functional unit construction (equivalent to an internal combustion monocylinder engine) the cylinder and the displacer, the cylinder and the power piston, the heater and the cooler heat exchangers and the heat regenerator, key components connected as shown on Fig. 2 [4].

In the case of using a crank mechanism, the indicator diagram of the engine looks like the one presented also in Fig. 1.

The optimization of the Stirling engine functioning is a problem of major interest. Several papers studying various aspects of the Stirling engine optimization (e.g. the optimization of the heat exchangers - cooler, heater and regenerator - for irreversible cycles) were published [1], [2], [3].

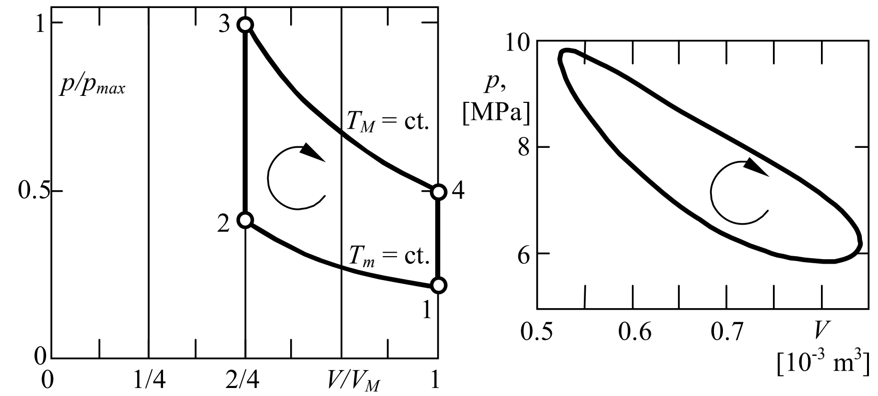

Fig. 1 - Direct Stirling thermodynamic cycle, for chosen compression ratio $V_M / V_m = 2$, and an indicator diagram, calculated for a numerical example

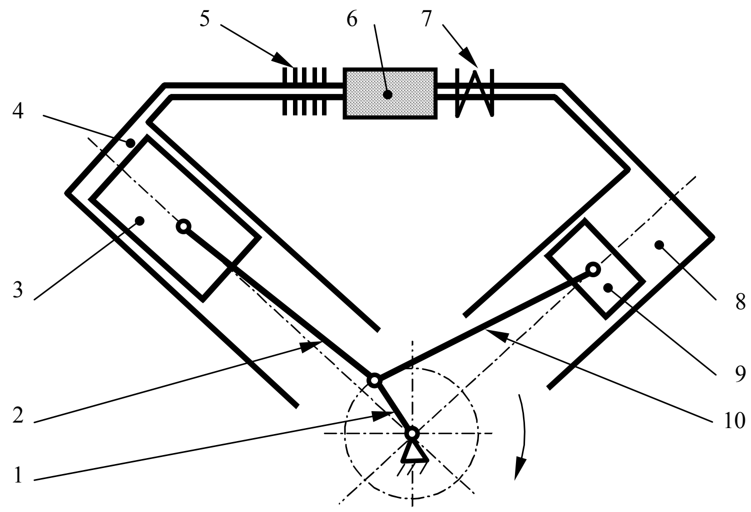

Fig. 2 - Alpha-type Stirling engine:
1 - crankshaft; 2 - displacer rod; 3 - displacer; 4 - expansion chamber; 5 - heater;
6 - regenerator; 7 - cooler; 8 - compression chamber; 9 - power piston; 10 - rod

2. Stirling Engine Performances Calculated with an Adiabatic Model

The adiabatic physico-mathematical model used for thermal performances calculation [7] is based on the following hypotheses: the working fluid is the ideal gas, all processes occur ideally, the temperature inside the heater is constant, the temperature inside the cooler is constant, the heat regenerator temperature is constant and equal to the logarithmic mean of cooler and heater temperatures, the momentary pressure is the same inside all engine chambers and the pistons move accordingly to known laws. The expansion and compression spaces do not exchange heat with the neighboring spaces, this particularity also giving the name of the model.

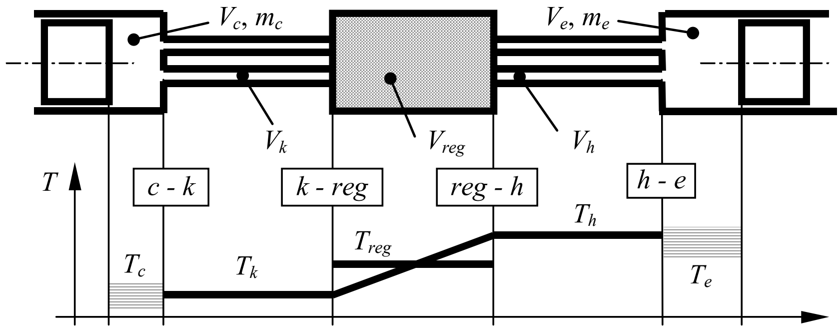

Fig. 3 - Schematic diagram of the alpha-type Stirling engine

The hypotheses concerning the temperatures inside the engine are shown as well in graphical manner on Fig. 3. On Fig. 3 letters V, T and m were used to mark the volumes, temperatures and masses while the subscripts e, c, k, reg, h refer to the expansion and the compression spaces, cooler, regenerator and heater (they all correspond to English initials, excepting k for cooler).

The model uses the differential equation of mass conservation for the working agent, the equation of state applied to the gas inside heat exchangers and the differential law of conservation of energy written for the adiabatic chambers.

According to the adopted hypotheses, inside the compression and expansion chambers the gas exchanges work with the surrounding environment (through piston movement) and enthalpy with the neighboring heat exchangers (cooler or heater). The internal energy of the gas inside the adiabatic chambers changes, as a consequence of mass and temperature variations. The heat exchanged by each of these two chambers is zero, conforming to the adiabatic hypothesis.

The adiabatic model produces five differential equations:

(1) $$dm_c = \frac{1}{R\,T_{c-k}}\left(\frac{V_c}{k}dp + p\,dV_c\right),$$

(2) $$dm_e = \frac{1}{R\,T_{h-e}}\left(\frac{V_e}{k}dp + p\,dV_e\right),$$

(3) $$dp = \frac{-k\,p\left(\dfrac{dV_c}{T_{c-k}} + \dfrac{dV_e}{T_{h-e}}\right)}{\dfrac{V_c}{T_{c-k}} + \dfrac{V_e}{T_{h-e}} + k\left(\dfrac{V_k}{T_k} + \dfrac{V_{reg}}{T_{reg}} + \dfrac{V_h}{T_h}\right)},$$

(4) $$dT_c = T_c\left(\frac{dp}{p} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c}\right),$$

(5) $$dT_e = T_e\left(\frac{dp}{p} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e}\right).$$

The composed subscripts *c-k* and *h-e* refer to the dimensions describing the separating sections between the compression and expansion chambers and their adjacent heat exchangers.

Eqs. (1), (2), (3), (4) and (5) form the system of differential equations of the adiabatic physico-mathematical model of the Stirling engine. The unknown functions are the pressure p, the masses m_c and m_e inside the compression and expansion chambers and the temperatures T_c and T_e in the same chambers. The system is nonlinear, because there are several terms in the differential equations that have an order higher than one. The system has variable coefficients and the conditional temperatures T_{c-k} and T_{h-e} of the agent that passes through the surfaces *c-k* and *h-e* depend on the sense of the gas flow. The conditional temperatures take the expressions:

(6)
$$T_{c-k} = T_c \text{ if } m_{c-k} > 0 \text{ (or } dm_c < 0\text{)};$$
$$T_{c-k} = T_k \text{ if } m_{c-k} < 0 \text{ (or } dm_c > 0\text{)};$$
$$T_{h-e} = T_h \text{ if } m_{h-e} > 0 \text{ (or } dm_e > 0\text{)};$$
$$T_{h-e} = T_e \text{ if } m_{h-e} < 0 \text{ (or } dm_e < 0\text{)}.$$

The system can be solved only by numerical integration. If the values of the unknown functions are adopted for a certain point in time, the problem is an initial value one and the numerical solution can be found with a Runge-Kutta method. The solution is obtained after several iterations, each of them using the previous one's results as initial values and thus getting closer to the result as the analysis goes on.

The useful effect of the Stirling engine is represented by the work L yielded per cycle, calculated with:

(7) $$L = \oint p(\alpha)\, d(V_t(\alpha)) = \int_0^{2\pi} p(\alpha)\left[\frac{d}{d\alpha}(V_c(\alpha))\right]d\alpha + \int_0^{2\pi} p(\alpha)\left[\frac{d}{d\alpha}(V_e(\alpha))\right]d\alpha\,,$$

where

- V_t is the total volume occupied by the working agent inside the machine and
- α is the crankshaft position angle.

The adiabatic efficiency of the Stirling engine can be calculated with the following general relation:

(8) $$\eta_{t\,ad} = \frac{L}{Q_h} = 1 + \frac{Q_k}{Q_h} = 1 - \frac{|Q_k|}{Q_h} = 1 + \frac{L_c}{L_e} = 1 - \frac{|L_c|}{L_e},$$

where $Q_k = L_c$ and $Q_h = L_e$ are the heats cyclically exchanged inside the cooler and the heater and the works exchanged inside the compression and expansion chambers and L is the work cyclically performed by the engine.

A closer picture of the real functioning of the engine can be obtained by taking into consideration various kinds of losses. Losses can be calculated through decoupled models. These models suppose that each loss is produced by an independent cause.

Results provided by the adiabatic model in this paper were corrected by taking into account two kinds of losses, determined with models presented by Urieli [7]. The decreasing of the work yielded inside the expanded chamber as consequence of pumping losses inside heat exchangers, as well as the decreasing of the yielded work caused by the heat exchange inside non-ideal cooler and heater were considered.

3. Optimization of the Diameter Ratio

The chosen optimization criteria were the maximum work yielded and the maximum efficiency of the alpha-type Stirling engine.

The paper analyzes a Stirling engine for which we know:

- the working agent (constant R);
- the temperatures T_k and T_h of the heat sources;
- the dimensions of the motion mechanism (identical for both displacer and power piston, that as a consequence share the same stroke);
- the mass of the working agent inside the machine (chosen).

In order to optimize the diameter ratio the maximum of the total volume occupied by the gas inside the machine was maintained constant. The volumes of the heat exchangers are constant and were calculated as ratios of the maximum volume of the expansion chamber. The maximum volume of the expansion chamber used for calculating the heat exchangers volumes was determined for the case when both pistons share the same diameter.

We use the diameter ratio:

(9) $$\varepsilon_d = \frac{d_d}{d_p},$$

where d_d is the diameter of the displacer piston and d_p is the diameter of the power piston.

At the optimization analysis the total volume occupied by the working agent inside the machine was considered constant. This condition means that, for engines working with the same mass of agent, the pressure inside the machine at ambient temperature is the same.

The numerical modeling of the adiabatic machine was used. All machine performances must be represented as functions of ε_d [5]. The maximum values of these functions were considered as optimums. Because the performances depend on the heat exchanger volumes also, the optimum values depend on these volumes too.

The isothermal physico-mathematical model of the Stirling engine was also used for calculation. The same hypotheses were used, except that the temperatures inside compression and expansion chambers were considered to be constant.

4. Numerical Example

An alpha-type Stirling engine (Fig. 1) featuring the following dimensions was chosen: $d_p = 0.073$ m; $d_d = 0.073$ m; $r = 0.0365$ m; $l_1 = l_2 = 0.15$ m; $f_{TDCp} = f_{BDCd} = 0.001$ m; $V_h = V_k = 0.1\ V_{e\ max}$; $V_{reg} = 1.2\ V_{e\ max}$, where $V_{e\ max}$ = maximum volume of the expansion chamber, d = cylinder diameter, r = crankshaft radius, l = rod length, f = dead space length. $V_{e\,max}$ determined for the chosen Stirling engine geometry is a constant value for the optimization calculations. *TDC* and *BDC* stand for top and bottom dead center.

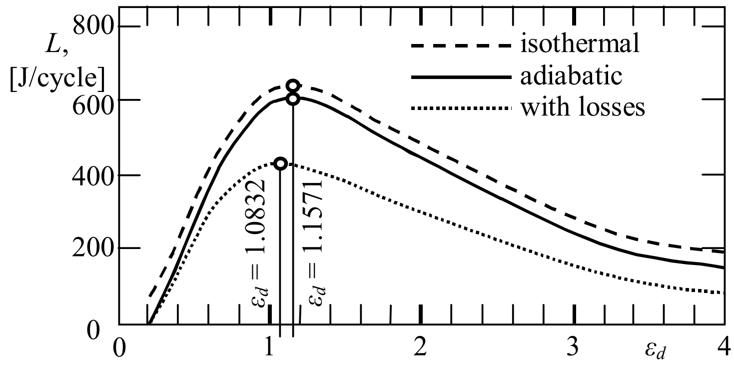

Fig. 4 – Variation of the work on ε_d ratio

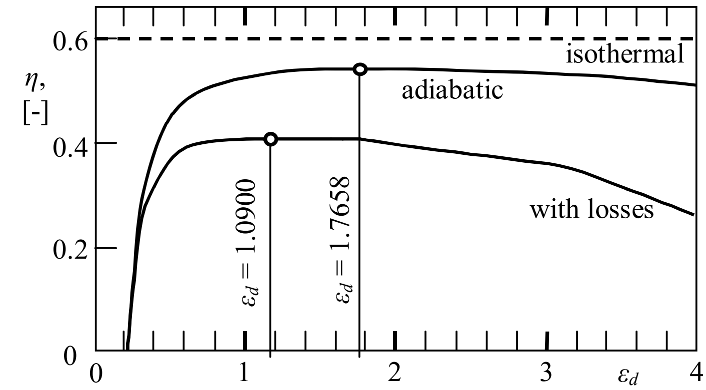

Fig. 5 – Variation of the adiabatic thermal efficiency η on ε_d ratio

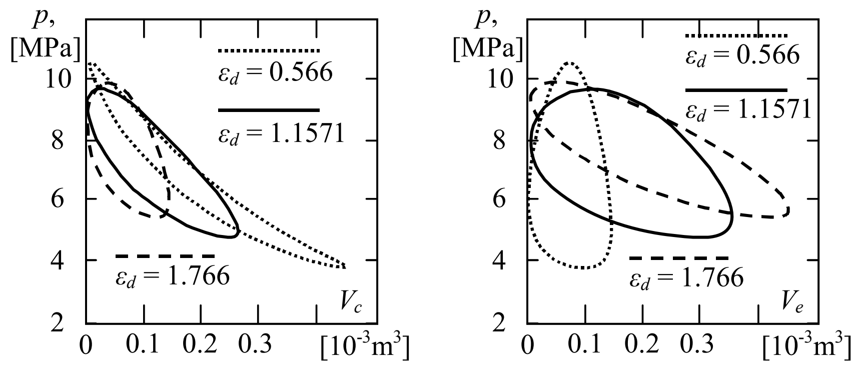

Fig. 6 – Indicator diagrams inside compression and expansion chambers for different diameter ration ε_d

The machine works with a total mass of hydrogen m = 0.0025 kg (R_{H2} = 4121 J/(kg K)) between temperatures T_h = 773 K and T_k = 310 K. The main dimensions of the engine were chosen from a project that aimed to build a Stirling engine with similar characteristics with the internal combustion engine of the Romanian-made Dacia 1300 vehicle.

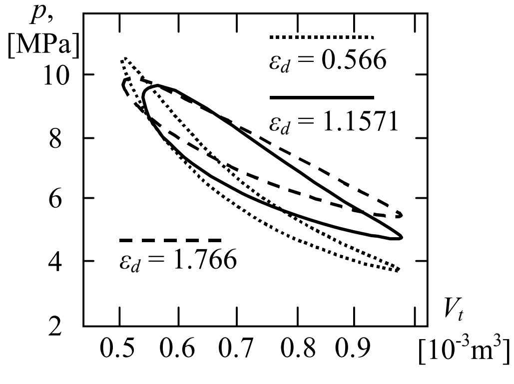

Fig. 7 – Indicator diagram inside the whole engine for different ε_d values

The following dimensions were adopted in order to calculate the work losses: cooler length 0.08 m, regenerator length 0.07 m, heater length 0.08 m, inside diameter of the cooler and heater pipes 0.002 m. The wire mesh of the regenerator have 400 wires of 0.0254 10^{-3} m in diameter per inch. The pressure drop for regenerator matrix is calculated based on experimental data taken from [8]. The revolution speed was considered 1000 rpm.

The most important numerical results obtained by optimization of the Stirling machine are presented in Fig. 4 and Fig. 5. The variations of the terms L and η as functions of ε_d ratio show that both functions reach their maximum values.

The influence of ε_d ratio over the functioning of the Stirling engine was stressed using the indicator diagrams $p(V)$. In Fig. 6 and Fig. 7 the indicator diagrams inside compression and expansion chambers (c and e) and inside the whole engine (t) for the optimum (adiabatic) value ε_d and for another two values of diameter ratio were presented. Values 0.566 and 1.766 were obtained by imposing d_p = 0.05 m, respectively d_d = 0.05 m.

5. Conclusions

The work cyclically performed by the alpha-type Stirling engine and the adiabatic thermal efficiency reach their maximum values for diameter ratios above 1; so, the diameter of the displacer piston must be greater than the diameter of the power piston.

The maximum value of the work depends on the physico-mathematical model used for numerical simulation (e.g. the isothermal model and the adiabatic model without or with losses taken into consideration).

Values of diameter ratios for which the work performed and the thermal efficiency are null can be calculated.

The thermal efficiency of the alpha-type Stirling engine is close to the optimum value for a large range of diameter ratios.

The difference between the maximum and minimum cycle pressure diminishes for greater displacer piston diameters.

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